Physics 1CH, Spring 2022

Sample Concept Questions I + Answers

For each question, you need to explain your answer.

1) A plane electromagnetic wave with angular frequency $\omega = 3.9 \times 10^{15}$ rad/s is traveling through a transparent medium with index of refraction $n = 1.2$. What is the shortest distance between two points on the wave that are separated by a phase difference of 45^o?

Phase velocity $v = c/n = 2.5 \times 10^8$ *m/s*

Frequency $\nu = \omega/2\pi = 6.2 \times 10^{14}$ *Hz*

Wavelength $\lambda = v/\nu = 4.0 \times 10^{-7}$*m = 400 nm*

Phase difference of 45^o *is 45/360 of one cycle or one wavelength*

Shortest distance $\Delta x = (45/360)$ ***400 nm = 50 nm***

2) Can electromagnetic waves travel through a perfect vacuum? What is traveling when the wave moves? Can sound waves travel through a perfect vacuum? Explain your answer.

- *Yes, EM waves can travel through a perfect vacuum. No medium is needed.*

- *When the wave moves, it is the E and B fields that are traveling. Also energy is traveling.*

- *Sound waves are compressional and require a medium. Sound cannot travel in a vacuum.*

[We've not covered sound yet, so don't worry too much about that part].

3) The index of refraction of the human cornea is about 1.38. If you can see clearly in air, why can't you see clearly underwater? Why do goggles help? Drawing a picture will help and you can consider an object at infinity to make things easier.

Your eyes are suited to make good images of objects in air, with $n_1 = 1.0$ *and* $n_2 = 1.38$. *However, in water* $n_1 = 1.33$. *Since focal strength is proportional to* $(n_2\text{-}n_1)$, *your eyes have much less focal strength in water (i.e. a long focal length). Goggles work by adding a layer of air before your eyes, which then can focus properly.*

4) The travel time of signals from satellites to receiving stations on Earth varies with the frequency of the signal. Explain why this is so.

This is an example of ***dispersion,*** *where the index of refraction, n, depends on wavelength (or frequency). From the statement of the problem, we know that the time travel varies with frequency. This means that the speed of light varies with frequency. Since the speed is c/n, this means that n varies with frequency and hence the atmosphere is a dispersive medium.*

5) Stars twinkle, but planets do not. Explain why this is. (A figure might be useful).

There are two things going on here. First, twinkling is caused by light interacting with molecules in the atmosphere through both scattering and diffraction. Second, stars are far away and appear as point objects to us on Earth – thus, there is one unique path for light to travel (or you can say that the light from a star is a very good plane wave) and when the bean of light is interrupted, the star twinkles. Planets are much closer to us and hence their light reaches us from a disk, not a point. (This is easily verified with even a modest-power binoculars). Hence the atmospheric effects average out and we see a steady brightness from a planet. (Of course, a planet's brightness is reduced relative to no atmosphere).

6) True or False? *When sunlight reflects off the top surface of a swimming pool, the reflected wave incurs a phase shift of $\pi/2$ radians*. Explain your answer.

Sunlight reflecting off water is an external reflection (from lower index to a higher index). Such a reflection incurs a phase shift of π radians.

7) What is the minimum size plane mirror required for a 1.8m tall person to be able to see a full length image of themselves? Explain your answer and provide a figure showing the size and location of the plane mirror relative to the person. Assume that the vertical distance from the person's eyes to the top of their head is 10cm.

***The mirror needs to be at least 90cm in size, with its lower edge 85cm off the ground**. To think about this problem, you need to realize that the person needs to see his/her/their feet and the top of his/her/their head. The person's eyes are 170cm off the ground, the feet are at 0cm and the top of the head at 180cm. Thus, if the mirror starts at 85cm off the ground, light from the feet will just make it into their eyes. Similarly, with the top of the mirror at 175cm, light from the top of the head will also just make it into the eyes.*

8) True or False? *A converging lens cannot form a real image from a virtual object*. Explain your answer (an equation or a figure might be useful).

***False.** Example, for o = -20cm and f =+20cm, the image I = +10cm. (What about if the lens were a diverging one?).*

9) You are outside viewing a rainbow that forms in a large semi-circular arc in the sky. Does the top of the arc appear red or violet to you? Or does it depend on your orientation to the rainbow? Explain your answer (with a figure, if you like).

***The top of the arc would be red.** Rainbows form via refraction of (white) sunlight by water droplets. Water is a dispersive medium. For normal dispersion of optical light, red light has the smallest index of refraction and blue light the largest. The top of the arc = least curvature, least amount of refraction, and smallest n.*

10) Why has a lens two focal points and a mirror only one?

Light can pass through a lens from either direction and thus there are two focal points. Light can hit a mirror from only one direction and hence only one focal point.

11) Give two reasons why circular water waves in a pond decrease in amplitude as they travel away from the source.

1) The amplitude decreases as the wave spreads outward to conserve energy. 2) There is dissipation in the water so that the total amount of energy in the wave decreases (i.e. energy is lost from dissipative forces).

12) A watchmaker uses diverging eyeglasses for driving, no glasses for reading, and converging glasses in occupational work. Is the watchmaker nearsighted for farsighted? Explain.

***Nearsighted**. A person who is nearsighted cannot see objects well at a distance and needs a diverging lens to provide correction. For reading the object is relatively close and nearsighted people can read without glasses. For watchmaking work, the object is very close and thus a magnifying class (converging lens) is needed.*

13) True or False? *In an electromagnetic wave in a vacuum, the electric and magnetic fields are in phase and the electric and magnetic field strengths are equal in magnitude*. Explain your answer.

*Two parts: a) E and B are in phase; this is **true** and follows from Maxwell's equations.*

*b) $E = B$; this is **false**. $E = cB$.*

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Sample Problems I + Answers

Numerical answers can be given to two significant figures.

1) Would you be willing to pay 25 cents for an object valued by a mathematician at \$ i^i ?
Explain your reasoning.

No. *It's equal to* $e^{(-\pi/2)}$ *dollars or 21 cents. Use the Euler identity to see that* $\exp^{(i\pi/2)}=i$ *and then take that to the i power. Conclusion: don't trust mathematicians.*

2) A plane electromagnetic wave is traveling in free space along the +*z* direction. The magnetic field associated with the wave is in the +*x* direction and can be expressed as:

$$\vec{B} = (2 \text{ x } 10^{-7}) \cos(kz - \omega t)\ \hat{\mathrm{i}}\ \text{T},$$

$$\text{with } \omega = 2\pi \text{ x } 10^{8} \text{ Hz}$$

(a) Write down an expression for the electric field associated with the wave. Remember to indicate the direction of the field and its units.

$\vec{S} = \vec{E} \text{ x } \vec{B}$ *gives the direction of the wave velocity, which is along* k. *With* $\vec{B}$ *along* $\hat{i}$, *this means that* $\vec{E}$ *is along* $-j$. *In magnitude* $E = cB$, *and hence:*

$$\vec{E} = -60 \cos(kz - wt)\ \hat{\mathrm{j}}\ \text{V/m}.$$

(b) What is the wavelength of the wave and which waveband is this EM wave in?

Calculating gives $\nu = 10^8$ *Hz and hence* $\lambda = 3$*m. This is in the radio band.*

(c) What is the average power passing through a square of side 50 cm positioned in the *xy* plane?

Calculating the average Poynting vector gives $\langle S \rangle = (15/\pi)$ W/m^2. *We get the full Poynting flux since the square is perpendicular to the wave velocity. The average power is 1.2 W.*

[Note to class: this problem is a bit too easy].

3) A coin lies at the bottom of a swimming pool that has depth *d* and index of refraction *n*.

(a) Determine a simple expression for the apparent depth of the coin (d_{app}) as a function of *d* and *n*, for light rays that are close to the normal. You can assume that the index of refraction of air is 1.0. Ideally, your expression should not contain any angles or trigonometric functions.

Evaluate d_{app} for *d* = 2.40m and *n* = 4/3. (A picture of the geometry would be useful here).

Consider light leaving the coin, going up through the pool and hitting the surface at an incident angle θ_1 *(with respect to the normal). Then it refracts upon existing and exits at an angle* θ_2 *(with respect to the normal). Let x be the lateral distance between the point where the light exits* *the pool and the point that is directly above the coin. Then $\tan(\theta_1)=x/d$ and $\tan(\theta_2)=x/d_{app}$. Using the small angle approximately $\sin(\theta) \sim \tan(\theta) \sim \theta$ and Snell's law, you get $d_{app} = d/n$.*

Using the numbers given, $d_{app} = 1.80m$.

(b) Is the expression for d_{app} derived in (a) valid for all viewing angles of the coin at the bottom? Evaluate d_{app} when viewing the coin at an angle of 50^o with respect to the normal of the top surface of the pool. Use the same numbers for *d* and *n* as in (a). What is the minimum value for d_{app}? Explain.

No, the expression we derived in (a) is not valid for all viewing angles because we used the small angle approximation. Using $\theta_2 = 50^o$ and plugging in to the formulae with no approximations, you get $d_{app} = 1.42m$. The minimum value occurs when you have total internal reflection. This happens for $\theta_1 = 48.6^o$, $\theta_2 = 90^o$, and $d_{app} = 0m$.

4) Two plane mirrors are positioned with their reflecting surfaces parallel to one another. If a small object is placed one-third of the way between the two mirrors, what are the positions of the five images that lie closest to the object?

Let 3L = mirror spacing. The object is L distance away from the right mirror and 2L distance away from the left mirror. Then use the standard rules about plane mirrors to determine the first images formed of the object in the right and left mirrors. Those images serves as objects for the next reflection, and so forth. The five closest (virtual) images are at:

1: L behind right mirror, 2L from object

2: 2L behind left mirror, 4L from object

3: 5L behind right mirror, 6L from object

4: 4L behind left mirror, 6L from object

5: 7L behind right mirror, 8L from object

5) An object and a viewing screen are separated by a fixed distance of 100 cm. At what location (or locations) between the object and the screen should a lens of focal length $f = +24$ cm be placed to produce a sharp image on the screen? Draw a diagram illustrating the geometry for the location (or locations).

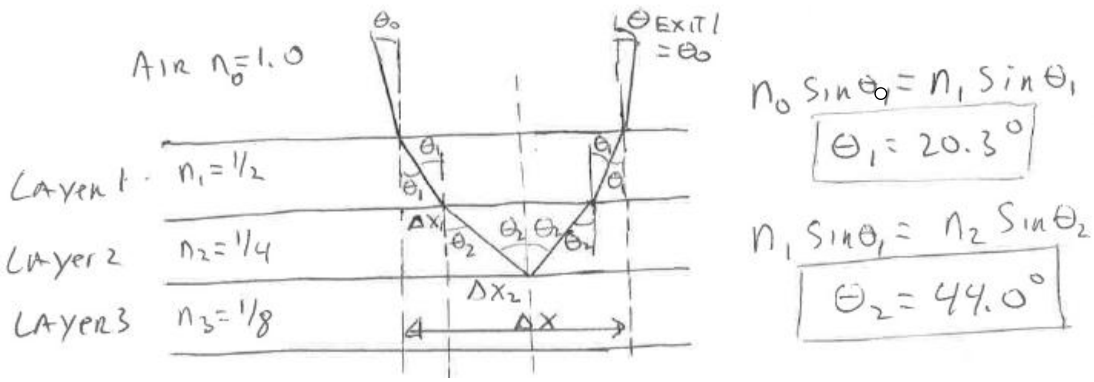
From the problem, we know that $o + i = 100$ cm. Using this relation and the lens equation, we get a quadratic equation for o or i. The solutions are:
$o = 40$ cm, $i = 60$ cm, $M = -1.5$; real inverted image.
$o = 60$ cm, $i = 40$ cm, $M = -2/3$; real inverted image.

6) Deep in the Amazon rainforest is a lake containing a mysterious transparent liquid. The local villagers know that the top 1m layer has an index of refraction n=1/2, the next 1m layer has an index of refraction n=1/4, then n=1.8, and so forth. The lake is very deep, and the index of refraction approaches 0 near the bottom. Standing on a ledge above the lake, you take a laser pointer from your pocket and direct it into the lake. Assume that the laser beam hits the planar

surface of the lake at an incident angle (measured with respect to the normal) of θ_0, where $0^o<\theta_0<90^o$.

a) What happens to the laser beam for $\theta_0=10^o$? Support your answer with a calculation that determines the path of the beam in the lake. Illustrate the path of the beam with a figure.

See figure below.



Between each layer, the critical angle is $Sin(\theta_c) = ½$ *or* $\theta_c = 30^o$. *So, when the light hits Layer 3 it will totally internally reflect. The light then refracts back out of the liquid with the same set of angles as it came in with. Hence* $\theta_{exit} = \theta_0$. *To get the lateral displacement* Δx, *we have* $Tan(\theta_1)=\Delta x_1/1m$ *or* $\Delta x_1=0.37m$. *Similarly* $\Delta x_2=0.97m$ *and the overall* $\Delta x=2(\Delta x_1+\Delta x_2)=2.68m$.

b) Now discuss what happens to the laser beam for all values of θ_0. Support your answer with a calculation and discuss the path of the beam for various ranges of θ_0.

The general expression for the refracted angle in the water, θ_m, *is:*

$$Sin(\theta_m) = 2^m\, Sin(\theta_0)\ ,\ m = 1,2,3\ldots \text{ (whatever is needed)}$$

So, for any value of $\theta_0 \neq 0$ *and large enough m, the RHS of the above equation is greater than 1 and the light will total internally reflect at the m'th surface. By symmetry* $\theta_{exit} = \theta_0$ *for all* θ_0. *To calculate the angle ranges, we note that T.I.R. occurs when* $2^m\,(Sin(\theta_0)_c) = 1.0$. *This gives us:*

m	$(\theta_0)_c$
1	30.0^o
2	14.5^o
3	7.18^o
4	3.58^o

So for $\theta_0 > 30^o$, *light reflects at top of 1st layer; for* $14.5^o < \theta_0 < 30^o$, *light reflects at top of 2nd layer, and so forth.*

7) On an optical bench is a lens with a focal length, $f_1 = +10$ cm, and a mirror of unknown type (convex, concave or planar) and unknown focal length. An upright object, O_1, is located 15 cm to the left of the lens. The mirror is located 50 cm to the right of the lens. The image formed by the mirror, I_2, is inverted and is of the same size when compared to the original object O_1.

a) Determine the position of the image formed by the lens, I_1. Is it real/virtual, upright/inverted, and what is its magnification?

$o_1 = +15.0cm$, $f_1 = +10.0cm$ → $i_1 = +30.0cm$, $M_1 = -i_1/o_1 = \mathbf{-2.0}$

Image is real, inverted and twice the size of the object.

b) Given what you know about the second image, determine the parameters of the mirror: is it convex, concave, or planar, and what is its radius of curvature, R? Determine the position of the image formed by the mirror, I_2. Is it real/virtual?

$o_2 = 50.0cm – 30.0cm = \mathbf{20.0cm}$

$M_{tot} = M_1 M_2 = -1.0$ → $\mathbf{M_2 = 0.5}$

$M_2 = -i_2 / o_2 = 0.5$ → $\mathbf{i_2 = -10.0cm}$

Using the mirror equation with o_2 and i_2, gives $f_2 = -20.0cm$; the mirror is ***convex***

$f_2 = -R/2$ → $\mathbf{R = +40.0cm}$

Since $i_2 < 0$, the I_2 image is ***virtual***

c) On the horizontal axis below, draw the lens and the mirror. Then provide a to-scale ray-trace for a least two rays from the initial object to the first image and then at least two rays from the second object to the second image. Indicate the positions of O_1, I_1, O_2, I_2, F_1, F_2, and C, where F_1 identifies the focal points of the lens, F_2 is the focal point of the mirror, and C is the center of the mirror. (You do not have to ray-trace back through the lens to form a third image).