Physics 1BH Winter 2022 Midterm #1

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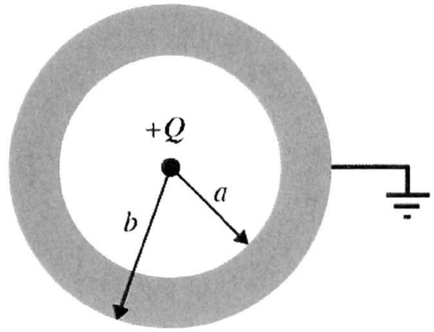
Instructions

- This exam is 50 minutes long.
- This exam is closed-notes, closed-book. No phones or calculators allowed.
- The standard equations sheet is allowed.
- You will independently be given points for your reasoning, your mathematical work, and for correctness of answers, so make sure to show your reasoning and work even if you're not certain you can compute the correct answer. Try to convey your solution strategy even if you can't execute it in time.
- If on a given problem you need some space to do scratch work, do that scratch work elsewhere. Only the work that is included on this test packet in the boxed space provided will be graded.
- **Put boxes around any final numerical or symbolic answers.**
- **DO NOT BOX MULTIPLE "FINAL" ANSWERS. If you do not want an answer graded, cross it out COMPLETELY with an 'X.'** Providing multiple "final" answers will result in a zero for that question.

Problem 1	10 pts
Problem 2	30 pts
Problem 3	30 pts
Problem 4	30 pts

1. **(10 points)** A point charge $+Q$ is located within a spherical conducting shell that is grounded (i.e. held at constant potential $\phi = 0$). Assume charges are in electrostatic equilibrium (no steady currents).



The conceptual questions below **do not** require explicit calculations. But please provide **brief explanations in words**.

a. What is the electric field within the material of the conducting shell ($a < r < b$)? Why?
b. What is the electric field outside of the conducting shell ($r > b$)? Why?
c. What is the total charge (if any) on the inner surface of the conducting shell ($r = a$)? On the outer shell ($r = b$)? Why?

a) The electric field within the material of the spherical conducting shell is zero. The electric field inside a conductor is always zero because the electrons will move around to reach equilibrium (charges can move around freely in conductors). Conductors are equipotentials.

b) 

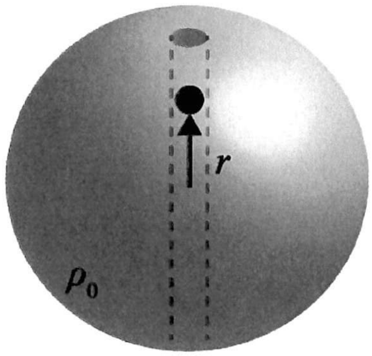
$$E_{out} \cdot \cancel{A} = \frac{Q_{enc}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0} \rightarrow E_{out} = \frac{\sigma}{\epsilon_0}$$

$$\sigma = \frac{+Q}{4\pi b^2} \rightarrow \boxed{E_{out} = \frac{1}{4\pi\epsilon_0} \cdot \frac{+Q}{b^2}}$$

The electric field outside a conductor is always σ/ϵ_0, calculated using a Gaussian surface. The charge on the outside of the shell is $+Q$, so the E field must be what is boxed

c) The charge on the inner surface is $-Q$ and on the outer surface $+Q$ because in any conductor, the E-field must be zero, so the charges must cancel to equal zero.

2. (**30 points**) An insulating sphere of radius a and constant positive volume charge density, ρ_0, has a ***very*** small hole through its axis. A particle of negative charge $-Q$ and mass M is allowed to move within the sphere along the hole, where r is a coordinate that can have positive and negative values. [The hole is so small that it does not affect the charge density.]



Show that the negative charge $-Q$ particle undergoes simple harmonic motion $\frac{d^2r}{dt^2} + \omega^2 r = 0$ when inside of the sphere ($-a < r < a$). What is the angular frequency of oscillation, ω?
Hint: *What is the force on charge -Q particle due to the constant charge density ρ_0 when it is inside of the sphere? This and Newton's 3rd law will give the equation of motion.*

$F = m\vec{a}$, $\vec{F_{el}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$

$q_1 = -Q$, $q_2 = \rho_0 (4/3\,\pi a^3)$

$\rho = \frac{\text{charge}}{\text{volume}}$

$$\vec{F_{el}} = -\frac{1}{\cancel{4\pi}\epsilon_0}\frac{Q\rho_0 (4/3)\cancel{\pi} a^3}{r^2}\hat{r}$$

$$= -\frac{1}{\epsilon_0}\frac{Q\rho_0 a^3}{3r^2}\hat{r}$$

$$\vec{a} = \frac{F}{m} = -\frac{Q\rho_0 a^3}{3M\epsilon_0 r^2}\hat{r} = \frac{d^2r}{dt^2}$$

Taylor Series:

$$x(t) \approx X_0 + \frac{dr}{dt}(1-r) + \frac{d^2r}{dt^2}$$

$$\omega = \frac{v}{r}$$

$$\omega^2 = \frac{v^2}{r^2}$$

$$\omega^2 r = \frac{v^2}{r}$$

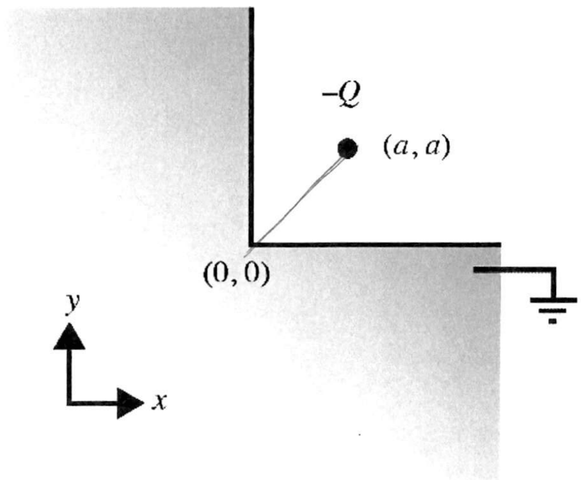
Use Taylor series to prove $\frac{d^2r}{dt^2} + \omega^2 r = 0$

$$\frac{d^2r}{dt^2} = -\omega^2 r$$

$$\cancel{-}\frac{Q \rho_0 a^3}{3 M \varepsilon_0 r^2} = \cancel{-}\omega^2 r$$

$$\boxed{\omega = \sqrt{\frac{Q \rho_0 a^3}{3 M \varepsilon_0 r^3}}}$$

3. (**30 points**) A negative charge $-Q$ is located at a position $x = a, y = a$ near the corner of a grounded ($\phi_0 = 0$) conducting surface that stretches to infinity along the planes defined by $x = 0, y = 0$ (and out of the page).

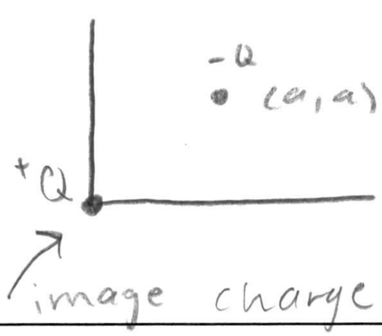


a. What is the potential $\phi(x, y)$ in the region defined by $x \geq 0, y \geq 0$? Verify that $\phi(0,0) = 0 = \phi_0$.
b. What is the force (magnitude and direction) on the $-Q$ charge? Does the direction make sense?
Hint 1: Symmetry and the constraint that $\phi = 0$ at the origin can tell you how many image charges to use (and their signs!).
Hint 2: You do ***not*** *need to take a (possibly messy) derivative to find the force!*

a) $\phi(x,y) = \frac{1}{4\pi\epsilon_0} \frac{-Q}{r}$, $r = \sqrt{x^2+y^2}$

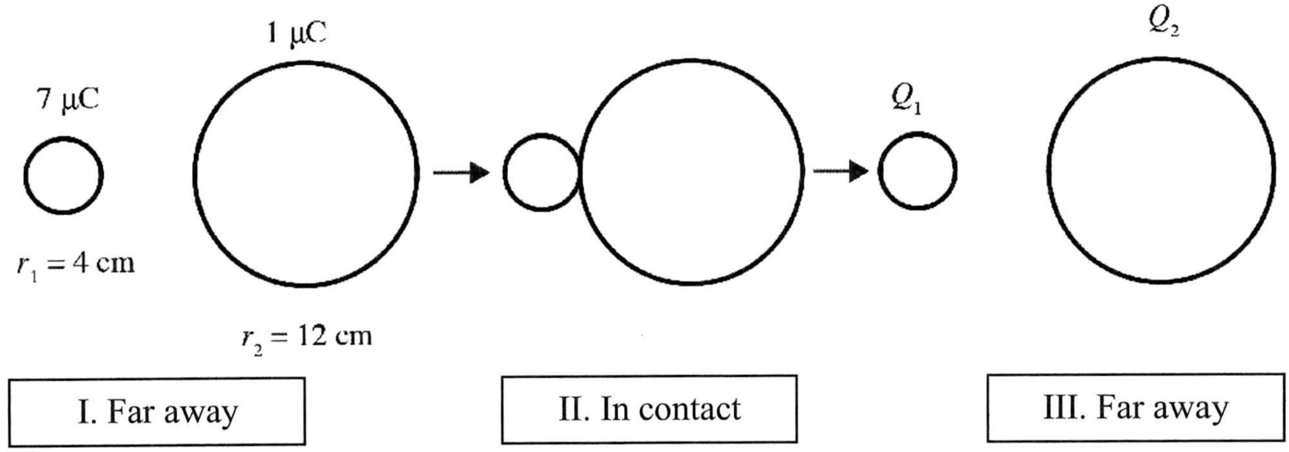
$$\boxed{\phi(x,y) = \frac{1}{4\pi\epsilon_0} \frac{-Q}{\sqrt{x^2+y^2}}}$$

b) Use mirror of images and symmetry to find force on $-Q$



4. (**30 points**) Two spherical conductors with ***different radii*** ($r_1 = 4$ cm, $r_2 = 12$ cm) are initially charged with positive excess charges $Q_1 = 7\ \mu C$ and $Q_2 = 1\ \mu C$, respectively, and located ***very far away from each other***. After bringing the conductors together into ***physical contact***, the conductors are once again separated very far away from each other (i.e. the conductors do not interact with each other). What is the final total charge on each conductor?

Hint: The potential on an isolated charged sphere of radius r (relative to infinity) is $\phi = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$.



$$\phi_1 = \frac{1}{4\pi\epsilon_0}\frac{Q_1}{r_1}, \quad \phi_2 = \frac{1}{4\pi\epsilon_0}\frac{Q_2}{r_2}$$

$\phi_{tot} = 0$ after contact, $Q_{tot} = 8\mu C$

$$\cancel{\frac{1}{4\pi\epsilon_0}}\frac{Q_{1f}}{4cm} = \cancel{\frac{1}{4\pi\epsilon_0}}\frac{Q_{2f}}{12cm}, \quad Q_{1f} + Q_{2f} = 8\mu C$$

$$Q_{2f} = 8\mu C - Q_{1f}$$

$3Q_{1f} = Q_{2f}$

$3Q_{1f} = 8\mu C - Q_{1f}$

$4Q_{1f} = 8\mu C$

$$\boxed{Q_{1f} = 2\mu C \rightarrow Q_{2f} = 6\mu C}$$

Problem	Score	Max pts
Problem 1		10 pts
Problem 2		30 pts
Problem 3		30 pts
Problem 4		30 pts
Total		100 pts