Midterm 1

Instructions: Please write your answers and show all necessary work on either the provided exam (if you wish to print it), separate sheets of paper, or a tablet computer. You do not need to print out the exam. You may use your textbook, notes, and resources on CCLE or elsewhere on the internet. You may also use a calculator or computer, including online resources such as CoCalc, Desmos, and WolframAlpha. However, as always, **you must show all of your work to receive full credit** for each problem, and **you must not get help from other people**. All work shown must be your own. If you have a question about the exam at any point during the exam period, you can email your question to LS30BS2020@gmail.com. When you are finished with the exam, take a photo of your answer to each question, and upload them to Gradescope. Note that you will upload a separate file for each question! We recommend that you start each question on a new page.

1. (10 points) The human gut is host to many different types of bacteria. These bacteria aid in digestion by breaking down indigestible food into more easily digestible components. The most prevalent of these bacteria are the two phyla Bacteroidetes (B) and Firmicutes (F). Imbalance of these bacteria in the gut can lead to inflammation in the gut, which triggers the release of inflammatory cytokines (C), increasing the risk of inflammatory bowel syndrome, diabetes, eating disorders, and even depression. This is referred to as the "gut-brain" axis.

   Use the following assumptions to write a set of differential equations that model these two bacteria and cytokines in the human body. You may make up parameters where necessary.

   - Bacteroidetes, in the absence of Firmicutes, grow logistically, with a natural per capita growth rate* of 0.2.
   - Firmicutes also grow logistically, with a natural per capita growth rate 0.15.
   - Because a human diet is roughly constant, the carrying capacities of Bacteroidetes and Firmicutes are 300 and 200 (per mL of blood), respectively.
   - Firmicutes and Bacteroidetes compete for resources. Whenever they interact with each other, the Bacteroidetes is destroyed. This happens with a rate constant of 0.09.
   - Cytokines are produced at a rate of 50 (per mL of blood) per day.
   - When the ratio of Firmicutes to Bacteroidetes gets too high, it triggers excess production of Cytokines. The production of cytokines in this way is proportional to the ratio of Firmicutes to Bacteroidetes.
   - Cytokines degrade at a per-molecule rate of 80% per day.
   - Cytokines can trigger an increased appetite. Therefore a high numbers of cytokines causes a sudden (steep) increase of Firmicute production, with a maximum of 40 (per mL of blood).

*Note: natural per capita growth rate is the same as natural growth rate or intrinsic growth rate.

Question 1 continues on the next page...

2. (10 points) Leopard sharks are small sharks commonly found along the California coast. They prey on many species, including red crabs. Use the following assumptions to write a model of the interactions between ***adult*** red crabs (C) and ***adult*** leopard sharks (S). You can make up parameters as necessary.

   - Because of their varied diet, sharks have a constant per capita birth rate of 0.1.
   - Newly born sharks take about 10 years to reach adulthood.
   - As adults, leopard sharks have a per capita death rate of 0.05.
   - Crabs have a per capita birth rate of 0.5 and take about 3 months ($\frac{1}{4}$ of a year) to reach adulthood.
   - Adult crabs die of natural causes at a per capita rate of 0.2.
   - The rate at which a single leopard shark eats crabs is a (non-sigmoid) saturating function of crab abundance, with a maximum of 0.7.

Question 2 continues on the next page...

3. (8 points) Consider the following Delay Differential Equation (DDE):

$$X(t)' = 2.5 \cdot X(t) \cdot \left(1 - \frac{X(t-1.5)}{10 + X(t-2)}\right)$$

Using the table on the right, use Euler's Method to simulate the DDE for 0.2 units of time, with a time step of 0.1, starting at time $t = 2.5$.

t	$X(t)$
0.0	49
0.1	44
0.2	40
0.3	37
0.4	33
0.5	29
0.6	32
0.7	36
0.8	41
0.9	47
1.0	48
1.1	46
1.2	43
1.3	41
1.4	40
1.5	46
1.6	51
1.7	55
1.8	57
1.9	53
2.0	50
2.1	48
2.2	51
2.3	54
2.4	58
2.5	63

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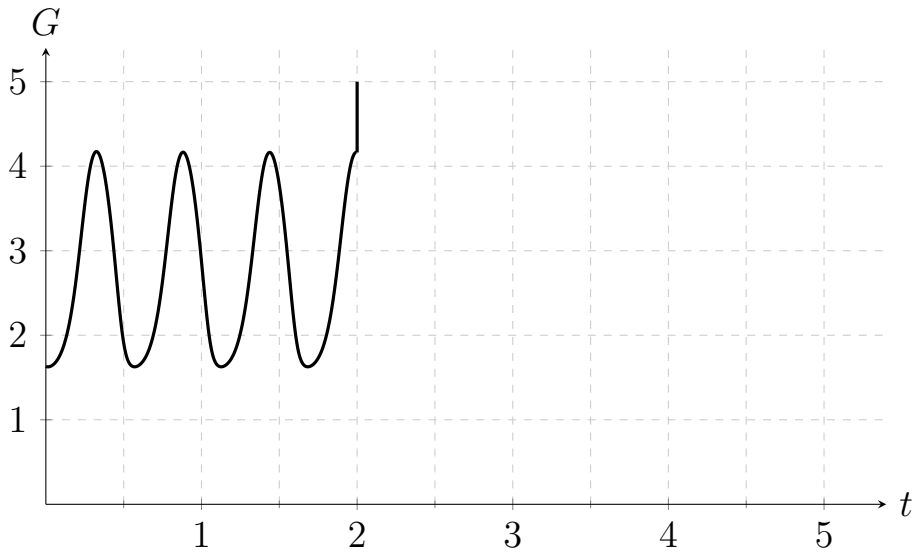
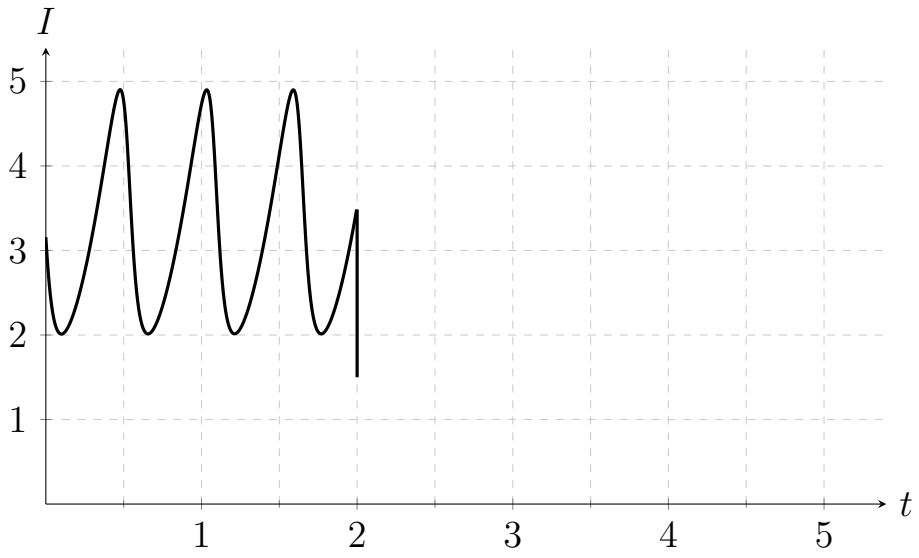
4. (a) (5 points) Will a system with an attractor ***always*** return to that attractor if perturbed? Justify your answer.

   (b) (5 points) Below are the time series of a limit cycle attractor in a 2-variable model. At time $t = 2$, we perturb the system, as shown. Sketch the rest of the graphs until $t = 5$.
   Note: If you can't print out this page, you can just sketch the graphs below onto your answer page, then add to them.





5. Cortisol (C) is a hormone made by the adrenal glands that regulates metabolism, the immune response, and helps mediate stressful situations. Its production is mediated by adrenocorticotropic hormone, secreted by the pituitary gland (P). The pituitary, in turn, is mediated by corticotropin releasing hormone, secreted by the hypothalamus (H). Finally, cortisol negatively regulates both the pituitary and the hypothalamus. This system can be modeled with the following equations:

$$\begin{cases} H' = \frac{1}{1+C^{10}} - 0.2H \\ P' = 0.8H + \frac{8}{8+C^3} - 0.2P \\ C' = 0.9P - 0.2C \end{cases}$$

(a) (4 points) Draw a cause/effect diagram for these three variables and identify the two feedback loops and their types.

Question 5 continues on the next page...

(b) (4 points) In a normal patient, the hormone levels oscillate. Why is it important that physiological oscillations such as this one are limit cycle oscillations rather than neutral ones?

(c) (3 points) Say a patient is exhibiting symptoms consistent with Cushing's disease, a disorder in which the pituitary produces too much adrenocorticotropic hormone (P). In this patient, this is caused by a pituitary tumor. After surgically removing the tumor, you observe that the pituitary is no longer affected by cortisol levels, effectively removing the link from C to P. Assuming all else is like normal, would you still expect to see oscillations? Justify your answer.

(d) (3 points) Would you expect to see any oscillations if you removed the link from C to H instead of from C to P? Justify your answer.

6. (8 points) You and a friend simulate a chaotic system with slightly different initial conditions. What aspects of your simulation results will be similar? What aspects will be different?

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7. (8 points) The Fibonacci sequence is a sequence in which each number in the sequence is the sum of the two previous ones. For example, the sequence can run as follows:

$$0,\ 1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13,\ 21,\ \ldots$$

The code below generates a Fibonacci sequence through iteration, but it is incomplete. Fill in the blanks to make the code correctly print out the first 20 numbers of the Fibonacci sequence.

```
val_list = [0, 1]

for i in _________:
    new_val = _________ + _________
    val_list.append(_________)

print val_list
```