Math 32b – Lecture 3 – Winter 2022
Midterm 1
Due via Gradescope no later than 8am Pacific time on Thursday Jan/27/2022.

Sign and submit the following honor statement:

> *I certify on my honor that I have neither given nor received any help, or used any non-permitted resources, while completing this evaluation.*

Signed:

Print name:

This exam contains 4 pages (including this cover page) and 3 problems. There are a total of 45 points available.

- Use extra pages as you need them.
- Attempt all questions.
- The work submitted must be entirely your own: you may not discuss with anyone else, except that...
- You may email the instructor `killip@math.ucla.edu` with any queries about what the questions are asking.
- This exam is open book. You may use your notes, the textbook, and any online resource that does not involve interaction with another person.
- **Posting problems to online forums or "tutoring" websites counts as interaction with another person so is strictly forbidden.**

1. (15 points) The region $\mathcal{R}$ defined by the inequalities

$$\sqrt{x^2+y^2} \leq z \leq \sqrt{1-x^2-y^2}$$

is occupied by a homogeneous solid of density ρ.

We wish to determine the moment of inertia I for this body under rotations about the z axis.

(a) Using cylindrical coordinates, write down the multiple integral that yields the value of I.

(b) Using spherical polar coordinates, write down the multiple integral that yields I.

(c) By using (a) and/or (b) compute I. (They are about equally difficult.)

2. (15 points) Consider the region $\mathcal{B}$ defined by

$$0 \leq x^2 \leq y \leq x \quad \text{and} \quad 0 \leq z \leq \frac{y}{x}$$

(a) Compute the volume of $\mathcal{B}$.

(b) Sketch (on x-y axes) the cross-section of $\mathcal{B}$ for a fixed $0 < z < 1$.

(c) Find a way to write the integral of a general $f(x, y, z)$ over $\mathcal{B}$ as a sum of several triple integrals all with z outermost.

3. (15 points) Let us model the location (X, Y) of successful shots on a goal by the following probability density:

$$p(x,y) = \begin{cases} \frac{1}{36}\, x^2\, y & : -3 \le x \le 3 \text{ and } 0 \le y \le 2 \\ 0 & : \text{otherwise} \end{cases}$$

Here x is measured horizontally, and y is measured vertically up from the ground.

(a) What is the probability that the ball passes through the middle third of the goal, that is, where $-1 \le X \le 1$?

(b) What is the median height of a successful shot?

(c) What is the average distance from the center line (i.e., $x = 0$) of the goal?