Physics 1CH, Spring 2022

Sample Concept Questions III + Answers

For each question, you need to explain your answer.

1) True or False? A circular disk gives the same diffraction pattern as a hole of the same size.

True. *By Babinet's Principle, the complement of an aperture has the same diffraction pattern as the aperture itself*

2) For each of the following pairs of sources, are they coherent or not coherent?

i) Two candles ***Incoherent.*** *Separate sources*.

ii) One candle and its image in a plane mirror ***Coherent.*** *Same source.*

iii) Two pinholes uniformly illuminated by the Sun ***Incoherent.*** *Two separate sources.*

iv) The two headlights of a car ***Incoherent.*** *Separate sources.*

3) Astronaut Rita is in a low-Earth orbit (altitude = 300 km) and passes over her home town. Can she use her unaided eyes to identify her (single-family) house? Consider visible light and use an estimate for the size of her house of 20 m and a pupil diameter of 4 mm. If she cannot resolve her house, explain why not. (Make sure to state the physics principle(s) involved and to show your work).

The physics principle is diffraction, which sets a limit on the angular resolution.
Angular size of the house is $\theta_H \sim 20m/300km = 6.7 \times 10^{-5}$.
Angular resolution (diffraction limit) $\Delta\theta = 1.22\ \lambda/D = 1.22\ (500nm)/4.0mm = 1.5 \times 10^{-4}$.
The angular size of the house is somewhat smaller than the angular resolution, so she cannot quite resolve her house.

4) True or False? Two events that are simultaneous in one reference frame are not simultaneous in another frame moving relative to the first.

True. *Events that are simultaneous in one frame will not be simultaneous in another frame moving relative to the first.*

5) In considering relativity, it is often said that "moving clocks run slow." Does this statement refer to the fact that motion alters the way a clock works? If no, what does it refer to?

No, motion does not affect the way that clocks work. What motion affects is the different progression of time in two clocks that have relative velocities with respect to one another. Suppose you have clock A at rest in frame S and clock A' at rest in S', with S' moving at constant velocity relative to S. Clock A' will measure the proper time of events in S', while A will measure a dilated time (longer time) of those same events in S. Thus, the moving clock as observed in S will appear to be running slow and that is the origin of the popular expression.

6) True or False? The Michelson-Morley experiment showed the light did not obey Galilean relativity. Explain your answer.

True. *If light obeyed Galilean relativity, the experiment would have seen a fringe shift because the velocity of the apparatus would add to (of subtract from) the speed of light moving through the ether. However, no fringe shift was observed, indicating that the speed of light was the same in all frames and violating Galilean relativity.*

7) True or False? In Compton scattering, the kinetic energy gained by the electron depends on the scattering angle of the photon, θ, but not on the wavelength of the incident light.

False. *The wavelength shift ($\Delta\lambda$) depends only on the scatting angle θ. But the KE of the electron can be determined from the change in KE of the photon:*

$K_e = E_\gamma - E_\gamma' = (\Delta\lambda / \lambda\lambda')hc$ *and this does depend on the wavelength of the incident light.*

8) "The mass of the electron is 0.511 MeV". Exactly what does this statement mean?

It means that the rest mass of the electron is 0.511 MeV/c^2. The statement is a simplification.

9) A particle with zero mass (e.g. a neutrino possibly) can transport momentum. However, by the expression p = γmv, the momentum is directly proportional to the mass and therefore should be zero if the mass is zero. Explain this contradiction.

The expression $p = \gamma mv$ simply does not hold for a massless particle. For such a particle the momentum is $p = E/c$. There is no contradiction.

10) Is the mass of a stable, composite particle (e.g. a gold nucleus) greater than, less than, or equal to the sum of the masses of its constituents? Explain your answer.

It will take energy to split up a stable composite particle to its constituents. Thus, such a particle has a lower rest energy than the sum of the rest energies of its constituents. Hence it also has a lower rest mass. The difference between the sum of the constituent rest energies and the rest energy of the composite particle is called the <u>binding energy</u>.

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Sample Problems III + Answers

Numerical answers can be given to two significant figures.

1) The distance between the first and fifth minima of a single-slit diffraction pattern is 0.350 mm with the screen 41.3 cm from the slit, using light having a wavelength of 546 nm.

(a) Calculate the diffraction angle θ of the first minimum.

$\theta_1 = 8.76 \times 10^{-5}$.

(b) Calculate the slit width.

b = 2.58 mm.

2) Light of wavelength is incident normally on two slits that have a slit separation of *a* and a slit width of *b*.

(a) For *a* = 2*b*, how many interference fringes lie in the central diffraction envelope?

The interference fringes for m = ±2 fall on top of the diffraction minima on either side of the central maximum. Hence there are 3 complete interference fringes in the central diffraction envelope, corresponding to m = -1, 0, 1.

(b) For *a* = *b*, the two slits coalesce into a single slit. Show that the irradiance formula for the double slit (with diffraction) reduces to what you expect for a single slit.

With interference and diffraction together, we have:

$I = I_{max} \cos^2(\alpha) \sin^2(\beta)/\beta^2$ *where*

$I_{max} = 4I_0$ *with two slits and*

I_0 *= the irradiance from a single slit*

$\alpha = ka \sin(\theta)/2$

$\beta = kb \sin(\theta)/2$

When a = b, a = b and we get:

$I = I_{max} \cos^2(\beta) \sin^2(\beta)/\beta^2$

We can simplify this using the trig identity sin(2x) = 2 sin(x) cos(x), to get:

$I = I_{max} \sin^2(2\beta) / (2\beta)^2$

This is clearly the diffraction pattern for a single slit of width 2b.

3) Compton used photons of wavelength 0.0711 nm in his scattering experiment.

(a) What is the energy of these photons?

(b) What is the wavelength and energy of a photon back-scattered (i.e. $\theta = 180^o$)?

(c) What is the kinetic energy of the electron scattered as in part (b)?

(a) $E_\gamma = hc / \lambda = 17.47$ *keV.*

(b) $\lambda' - \lambda = (h / m_e c) (1 - \cos(\theta)) = 2 h / m_e c = 0.00486$ *nm,* $\lambda' = 0.0760$ *nm,* $E_\gamma' = 16.35$ *keV.*

(c) $E_e' = 17.47$ *keV – 16.35 keV = 1.1v2 keV.*

4) A spaceship, at rest in a certain reference frame S, is given a speed increment of 0.500c. It is then given a further 0.500c increment in this new frame, and this process is continued until its speed with respect to its original frame S exceeds 0.999c. How many increments does it require?

Successively apply the velocity transformation:

$$u_x = \frac{u_x' + V}{1 + u_x' V / c^2}$$

where u_x *is the speed in S,* u'_x *is the speed in* S' *and V = 0.500c. The successive values of* u_x *are 0.500c, 0.800c, 0.929c, 0.976c, 0.992c, 0.997c and 0.999c. So, seven increments are needed.*

5) Find the speed parameter (β) of a particle that takes 2 years longer than light to travel a distance of 6.0 light-years.

Since it takes light 6 years to travel 6 light-years, the particle takes 8 years. This means it has a β *factor of* $\beta = 6/8 = 0.75$*.*

6) A particle with a rest mass of 1 MeV/c^2 and a kinetic energy of 2 MeV collides with a stationary particle of rest mass 2 MeV/c^2. After the collision, the particles stick together.

a) Determine the speed of the moving particle before the collision and the initial total momentum of the system.

Express your answers in appropriate units (i.e. speed in units of the speed of light and momentum in units of MeV/c).

b) Determine the speed of the particles and the rest mass of the system after the collision. Again, express your answers in appropriate units.

We write down the expression for the initial energy E and final energy E' and similarly the expression for the initial momentum p and the final momentum p' (we can use p as a scalar since the collision is in 1D). Then we do these steps:

- *We calculate* γ, β *and v for* m_1*, given its energy.* $\gamma = 3.0$, $\beta = 0.943$, $v_1 = 0.943c$.
- *We determine* $p = p_1 = \gamma m_1 v_1 = 2.83$ *MeV/c and set it equal to p' (cons. of momentum).*
- *Use E' = E (cons of energy); hence with p', we determine M (final mass) = 4.12 MeV.*
- *Since* $E' = \gamma' Mc^2$*, we can determine* $\gamma' = 1.21$, $\beta' = 0.567$ *and* $v' = 0.563c$.

7) The charged pion particle (π) has a rest mass of 273 m_e, where m_e is the rest mass of the electron. This problem consists of two separate, and unrelated, parts.

a) Suppose the pions in a beam all have a kinetic energy of 500 MeV. If the mean proper lifetime for the pion is 26 ns, how long, on average, will the pions travel before decaying?

The pion lifetime (proper time) is $T_o = 2.6 \times 10^{-8}$ s. Given its kinetic energy and rest mass, the pion has $\gamma = 4.584$ and $\beta = 0.9759$. The mean distance it travels is $<d> = vt$, but $v = \beta c$ and $t = \gamma T_o$ (i.e. the dilated time). Hence $<d> = \gamma \beta c T_o = 34.9$ m.

b) Now consider the decay of a pion at rest into a muon and a neutrino. The muon (μ) has a rest mass of 207 m_e. For this problem, you can assume that the neutrino (ν) has zero mass and so, like the photon, it moves at the speed of light and its energy is all kinetic. Determine the kinetic energies of the muon and neutrino.

[Hint: the math will be easier if you try to solve for the energy of the neutrino first, i.e. by substituting for the energy of the muon. The problem can also be solved with four-vectors, but they are not required].

Naturally, you need to use conservation of energy and momentum. You have the following pieces of information:

(1) $E_\pi = m_\pi c^2$ (initial energy)
(2) $E_\nu = P_\nu c$ (since $m_\nu = 0$)
(3) $E_\mu^2 = (P_\mu c)^2 + (m_\mu c^2)^2$ ($= P_\nu^2 c^2 + m_\mu^2 c^4$, using (5) below)

and you also have conservation of energy and conservation of momentum

(4) $E_\pi = E_\mu + E_\nu$

(5) $P = 0$ (initial, in the π rest frame), hence $P' = 0$ (final) and hence $P_\mu = P_\nu$

Using (1),(2),(4) to solve for E_μ, squaring that and setting equal to (3) gives $P_\mu = 29.8$ MeV/c. This means that $E_\nu = 29.8$ MeV and obviously its energy is all kinetic, $K_\nu = 29.8$ MeV.

Knowing P_μ and using (3), we solve for $E_\mu = 109.8$ MeV and hence $K_\mu = 4.1$ MeV.

In the review session, we can show you the four-vector approach. It's faster!