Midterm 2
Physics 1AH, Fall 2021
Tuesday, November 16, 12:00 pm

Exam Format

- There are three problems on this exam, each with multiple parts. The point values for each part of each problem are indicated on the exam.
- You will have 1 hour to complete this exam.
- Please show your work on each problem.
- Unless stated otherwise, write all answers in terms of the symbols given in the problem and the physical constant $g \approx +9.8 \text{ m/s}^2$.

Completing Your Exam

- Write your name and UID on this page.
- This exam has **18** pages. Once the exam starts, please check your exam to make sure all pages are included.
- Each part of each problem appears at the top of a page. This exam is printed double-sided, so check both the front and back of each page.
- Write all answers legibly inside the designated boxes.
- This exam is printed with ample space for scratch work. If you need extra scratch paper, raise your hand (do not leave your seat) and a proctor will bring some. Write your name, UID, and the problem and part number on each piece of scratch paper you use.

Resources

- You may reference your optional double-sided 8.5-by-11 sheet of notes during the exam. You may not use any other resources (including print and electronic resources) during the exam. Calculators are not allowed.
- You may not communicate with anyone except the course staff during the exam.
- Some students will take this exam late. After completing the exam, do not discuss the exam with anyone who has not yet taken it.

Name Katherine Callahan

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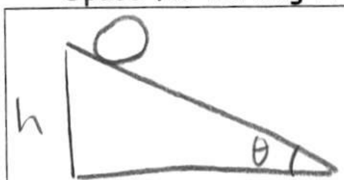
Problem 1 (Three parts (a)-(c); 40 points)

A wheel of radius R and mass m rolls down an incline, which is at an angle θ with respect to the horizontal. Assume the wheel is a circular disk of uniform density. Assume that the wheel starts from rest at height h. Assume that the wheel rolls without sliding.

Problem 1(a) (10 points)

What is the speed of the wheel when it reaches the bottom of the incline? Use conservation of energy to obtain your answer.

Space for working Problem 1(a)



$E_i = mgh$

$E_f = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$I = \frac{1}{2}mR^2$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \quad , \omega = v/r$

$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 + \frac{1}{2}\left(\frac{1}{2}\cancel{m}\cancel{R^2}\right)\left(v^2/\cancel{R^2}\right)$

$gh = \frac{v^2}{2} + \frac{v^2}{4}$

$gh = \frac{3v^2}{4}$

$v^2 = \frac{4gh}{3}$

$v = \pm\sqrt{\frac{4gh}{3}}$, speed is positive so

$\boxed{v = \sqrt{\frac{4gh}{3}}}$

Additional space for working Problem 1(a)

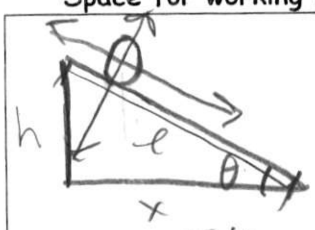
Problem 1(b) starts on the next page→

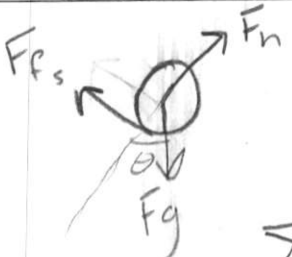
A wheel of radius R and mass m rolls down an incline, which is at an angle θ with respect to the horizontal. Assume the wheel is a circular disk of uniform density. Assume that the wheel starts from rest at height h. Assume that the wheel rolls without sliding.

Problem 1(b) *(10 points)*

Find the speed of the wheel at the bottom of the incline by calculating the equations of motion.

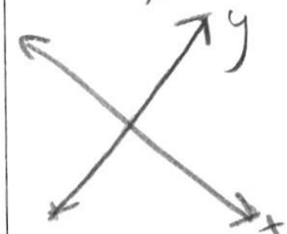
Space for working Problem 1(b)





$F_n = mg\cos\theta$

$F_{fs} \le \mu mg\cos\theta$



$\sum F_x = mg\sin\theta - F_{fs}$

$\sum \tau = F_{fs} \cdot R$, $\tau = I\alpha$

$\mu \cancel{m} g\cos\theta \cdot \cancel{R} = 1/2\, \cancel{m} R^2 \alpha$, $\alpha = a/r$

$\mu g\cos\theta = 1/2\, R(a/\cancel{R})$

$a = 2\mu g\cos\theta$

$\cancel{m}\ddot{x} = \cancel{m} g\sin\theta - \mu \cancel{m} g\cos\theta$

$\ddot{x} = g\sin\theta - \mu g\cos\theta$ $\quad gt\sin\theta$, $1/2\, gt^2\sin\theta$

$v(t) + C = \int_0^t 2\mu g\cos\theta \, dt$

$V(t) = 2t\mu g\cos\theta + V_i$, $V_i = 0$

$V(t) = 2t\mu g\cos\theta$

$x(t) + C = \int_0^t 2t\mu g\cos\theta \, dt$

$x(t) = t^2\mu g\cos\theta + x_i$, let $x_i = 0$

Additional space for working Problem 1(b)

$x(t) = t^2 \mu g \cos\theta$, $x(t_f) = \ell$, $\sin\theta = h/\ell$

$\ell = t_f^2 \mu g \cos\theta$ $\qquad \ell = h/\sin\theta$

$$t_f = \sqrt{\frac{\ell}{\mu g \cos\theta}} = \sqrt{\frac{h}{\mu g \sin\theta \cos\theta}}$$

$$v(t_f) = 2 t_f \mu g \cos\theta$$

$$= 2\mu g \cos\theta \sqrt{\frac{h}{\mu g \sin\theta \cos\theta}}$$

$$= 2\sqrt{\frac{h \mu^2 g^2 \cos^2\theta}{\mu g \sin\theta \cancel{\cos\theta}}}$$

$$\boxed{v(t_f) = \sqrt{4 g h \mu \tan\theta}}$$

$x(t_f) = \ell$, $\ell = 1/2\, g t_f^2 \sin\theta$

$$t_f = \sqrt{\frac{2\ell}{g \sin\theta}}$$

$$v(t_f) = g\sin\theta \sqrt{\frac{2\ell}{g\sin\theta}} = g\sin\theta\sqrt{\frac{2h}{g\sin^2\theta}}$$

Problem 1(c) starts on the next page→

A wheel of radius R and mass m rolls down an incline, which is at an angle θ with respect to the horizontal. Assume the wheel is a circular disk of uniform density. Assume that the wheel starts from rest at height h. Assume that the wheel rolls without sliding.

Problem 1(c) *(20 points)*

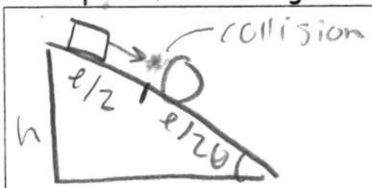
Now, assume that a block of the same mass m slides down the incline, starting from rest at height h, and assume that the incline has a coefficient of friction μ.

energy not conserved

When the block is halfway down the incline, it collides elastically with the wheel. The wheel was at rest an instant before the collision. After the collision, the wheel rolls down from this point. Assume the collision occurs instantaneously and ignore all effects of friction during the collision.

What is the speed of the wheel immediately after the collision? What is its final speed at the bottom of the incline?

Space for working Problem 1(c)



$\Sigma F_x = mg\sin\theta - f$

$N = mg\cos\theta$

$f = \mu mg\cos\theta$

$\Sigma F_x = mg\sin\theta - \mu mg\cos\theta$

$\cancel{m}\ddot{x} = \cancel{m}g\sin\theta - \mu \cancel{m}g\cos\theta$

$\frac{dv}{dt} = \ddot{x} = g\sin\theta - \mu g\cos\theta$

$dv = (g\sin\theta - \mu g\cos\theta)\,dt$

$v(t) + C = \int_0^t (g\sin\theta - \mu g\cos\theta)\,dt$

$v(t) = gt\sin\theta - \mu gt\cos\theta + V_i, \; V_i = 0$

$v(t) = gt\sin\theta - \mu gt\cos\theta$

$x(t) + C = \int_0^t (gt\sin\theta - \mu gt\cos\theta)\,dt$

Additional space for working Problem 1(c)

$$x(t) = \frac{gt^2\sin\theta}{2} - \frac{\mu g t^2\cos\theta}{2} + x_i, \quad \text{let } x_i = 0$$

$$x(t) = \frac{gt^2\sin\theta}{2} - \frac{\mu g t^2\cos\theta}{2}, \quad x(t_1) = \ell/2$$

$$\ell/\cancel{2} = \frac{g t_1^2\sin\theta}{\cancel{2}} - \frac{\mu g t_1^2\cos\theta}{\cancel{2}}$$

$$\ell = t_1^2(g\sin\theta - \mu g\cos\theta)$$

$$t_1 = \sqrt{\frac{\ell}{g\sin\theta - \mu g\cos\theta}}, \quad \ell = h/\sin\theta$$

$$= \sqrt{\frac{h}{\sin\theta(g\sin\theta - \mu g\cos\theta)}}$$

Problem 2 starts on the next page→

Problem 2 (Three parts (a)-(c); 30 points)

An object of mass m is moving at velocity $\vec{v_1}$ (to the right and upwards), and an object of mass $2m$ is moving at velocity $\vec{v_2}$ (to the left and upwards). You may assume that the y-component of the velocities is the same, and the x-component is equal and in opposite directions. After they collide, they are moving at velocities $\vec{v_3}$ and $\vec{v_4}$, respectively, again with equal y components. Write all answers in terms of m and the components of $\vec{v_1}$, which are $v_{1,x}$ and $v_{1,y}$.

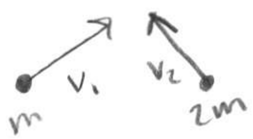
Problem 2(a) (20 points)

Assuming that the collision was elastic, what are the two velocities $\vec{v_3}$ and $\vec{v_4}$?

Space for working Problem 2(a)

$V_{1y} = V_{2y}$, $V_{1x} = -V_{2x}$ $\quad ||V_1|| = ||V_2||$

$= V_y$



$\cancel{m} V_y + 2\cancel{m} V_y = \cancel{m} V_{fy} + 2\cancel{m} V_{fy}$

$\cancel{3} V_y = \cancel{3} V_f \rightarrow V_y = V_{fy}$

↳ constant y velocity

$\cancel{m} V_{1x} + 2\cancel{m} V_{2x} = \cancel{m} V_{1xf} + 2\cancel{m} V_{2xf}$

$V_{1x} - 2V_{1x} = V_{1xf} + 2V_{2xf}$

$-V_{1x} = V_{1xf} + 2V_{2xf}$ (1)

$\frac{1}{2} \cancel{m} V_1^2 + \frac{1}{2}(2\cancel{m}) V_2^2 = \frac{1}{2} \cancel{m} V_{1f}^2 + \cancel{\frac{1}{2}}(2\cancel{m}) V_{2f}^2$

$\frac{1}{2} V_1^2 + V_1^2 = \frac{1}{2} V_{1f}^2 + V_{2f}^2$

$3/2\, V_1^2 = 1/2\, V_{1f}^2 + V_{2f}^2$

$V_1^2 = V_{1x}^2 + V_{1y}^2$

$V_{1f}^2 = V_{1xf}^2 + V_{1yf}^2$

$V_{2f}^2 = V_{2xf}^2 + V_{2yf}^2$

Additional space for working Problem 2(a)

$3/2\,(V_{1x}^2 + V_{1y}^2) = 1/2\,(V_{1xf}^2 + V_{1yf}^2) + (V_{2fx}^2 + V_{2yf}^2)$

$V_{2yf} = V_y,\ V_{1yf} = V_y,\ V_{1y} = V_y$ (y velocity is constant)

$3/2\,V_{1x}^2 + \cancel{3/2\,V_y^2} = 1/2\,V_{1xf}^2 + \cancel{1/2\,V_y^2} + V_{2fx}^2 + \cancel{V_y^2}$

$3/2\,V_{1x}^2 = 1/2\,V_{1xf}^2 + V_{2fx}^2$ (2)

(1) $2V_{2xf} = -V_{1x} - V_{1xf}$

$V_{2xf} = \dfrac{-V_{1x} - V_{1xf}}{2}$

$3/2\,V_{1x}^2 = 1/2\,V_{1xf}^2 + \left(\dfrac{-V_{1x} - V_{1xf}}{2}\right)^2$

$3/2\,V_{1x}^2 = 1/2\,V_{1xf}^2 + \dfrac{V_{1x}^2 + 2V_{1x}V_{1xf} + V_{1xf}^2}{4}$

$V_4^2 = V_{2xf}^2 + V_{2y}^2$

$V_4^2 = V_{1x}^2 + 2V_{1x}$

Problem 2(b) starts on the next page→

An object of mass m is moving at velocity $\overrightarrow{v_1}$ (to the right and upwards), and an object of mass $2m$ is moving at velocity $\overrightarrow{v_2}$ (to the left and upwards). You may assume that the y-component of the velocities is the same, and the x-component is equal and in opposite directions. After they collide, they are moving at velocities $\overrightarrow{v_3}$ and $\overrightarrow{v_4}$, respectively, again with equal y components. Write all answers in terms of m and the components of $\overrightarrow{v_1}$, which are $v_{1,x}$ and $v_{1,y}$.

<u>Problem 2(b)</u> (5 points)

If the collision was perfectly inelastic, i.e., the objects stick together, what is the final velocity $\overrightarrow{v_3}$ after the collision?

Space for working Problem 2(b)

$$\cancel{m} V_{1x} + 2\cancel{m} V_{2x} = 3\cancel{m} V_3$$

$$V_{1x} - 2V_{1x} = 3V_3$$

$$-V_{1x} = 3V_3$$

$$\boxed{V_3 = -\frac{V_{1x}}{3}}$$

Problem 2(c) *(5 points)*

For the case in (b), how much energy has been dissipated due to the collision?

Space for working Problem 2(c)

$E_i = 1/2\, m V_1^2 + 1/2\, m V_2^2$

$E_f = 1/2\, m V_3^2$

$1/2\, m V_1^2 + 1/2\, m V_2^2 = 1/2\, m V_3^2 + E_D$

$1/2\, m (V_{1x}^2 + V_y^2) + 1/2\, m ((-V_{1x})^2 + V_y^2) = 1/2\, m \left(\frac{-V_{1x}}{3}\right)^2 + E_D$

$E_D = 1/2\, m V_{1x}^2 + 1/2\, m V_{1y}^2 + 1/2\, m V_{1x}^2 + 1/2\, m V_{1y}^2 - 1/2\, m \frac{V_{1x}^2}{9}$

$E_D = m V_{1x}^2 - \frac{m V_{1x}^2}{18} + m V_{1y}^2$

$\boxed{E_D = \frac{17 m V_{1x}^2}{18} + m V_{1y}^2}$

Problem 3 starts on the next page→

Problem 3 (5 parts (a)-(e); 30 points)

An object of mass m is sitting on a flat surface, with a friction coefficient μ. It is connected to the wall by a spring of stiffness k. It is initially displaced to a position x_0 away from equilibrium, and then released.

Problem 3(a) (5 points)

How much energy is dissipated due to friction, by the time the object comes to the first stop (i.e. completes the first half-cycle)?

Space for working Problem 3(a)

Additional space for working Problem 3(a)

Problem 3(b) starts on the next page→

An object of mass m is sitting on a flat surface, with a friction coefficient μ. It is connected to the wall by a spring of stiffness k. It is initially displaced to a position x_0 away from equilibrium, and then released.

Problem 3(b) *(5 points)*

Find the equation for the distance from equilibrium at the end of the n^{th} half-cycle.

Space for working Problem 3(b)

Problem 3(c) *(5 points)*

How many half-cycles will the object complete before it comes to a full stop? Assume this ratio is an integer.

Space for working Problem 3(c)

Problem 3(d) starts on the next page→

An object of mass m is sitting on a flat surface, with a friction coefficient μ. It is connected to the wall by a spring of stiffness k. It is initially displaced to a position x_0 away from equilibrium, and then released.

Problem 3(d) *(5 points)*

What is the energy dissipated during the n^{th} half-cycle?

Space for working Problem 3(d)

Problem 3(e) *(10 points)*

What is the total energy dissipated by the time the object comes to a full stop? Use your answer in (d) and sum over the cycles.

Space for working Problem 3(e)

Additional space for working Problem 3(e)