20W-LIFESCI30B-2 Midterm Exam

CHRISTINA KILKEARY

TOTAL POINTS

56 / 60

QUESTION 1

1 LSC Foods and Fertilizers 10 / 10

✓ **- 0 pts Correct**

- 1 pts incorrect number of new entry-level workers

- 0.5 pts Partially correct incoming entry-level workers

- 1 pts incorrect number of new staff supervisors

- 0.5 pts partially incorrect number of new staff supervisors

- 1 pts incorrect number of new hired managers

- 0.5 pts partially incorrect number of new hired managers

- 2 pts incorrect number of new demoted entry-level workers

- 1 pts mostly incorrect number of new demoted entry-level workers

- 0.5 pts partially incorrect number of new demoted entry-level workers

- 0.5 pts incorrect time delay(demoted entry-level workers)

- 0.25 pts patially incorrect time delay (demoted entry-level workers)

- 2 pts incorrect number of supervisors leave

- 1 pts mostly incorrect number of supervisors leave

- 0.5 pts partially incorrect number of supervisors leave

- 2 pts incorrect number of new promoted managers

- 1 pts mostly incorrect number of new promoted managers

- 0.5 pts partially incorrect number of new promoted managers

- 0.5 pts incorrect time delay(promoted managers)

- 0.25 pts patially incorrect time delay (promoted managers)

- 1 pts incorrect number of new retired managers

- 0.5 pts partially incorrect number of new retired managers

QUESTION 2

2 Penguin Life 9 / 10

- 0 pts Correct

✓ **- 1 pts Incorrect entry**

- 0.5 pts 20% of eggs "never" hatch

- 1 pts Incorrect Pattern

QUESTION 3

LCA 10 pts

3.1 Models to Phase Portraits 2 / 2

✓ **- 0 pts Correct**

- 0.5 pts Incorrect model-graph and phase potrait match for A- Should be Unstable Spiral (Spring and Friction in same direcion)

- 0.5 pts Incorrect model-graph and phase potrait match for B- Should be LCA with oscillations

- 0.5 pts Incorrect model-graph and phase potrait match for C- Should be Stable Spiral (Spring, Friction opposing direction)

- 0.5 pts Incorrect model-graph and phase potrait match for D- Should be friction less spring

3.2 N-Shaped Friction Does What? 2 / 2

✓ **- 0 pts Correct**

- 2 pts Incorrect Match for all portions of the figure

- 1 pts Incorrect Match for portions 1-A, 4-D

3.3 How Do I Play the Clarinet? 6 / 6

✓ **- 0 pts Correct**

- 1 pts Did not refer to N-Shaped Friction

- 2 pts Did not verbalize how intermediate velocities

create acceleration, and the higher velocities create deceleration

- **2 pts** Did not refer to Limit Cycle Attractor
- **1 pts** Did not explain why LCA produces stable behavior
- **1 pts** Incorrect explanation of how intermediate velocities create acceleration, and the higher velocities create deceleration
- **0.5 pts** Incomplete explanation of why an LCA produces stable behavior
- **0.5 pts** Incomplete explanation of how intermediate velocities create acceleration, and the higher velocities create deceleration
- **0.5 pts** Incorrect explanation of why an LCA produces stable behavior

QUESTION 4

4 Rick's Gonna Do It, Y'All! 10 / 10

✓ **- 0 pts Correct**

- **2 pts** wrong time delay values on initial step
- **1.5 pts** inconsistent time delay values between iterations
- **1.5 pts** incorrect solutions for D'(t) and/or Dnext
- **1 pts** only error in final step
- **2 pts** incorrect start time (t = 0)
- **1 pts** Extra calculation/missing step
- **5 pts** improper setup/failed to account for time delay
- **10 pts** No attempt
- **8 pts** only 1 iteration attempted

QUESTION 5

Linear Functions Love Linear Combinations! 10 pts

5.1 Linear Combinations 5 / 5

✓ **- 0 pts Correct**

- **1 pts** Incorrect solution of equations (a=4, b=3)
- **3 pts** Incomplete solution

5.2 Linear Functions 5 / 5

✓ **- 0 pts Correct**

- **1 pts** Incorrect solution carried over
- **2 pts** Incorrect expansion of f (4*(3,7) + 3*(6,2))
- **2 pts** Did not plug in f(3,7) or f(6,2) correctly
- **5 pts** No answer/Incorrect answer
- **4 pts** No linear combination established
- **3 pts** Incorrect expression for f(15,17)
- **1 pts** Incorrect calculation

QUESTION 6

Linear Functions ARE Matrices. Well, sort of. 10 pts

6.1 Matrix of a Linear Function 1 / 4

- **0 pts** Correct
- **1 pts** Incorrect first basis

✓ **- 2 pts Incorrect second basis**

✓ **- 1 pts Incorrect third basis**

- **1 pts** Incorrect order
- **2 pts** Incorrect dimension
- **4 pts** Incorrect/ No answer

6.2 Composition One Way - It Works 3 / 3

✓ **+ 3 pts (almost) correct**

+ **1 pts** yes, it exists
+ **1 pts** matrix representing of f o g
+ **1 pts** matrix multiplication, i.e., correct result matrix size
+ **0 pts** wrong answer, i.e., no

6.3 Composition the Other Way - It Does NOT Work 3 / 3

✓ **+ 3 pts correct reasoning**

+ **2 pts** correct answer, i.e., no
+ **0 pts** wrong answer, i.e., yes

LS 30B: MATHEMATICS FOR LIFE SCIENTISTS
WINTER 2020 - LECTURES 2 and 3
Jukka Keränen

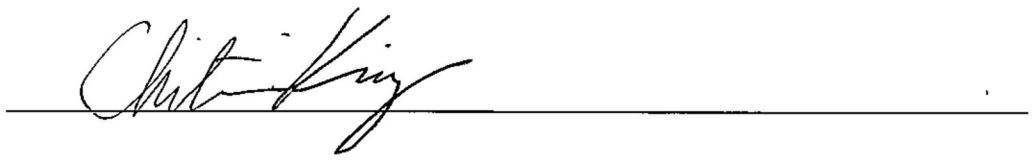
MIDTERM EXAMINATION

Your Name Christina Kilkeary

The Last Six Digits of Your Student ID number | 4 | 1 | 6 | 4 | 5 | 4 |

Your TA Section 2A (Neel)

By signing below, you confirm that you did not cheat on this exam. No exam booklet without a signature will be graded.



INSTRUCTIONS

- Please do **not** open this booklet until you are told to do so.
- In addition to basic writing instruments, you are allowed to use a non-programmable calculator.
- Your cell phone must be **turned off completely** and stowed away where you cannot see it.
- No books or notes.
- If you have a question at any time during the exam, please raise your hand.
- You will receive points only for work written on the numbered pages. Please use the reverse side as scratch paper.
- Make sure to write legibly. **Illegible work will not be graded.**
- Make sure to show all your work and justify your answers fully.

SCORE

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

TOTAL _____

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1. Some of you are already familiar with my favorite company, LSC Foods and Fertilizers, which has three categories of employees: entry-level workers (W), staff supervisors (S), and managers (M). The dynamics of the employee population of LSC Foods and Fertilizers can be modeled by the following assumptions. All rates are per-year rates.

- Every employee is either an entry-level worker, a staff supervisor, or a manager.

① • Every year, the company hires a fixed number of new entry-level workers, a.

② • Every year, the company hires one new manager (from outside).

③ • The probability that any one entry-level worker is promoted to a staff supervisor is b. (per capita)

④ • The supervisors help each other perform their duties more effectively. Thus, the probability that any one supervisor is promoted to a manager is an increasing sigmoid of the number of supervisors, with a half-saturation density of 50 and a maximum per-capita rate of 0.1. However, due to the excessive and arcane bureaucracy of the company, promotion decisions are made with a time delay of 2 years. (per capita)

⑤ • Every year, some supervisors are demoted back to entry-level workers. The per-capita rate at which supervisors are demoted is a decreasing sigmoid of the number of entry-level workers, with a half-saturation density of 100 and a maximum per-capita rate of 0.2. Again, due to excessive bureaucracy, demotion decisions are made with a time delay of 1 year.

⑥ • Every year, some supervisors leave the company due to lack of prospects for a promotion. The per-capita rate at which supervisors leave is a non-sigmoid saturating function of the number of managers, with a half-saturation density of 10 and a maximum per-capita rate of 0.3. There is no time delay here.

⑦ • Every year, d managers retire.

(10 pts) By using the assumptions above, fill in the flow rates in the diagram provided on the next page. That is: each blank space enclosed by a dashed rectangle should get one expression.

(You can use the space below for your scratch work, but you will receive points only for what you write on the next page.)

① $W' = +a$

② $M' = +1$

③ $W' = -bW(t)$

$S' = +bW(t)$

④ $S' = -0.1\left(\frac{S^n(t-2)}{50^n + S^n(t-2)}\right) S(t-2)$

$M' = +0.1\left(\frac{S^n(t-2)}{50^n + S^n(t-2)}\right) S(t-2)$

⑤ $S' = -0.2\left(\frac{100^m}{100^m + W^m(t-1)}\right) S(t-1)$

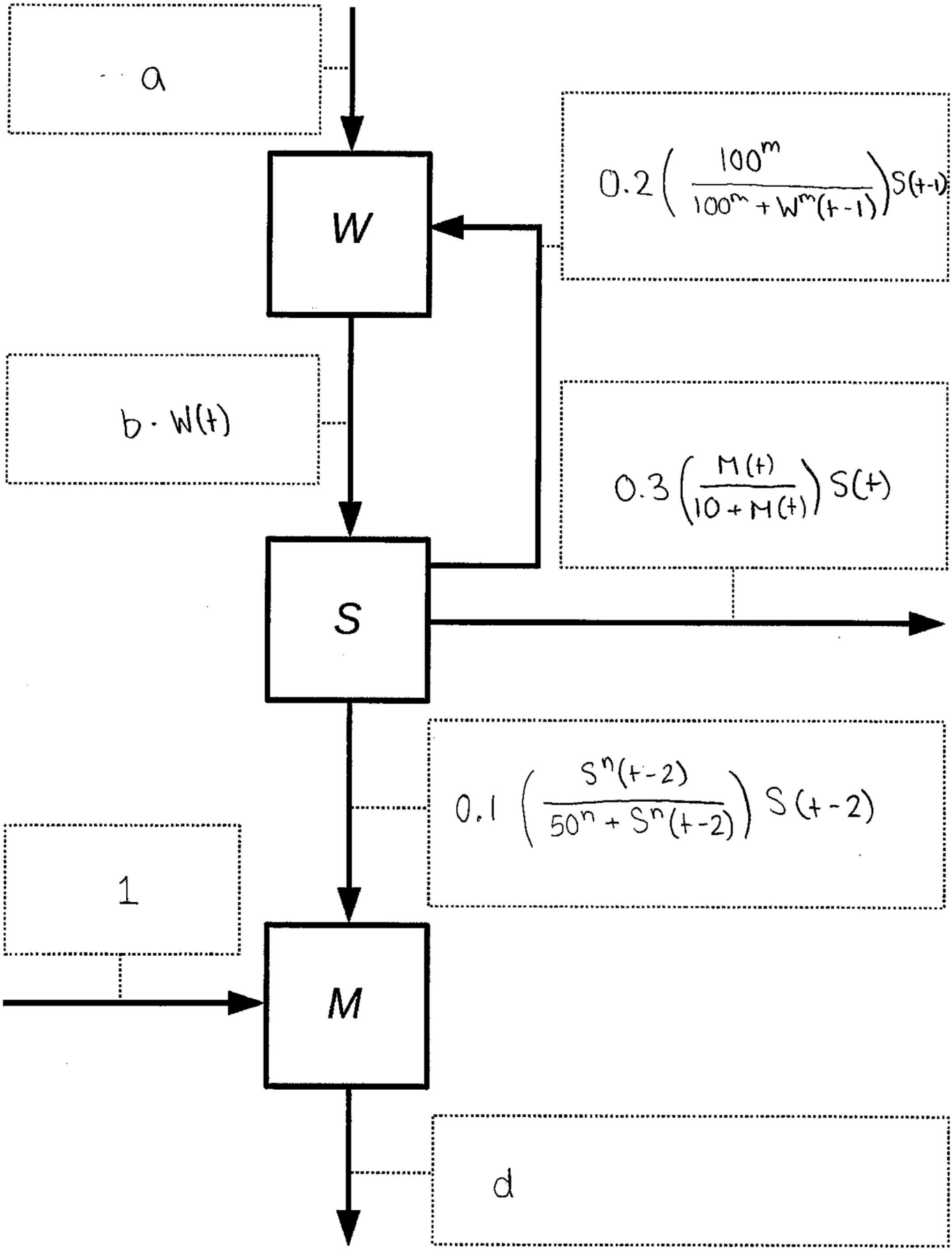
$W' = +0.2\left(\frac{100^m}{100^m + W^m(t-1)}\right) S(t-1)$

⑥ $S' = -0.3\left(\frac{M(t)}{10 + M(t)}\right) S(t)$

⑦ $M' = -d$

The Last **Six** Digits of Your Student ID number

4	1	6	4	5	4



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2. (10 pts) Female penguins have three distinct life stages: eggs (E), chicks (C), and reproductively mature adults (A). Write a discrete-time matrix model of the female penguin population, by using the following assumptions. All rates are per year rates.

- On average, adults lay 0.5 female eggs each year.
- Normally, an egg takes none stay eggs less than a year to hatch, but, sadly, 20% of eggs never hatch. 80% survive to chicks
- Penguins remain chicks for 2 years. Thus, about 50% of chicks mature into adults each year.
- 10% of chicks die each year. 90% survive
- 90% of adults survive from one year to the next.

$$E_{N+1} = 0\,E_N + 0\,C_N + 0.5\,A_N$$

$$C_{N+1} = 0.80E_N + 0.90\,C_N + 0\,A_N$$

$$A_{N+1} = 0\,E_N + 0.50\,C_N + 0.90A_N$$

matrix:

$$\begin{pmatrix} 0 & 0 & 0.5 \\ 0.8 & 0.9 & 0 \\ 0 & 0.5 & 0.9 \end{pmatrix}$$

vector:

$$\begin{pmatrix} E_{N+1} \\ C_{N+1} \\ A_{N+1} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0.5 \\ 0.8 & 0.9 & 0 \\ 0 & 0.5 & 0.9 \end{pmatrix} \begin{pmatrix} E_N \\ C_N \\ A_N \end{pmatrix}$$

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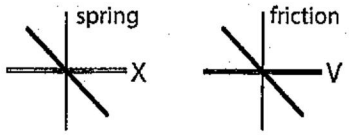
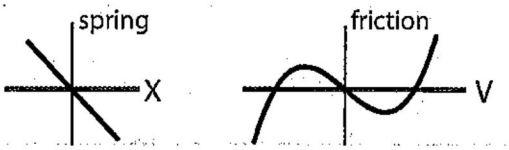
3. *a*) (2 pts) Consider an oscillating spring, with different types of forces of friction acting on it. Match up the models and graphs A) - D) with the phase portraits 1) - 4).

A) $X' = V, \quad V' = -X + V$



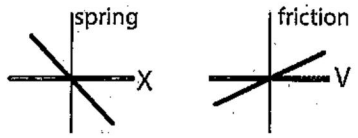
A) correponds to 4

B) $X' = V, \quad V' = -X - (V^3 - V)$



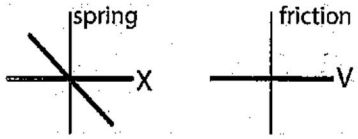
B) correponds to 3

C) $X' = V, \quad V' = -X - V$



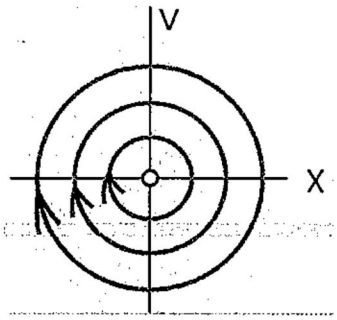
C) correponds to 2

D) $X' = V, \quad V' = -X$

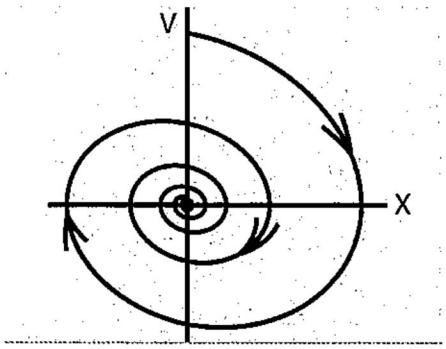


D) correponds to 1

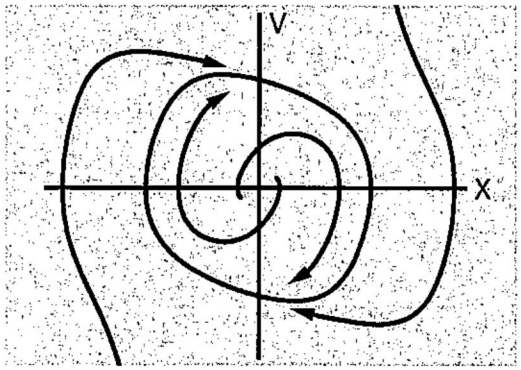
1)



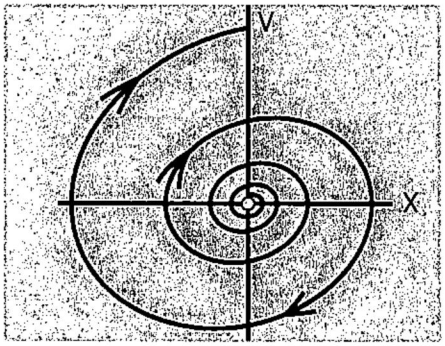
2)



3)



4)



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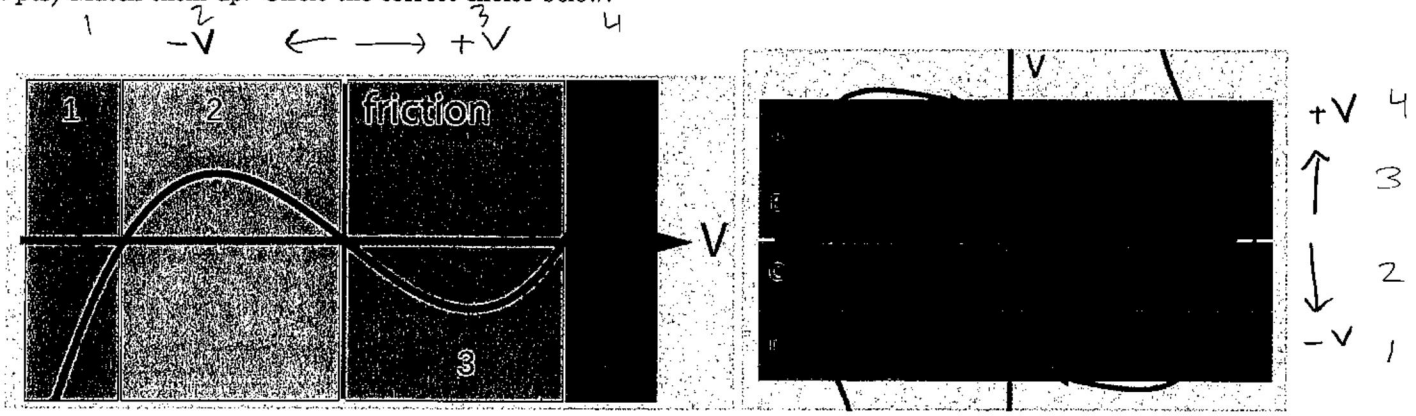
b) (2 pts) Match them up! Circle the correct choice below.



- 1-A, 2-B, 3-C, 4-D
- 1-B, 2-A, 3-D, 4-C
- 1-D, 2-C, 3-B, 4-A
- 1-A, 2-C, 3-B, 4-D

c) (6 pts) One of the models on the previous page, along with its corresponding phase portrait, can be used to explain how it is possible to hold a note on a reed instrument such as the clarinet. Outline that explanation.

(*Hint.* You might find the two graphs in part *b*) helpful in your discussion.)

Model and graph B) as well as its corresponding phase portrait 3) can help explain how its possible to hold a note on a reed instrument.

The reed inside the instrument is like a limit cycle attractor.
When a musician provides air flow, it speeds up the reed in the instrument.
When the same reed starts moving too fast, the air flow provided by the musician works to slow it down.
Because there is a tendency towards stable oscillation of the reed, small changes in air flow that humans are prone to don't affect the instrument and player's ability to hold a note.

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Euler's

4. (10 pts) Grandpa Rick has done goofed! He's forgotten the value at time $t = 0.3$ of a super-important state variable $D(t)$. Luckily, he knows that

$D_{next} = D_{now} + D'_{now} \cdot (\text{step size})$

- for all $t \leqslant 0$,

$$D(t) = t + 10,$$

- for all $t \geqslant 0$, the dynamics of $D(t)$ can be modeled with

$$D'(t) = 5D^2(t - 0.2) - 4D(t - 0.1)$$

Use Euler's method with a step size of 0.1 to approximate $D(0.3)$ and help Rick ~~destroy~~ save the world!

Rick *loves* precision, so you should not round your numbers.

(*Hint.* To make your work easier, the first four columns in your tally should be t, $D(t - 0.2)$, $D(t - 0.1)$, and $D(t)$.)

t	$D(t-0.2)$	$D(t-0.1)$	$D(t)$	$D'(t)$
0	$D(-0.2) = 9.8$	$D(-0.1) = 9.9$	10	440.6
0.1	$D(-0.1) = 9.9$	$D(0) = 10$	54.06	450.05
0.2	$D(0) = 10$	$D(0.1) = 54.06$	99.065	283.76
0.3			127.441	

$$D(0.1) = D(0) + \left(5D^2(-0.2) - 4D(-0.1)\right)(0.1)$$
$$= 10 + \left(5(9.8)^2 - 4(9.9)\right)(0.1)$$
$$= 10 + (480.2 - 39.6)(0.1)$$
$$= 10 + 44.06$$
$$D(0.1) = 54.06$$

$$D(0.2) = D(0.1) + \left(5D^2(-0.1) - 4D(0)\right)(0.1)$$
$$= 54.06 + \left(5(9.9)^2 - 4(10)\right)(0.1)$$
$$= 54.06 + (490.05 - 40)(0.1)$$
$$= 54.06 + (450.05)(0.1)$$
$$D(0.2) = 99.065$$

$$D(0.3) = D(0.2) + \left(5D^2(0) - 4D(0.1)\right)(0.1)$$
$$= 99.065 + (5 \cdot 100 - 4 \cdot 54.06)(0.1)$$
$$= 99.065 + (283.76)(0.1)$$
$$D(0.3) = 127.441$$

(0 pts) What do you think $D(t)$ represents?

honestly no clue.

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5. *a*) (5 pts) Let $\mathbf{U} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$ and $\mathbf{V} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$. Find real numbers a and b such that

$$a \cdot \mathbf{U} + b \cdot \mathbf{V} = \begin{pmatrix} 30 \\ 34 \end{pmatrix}.$$

(*Hint.* You can find a and b by solving a system of two linear equations with a and b as the variables.)

$$a \cdot \begin{pmatrix} 3 \\ 7 \end{pmatrix} + b \cdot \begin{pmatrix} 6 \\ 2 \end{pmatrix} = \begin{pmatrix} 30 \\ 34 \end{pmatrix}$$

$$\begin{cases} 3a + 6b = 30 \\ 7a + 2b = 34 \end{cases}$$

$3a = 30 - 6b$

$a = 10 - 2b$

$7(10-2b) + 2b = 34$

$70 - 14b + 2b = 34$

$36 = 12b$

$\boxed{b = 3}$

$3a + 6(3) = 30$

$3a + 18 = 30$

$3a = 12$

$\boxed{a = 4}$

b) (5 pts) Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear function for which

$$f\left(\begin{pmatrix} 3 \\ 7 \end{pmatrix}\right) = \begin{pmatrix} 3 \\ 3 \end{pmatrix}, \quad f\left(\begin{pmatrix} 6 \\ 2 \end{pmatrix}\right) = \begin{pmatrix} -4 \\ -4 \end{pmatrix}.$$

Find $f\left(\begin{pmatrix} 15 \\ 17 \end{pmatrix}\right)$.

(*Hint.* You will find part *a*) helpful here.)

$$a\begin{pmatrix} 3 \\ 7 \end{pmatrix} + b\begin{pmatrix} 6 \\ 2 \end{pmatrix} = \begin{pmatrix} 15 \\ 17 \end{pmatrix}$$

$$\begin{cases} 3a + 6b = 15 \\ 7a + 2b = 17 \end{cases}$$

$3a + 6b = 15$

$3a = 15 - 6b$

$a = 5 - 2b$

$7(5-2b) + 2b = 17$

$35 - 14b + 2b = 17$

$18 = 12b$

$18/12 = 3/2 = b$

$3a + 6(3/2) = 15$

$3a + 9 = 15$

$3a = 6$

$a = 2$

$$f\left(\begin{pmatrix} 15 \\ 17 \end{pmatrix}\right) = f\left(2\begin{pmatrix} 3 \\ 7 \end{pmatrix} + \frac{3}{2}\begin{pmatrix} 6 \\ 2 \end{pmatrix}\right)$$

$$= 2f\left(\begin{pmatrix} 3 \\ 7 \end{pmatrix}\right) + \frac{3}{2}f\left(\begin{pmatrix} 6 \\ 2 \end{pmatrix}\right)$$

$$= 2\begin{pmatrix} 3 \\ 3 \end{pmatrix} + \frac{3}{2}\begin{pmatrix} -4 \\ -4 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ 6 \end{pmatrix} + \begin{pmatrix} -6 \\ -6 \end{pmatrix}$$

$$\boxed{f\left(\begin{pmatrix} 15 \\ 17 \end{pmatrix}\right) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}}$$

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6. We have two linear functions, $f : \mathbb{R}^2 \to \mathbb{R}^4$ and $g : \mathbb{R}^3 \to \mathbb{R}^2$. The matrix representing f is

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 4 & 3 \\ 2 & 1 \end{pmatrix}.$$

a) (4 pts) Suppose that

$$g\left(\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \quad g\left(\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad g\left(\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} 6 \\ 5 \end{pmatrix}.$$

Find the matrix representing g.

matrix representing g: $\begin{pmatrix} 3 & 2 & 6 \\ 5 & 1 & 5 \end{pmatrix}$

b) (3 pts) Find the matrix representing $f \circ g$ or explain in terms of function composition why it does not exist.

f(g) $R_3 \to R_4$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 4 & 3 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 3 & 2 & 6 \\ 5 & 1 & 5 \end{pmatrix} = \begin{pmatrix} 3+10 & 2+2 & 6+10 \\ 9+20 & 6+4 & 18+20 \\ 12+15 & 8+3 & 24+15 \\ 6+5 & 4+1 & 12+5 \end{pmatrix}$$

$$f \circ g = \begin{pmatrix} 13 & 4 & 16 \\ 29 & 10 & 38 \\ 27 & 11 & 39 \\ 11 & 5 & 17 \end{pmatrix}$$

c) (3 pts) Find the matrix representing $g \circ f$ or explain in terms of function composition why it does not exist.

g(f)

does not exist

For this to exist the rows of f would need to match the columns of g which they do not as $4 \neq 3$. This is because in composition the outputs of $f(R^4)$ would need to be compatible with the input of g (R^3).