Math 32A: Quiz 1 Solutions

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1. Let $v_1 = \langle 1, 3, 5\rangle$ and $v_2 = \langle 2, -4, 6\rangle$.

   (a) Find the projection of v_1 along v_2 and the component of v_1 along v_2.

   We have

$$\begin{aligned}
\text{proj}_{v_2} v_1 &= \left(\frac{v_1 \cdot v_2}{\|v_2\|^2}\right) v_2 \\
&= \left(\frac{\langle 1,3,5\rangle \cdot \langle 2,-4,6\rangle}{\left(\sqrt{(2)^2 + (-4)^2 + (6)^2}\right)^2}\right) v_2 \\
&= \left(\frac{(1)(2) + (3)(-4) + (5)(6)}{4 + 16 + 36}\right) v_2 \\
&= \left(\frac{2 - 12 + 30}{56}\right) v_2 \\
&= \frac{20}{56} v_2 \\
&= \frac{5}{14} v_2.
\end{aligned}$$

   Thus

$$\text{proj}_{v_2} v_1 = \langle \frac{5}{7}, -\frac{10}{7}, \frac{15}{7}\rangle.$$

   The component of v_1 along v_2 is

$$\begin{aligned}
\frac{v_1 \cdot v_2}{\|v_2\|} &= \frac{\langle 1,3,5\rangle \cdot \langle 2,-4,6\rangle}{\sqrt{(2)^2 + (-4)^2 + (6)^2}} \\
&= \frac{20}{\sqrt{56}} \\
&= \frac{20}{2\sqrt{14}} \\
&= \frac{10}{\sqrt{14}}.
\end{aligned}$$

(b) Is the angle between v_1 and v_2 acute, right, or obtuse? (Recall that, according to our conventions, $0 \leq \theta \leq \pi$.)

We have

$$v_1 \cdot v_2 = \langle 1, 3, 5\rangle \cdot \langle 2, -4, 6\rangle = (1)(2) + (3)(-4) + (5)(6) = 2 - 12 + 30 = 20.$$

It follows

$$\|v_1\|\|v_2\| \cos\theta = 20 > 0.$$

Since $\|v_1\| > 0$ and $\|v_2\| > 0$, we conclude that

$$\cos\theta > 0.$$

Because $0 \leq \theta \leq \pi$, we must have

$$0 \leq \theta < \frac{\pi}{2}.$$

As v_1 and v_2 are not parallel to each other, we can exclude the possibility that $\theta = 0$, so it must be that

$$0 < \theta < \frac{\pi}{2}.$$

Therefore θ is acute.

2. Let $v = \langle 1, 1, 0\rangle$ and $w = \langle 1, 2, 3\rangle$. Calculate

$$v \cdot w - w \cdot v$$

and

$$v \times w - w \times v.$$

We have

$$\begin{aligned} v \cdot w - w \cdot v &= \langle 1, 1, 0\rangle \cdot \langle 1, 2, 3\rangle - \langle 1, 2, 3\rangle \cdot \langle 1, 1, 0\rangle \\ &= [(1)(1) + (1)(2) + (0)(3)] - [(1)(1) + (2)(1) + (3)(0)] \\ &= 3 - 3 \\ &= 0. \end{aligned}$$

Alternatively, we may use the commutativity of the dot product $v \cdot w = w \cdot v$ to deduce

$$v \cdot w - w \cdot v = v \cdot w - v \cdot w = 0.$$

To compute the second quantity, we use the anticommutativity of the cross product $v \times w = -w \times v$ to determine that

$$v \times w - w \times v = v \times w + v \times w = 2(v \times w).$$

We have

$$\begin{aligned} v \times w &= \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 1 & 2 & 3 \end{vmatrix} \\ &= \begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix} i - \begin{vmatrix} 1 & 0 \\ 1 & 3 \end{vmatrix} j + \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} k \\ &= ((1)(3) - (0)(2))i - ((1)(3) - (0)(1))j + ((1)(2) - (1)(1))k \\ &= 3i - 3j + k \\ &= \langle 3, -3, 1 \rangle. \end{aligned}$$

So

$$v \times w - w \times v = 2\langle 3, -3, 1 \rangle = \langle 6, -6, 2 \rangle.$$