

Midterm 1

Math 31B, Winter 2022

Midterm Exam: Friday, February 4.

Please read the policies for online midterms posted on Canvas, under Modules – Syllabus and Policies.

You may solve these problems by any method you like (as long as the method is correct).

1. Evaluate

$$\frac{d}{dx} \ln \sqrt{x}.$$

$$\frac{d}{dx} \ln \sqrt{x} = \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2x}$$

2. Evaluate

$$\lim_{x \rightarrow 2} \left(\frac{x^2 - 4x + 4}{x^2 - 2x} \right).$$

$$\lim_{x \rightarrow 2} \frac{x^2 - 4x + 4}{x^2 - 2x} = \frac{2^2 - 4(2) + 4}{2^2 - 2(2)} = \frac{0}{0}$$

$$\lim_{x \rightarrow 2} \frac{2x - 4}{2x - 2} = \frac{0}{2} = 0$$

CHECK HYPOTHESIS FOR L'HÔPITAL'S:

- numerator and denominator are differentiable
- at $x \approx 2$, limit is indeterminate $\frac{0}{0}$
- derivative of denominator is $2x - 2$ (non-zero $\approx x \approx 2$)

3. Evaluate

$$\int_{-1}^0 e^{2x} dx.$$

$$u = 2x$$

$$du = 2dx$$

$$\frac{1}{2} du = dx$$

$$\frac{1}{2} \int_{-1}^0 e^u du$$

$$\frac{1}{2} [e^{2x}]_{-1}^0 = \frac{1}{2} (e^0 - e^{-2}) = \frac{1}{2} \left(1 - \frac{1}{e^2} \right) = \frac{1}{2} - \frac{1}{2e^2}$$

4. Evaluate

$$\int x \sin(x) dx.$$

$$u = x$$

$$dv = \sin x dx$$

$$du = dx$$

$$v = -\cos x$$

$$\int x \sin x dx = -x \cos x + \int \cos x dx$$

$$= -x \cos x + \sin x + C$$

5. Evaluate

$$\int \frac{x^2 + 2x - 1}{x^2 - x} dx.$$

$$(x^2 - x)(1) = x^2 - x$$

$$(x^2 - x)(1) + 3x - 1 = x^2 + 2x - 1$$

$$\frac{x^2 + 2x - 1}{x^2 - x} = 1 + \frac{3x - 1}{x^2 - x}$$

$$\frac{3x-1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1} = \frac{A(x-1) + B(x)}{x(x-1)}$$

$$\begin{aligned} A(x-1) + B(x) &= 3x-1 \\ \text{at } x=1 \dots &\quad \text{at } x=0 \dots \\ B=2 &\quad -A=-1 \\ &\quad A=1 \end{aligned}$$

$$\int \frac{x^2 + 2x - 1}{x^2 - x} dx = \int 1 + \frac{3x-1}{x^2-x} dx = \int 1 dx + \int \frac{3x-1}{x(x-1)} dx = x + \int \frac{3x-1}{x(x-1)} dx = x + \int \frac{1}{x} + \frac{2}{x-1} dx$$

$$= x + \ln|x| + 2\ln|x-1| + C$$