20W-LIFESCI30B-2 Final Exam

CHRISTINA KILKEARY

TOTAL POINTS

87.5 / 88

QUESTION 1

1 Modeling Problem I 10 / 10

✓ - **0 pts** Correct

- **1 pts** Coeff of H^2 should be 0.2*2
- **1 pts** Wrong coefficient

QUESTION 2

Modeling Problem II 10 pts

2.1 Part a) 8 / 8

✓ - **0 pts** Correct

- **0.5 pts** Wrong sign on one of the terms
- **0.5 pts** No time delay for B in Aprime and Bprime
- **0.5 pts** Wrong/missing exponent
- **1 pts** One wrong/missing term in Aprime/Bprime/Cprime
- **1 pts** Extra term in Aprime/Bprime/Cprime
- **2 pts** Too many delays
- **2 pts** Two wrong terms in Aprime/Bprime/Cprime
- **2 pts** Missing final equations
- **3 pts** Incorrect Aprime equation
- **3 pts** Incorrect Bprime equation
- **2 pts** Incorrect Cprime equation
- **8 pts** Completely Incorrect

2.2 Part b) 1.5 / 2

- **0 pts** Correct
- **1 pts** Insufficient description
- **1 pts** Wrong flow diagram
- **1 pts** No mention of "long loop"
- **1 pts** No mention of "short loop"

✓ - **0.5 pts** **Wrong/incomplete Conclusion**

- **2 pts** Incorrect

QUESTION 3

Eigenvalues and Diagonalization 10 pts

3.1 Part a) 5 / 5

✓ - **0 pts** Correct

- **1 pts** Error in finding linearity
- **0.5 pts** Matrix error
- **1 pts** 1 wrong eigenvalue
- **2 pts** No work for finding linearity
- **5 pts** No work
- **2 pts** 2 wrong eigenvalues
- **1 pts** Calculation mistake

3.2 Part b) 5 / 5

✓ - **0 pts** Correct

- **2.5 pts** Eigenvector for 4 - sqrt(15)
- **2.5 pts** Eigenvector for 4 + sqrt(15)
- **2.5 pts** Calculation errors for both eigenvectors
- **1.25 pts** Calculation error for one eigenvector
- **1 pts** Writing final eigenvectors incorrectly (fractions)
- **1 pts** Writing final eigenvectors incorrectly (numbers/placement in vector)
- **1 pts** Writing final eigenvectors incorrectly (incorrect variable usage)
- **5 pts** No attempt
- **1 pts** Not writing final answer in vector form

QUESTION 4

Linear Discrete-Time Models I 10 pts

4.1 Part a) 4 / 4

✓ - **0 pts** Correct

- **2 pts** Partially incorrect/ incomplete
- **4 pts** No diagram

4.2 Part b) 6 / 6

✓ **- 0 pts** **Correct**
- **2 pts** one to two incorrect values
- **4 pts** three to six incorrect values
- **6 pts** six to nine incorrect values

QUESTION 5

Linear Discrete-Time Models II 10 pts

5.1 Part a) 2 / 2

✓ **- 0 pts** **Correct**
- **1 pts** Partially Correct: Tried to explain
- **2 pts** Incorrect: Stable Node not Specified or explanation incorrect

5.2 Part b) 2 / 2

✓ **- 0 pts** **Correct: Good attempt towards answer**
- **1 pts** Partial Explanation Given or Calculation Done: Some attempt made
- **2 pts** Incorrect

5.3 Part c) 3 / 3

✓ **- 0 pts** **Correct: Good attempt**
- **1 pts** Calculation of Xo only: No explanation
- **2 pts** Incorrect calculation or explanation only
- **3 pts** Incorrect: Explanation and Calculation

5.4 Part d) 3 / 3

✓ **- 0 pts** **Correct: Good Attempt**
- **1 pts** Partial Calculation & Explanation
- **2 pts** Some incorrect or no calculation. Explanation only
- **3 pts** Incorrect

QUESTION 6

6 Linear Differential Equations 10 / 10

✓ **- 0 pts** **Correct**
- **1 pts** Incorrect Matrix
- **2 pts** Incorrect Eigenvalues
- **1 pts** Incorrect Eigenvector
- **1 pts** Incorrect Eigenvector
- **1 pts** Incorrect Graph Setup
- **1 pts** Incorrect Eigenaxis
- **1 pts** Incorrect Eigenaxis
- **1 pts** Incorrect Flow Direction of Axis
- **1 pts** Incorrect Flow Direction of Axis
- **1 pts** Partially Incorrect Eigenvalues

QUESTION 7

Tangent Plane and Linear Approximation 10 pts

7.1 Part a) 2 / 2

✓ **- 0 pts** **Correct**
- **1 pts** Incorrect equation

7.2 Part b) 3 / 3

✓ **- 0 pts** **Correct**
- **1.5 pts** 1 incorrect eq + answer
- **1 pts** No final answer

7.3 Part c) 3 / 3

✓ **- 0 pts** **Correct**
- **1.5 pts** Partially correct equation

7.4 Part d) 2 / 2

✓ **- 0 pts** **Correct**

QUESTION 8

The Jacobian 10 pts

8.1 Part a) 4 / 4

✓ **- 0 pts** **Correct**
- **1 pts** Missing point (3,6)
- **0.5 pts** Incorrect point (0,15)
- **0.5 pts** Incorrect point (3,6)
- **0.5 pts** Incorrect point (12,0)

8.2 Part b) 4 / 4

✓ **- 0 pts** **Correct**
- **1 pts** Incorrect stability for (12,0)
- **4 pts** No answer
- **1 pts** No stability for (3,6)
- **1 pts** Incorrect stability for (0,15)
- **1 pts** Incorrect stability for (3,6)

8.3 Part c) **2 / 2**

✓ **- 0 pts** **Correct**

- 1 pts Insufficient explanation

- 2 pts Incorrect

QUESTION 9

9 1-D Optimization **4 / 4**

✓ **+ 2 pts** **correct critical points {sqrt(2), -sqrt(2), -3/4}**

✓ **+ 2 pts** **correct test {min, min, max}**

+ 0 pts wrong

QUESTION 10

10 2-D Optimization **4 / 4**

✓ **- 0 pts** **All correct**

- 1 pts Failed to show correct critical points: (0,0), (2,0)

- 1 pts Failed to show correct gradients

- 1 pts Failed to show correct Jacobian of the gradient

- 1 pts Failed to analyze each critical point

- 4 pts All incorrect/ Blank page

LS 30B: MATHEMATICS FOR LIFE SCIENTISTS
WINTER 2020 - LECTURES 2 and 3
Jukka Keränen

FINAL EXAM ANSWER SHEET

Your Name Christina Kilkeary

The Last **Six** Digits of Your Student ID number

4	1	6	4	5	4

Your TA Section Section 2A

By signing below, you confirm that you did not cheat on this exam. No exam booklet without a signature will be graded.

Christin Kily

INSTRUCTIONS

- This booklet **only** contains spaces for your answers.
- **The actual questions will be provided on our CCLE site at 6:00 PM (PDT) on Monday 3/16.**
- Please write your answer to each question in the corresponding space in this booklet.
- **You are allowed to use any and all resources, including talking to your classmates.**
- If you have a question at any time between 6:00 PM on Monday and 9:30 PM on Tuesday, please email your TA.
- You will receive points only for work written on the numbered pages. Please use the reverse side as scratch paper.
- Make sure to write legibly. **Illegible work will not be graded.**
- Make sure to show all your work and justify your answers fully.
- When you are done, please **scan this page and the 17 numbered pages (in order)** with your phone
- and upload them as a single document to Gradescope.
- **You must upload your scans to Gradescope by 9:30 PM (PDT) on Tuesday 3/17.**

SCORE

1. ______ 6. ______

2. ______ 7. ______

3. ______ 8. ______

4. ______ 9. ______

5. ______ 10. ______

TOTAL ______

The Last **Six** Digits of Your Student ID number | 4 | 1 | 6 | 4 | 5 | 4 |

1. **Modeling Problem I** (10 pts)

nerds (N) hipsters (H) regular folks (R)

- $N' = -N \cdot \left(\frac{200^n}{200^n + H^n}\right) \cdot (.10)$

  $H' = +N \cdot \left(\frac{200^n}{200^n + H^n}\right) \cdot (.10)$

- $H' = -H \left(\frac{N^m}{500^m + N^m}\right) \cdot (.15)$

  $N' = +H \left(\frac{N^m}{500^m + N^m}\right) \cdot (.15)$

- $H' = -2(.20)H^2 = -.4H^2$

  $R' = +2(.20)H^2 = +.4H^2$

- $R' = -(.15)R \cdot N \cdot H$

  $N' = +(.10)R \cdot N \cdot H$

  $H' = +(.05)R \cdot N \cdot H$

- $N' = -(.03)N \cdot H$

  $H' = -(.03)N \cdot H$

- $N' = +3141$

- $H' = +h \cdot (R/H)$

- $R' = +k \cdot (R \cdot N \cdot H)$

- $N' = -0.3N$

  $R' = -0.25R$

  $H' = -0.2H$

where $m > 1$ and $n > 1$ (since sigmoid)

$$N' = -0.1N\left(\frac{200^n}{200^n + H^n}\right) + .15H\left(\frac{N^m}{500^m + N^m}\right) + .1RNH - 0.03NH + 3141 - 0.3N$$

$$H' = 0.1N\left(\frac{200^n}{200^n + H^n}\right) - .15H\left(\frac{N^m}{500^m + N^m}\right) - .4H^2 + 0.05RNH + h\left(R/H\right) - 0.2H - 0.03NH$$

$$R' = 0.4H^2 - 0.15RNH + k(RNH) - 0.25R$$

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2. Modeling Problem II

three hormones in the body called A, B, and C

a) (8 pts)

$A \rightarrow B$

- $A'(t) = - \quad B(t-0.7) \cdot \left(\frac{A^n(t-0.7)}{h_1^n + A^n(t-0.7)} \right) \cdot (2.5)$

  $B'(t) = + \quad B(t-0.7) \cdot \left(\frac{A^n(t-0.7)}{h_1^n + A^n(t-0.7)} \right) \cdot (2.5)$

- $C'(t) = + \left(\frac{B^m(t)}{5^m + B^m(t)} \right) \cdot (1.5)$

- $A'(t) = + \left(\frac{C(t)}{h_2 + C(t)} \right) \cdot (1)$

- $A'(t) = -k_1 A(t)$

  $B'(t) = -k_2 B(t)$

  $C'(t) = -k_3 C(t)$

where $m > 1$ (since sigmoid)

where $n \geq 1$ (since sigmoid not specified)

$$A'(t) = \left(\frac{C(t)}{h_2 + C(t)} \right) - 2.5 \cdot B(t-0.7) \left(\frac{A^n(t-0.7)}{h_1^n + A^n(t-0.7)} \right) - k_1 A(t)$$

$$B'(t) = 2.5\, B(t-0.7) \left(\frac{A^n(t-0.7)}{h_1^n + A^n(t-0.7)} \right) - k_2 B(t)$$

$$C'(t) = 1.5 \left(\frac{B^m(t)}{5^m + B^m(t)} \right) - k_3 C(t)$$

Modeling Problem II (continued)

b) (2 pts)

Yes, I expect this model to exhibit oscillations beause there is explicit time delay in this loop as well as negative feedback.

the more A you have the more production of B but more the production of B takes from A

Thus, as long as the steepness of the sigmoid functions are sufficiently high, there will be oscillations.

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3. Eigenvalues and Diagonalization

a) (5 pts)

$$f\left(\begin{pmatrix}1\\0\end{pmatrix}\right) = \begin{pmatrix}-1\\5\end{pmatrix} \quad \text{and} \quad f\left(\begin{pmatrix}1\\1\end{pmatrix}\right) = \begin{pmatrix}-3\\14\end{pmatrix}$$

$$f\left(\begin{pmatrix}0\\1\end{pmatrix}\right) = f\left(\begin{pmatrix}1\\1\end{pmatrix}\right) - f\left(\begin{pmatrix}1\\0\end{pmatrix}\right) = \begin{pmatrix}-3\\14\end{pmatrix} - \begin{pmatrix}-1\\5\end{pmatrix} = \begin{pmatrix}-2\\9\end{pmatrix}$$

$$f = \begin{bmatrix}-1 & -2\\ 5 & 9\end{bmatrix}$$

$$\begin{bmatrix}-1-\lambda & -2\\ 5 & 9-\lambda\end{bmatrix} = 0$$

$$(-1-\lambda)(9-\lambda) - (-2)(5) = 0$$

$$\lambda^2 - 8\lambda - 9 + 10 = 0$$

$$\lambda^2 - 8\lambda + 1 = 0$$

$$\lambda = \frac{8 \pm \sqrt{64-4(1)}}{2} = \frac{8 \pm \sqrt{60}}{2} = \frac{8 \pm 2\sqrt{15}}{2} = 4 \pm \sqrt{15}$$

$\boxed{\text{the eigenvalues of } f \text{ are } \lambda = 4 \pm \sqrt{15}}$

b) (5 pts) (the entries of your vectors should not have fractions in them)

$\lambda_1 = 4 + \sqrt{15}$

$Mv = \lambda v$

$(M - \lambda I)v = 0$

$$\left[\begin{pmatrix}-1 & -2\\ 5 & 9\end{pmatrix} - (4+\sqrt{15})\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\right] v = 0$$

$$\left[\begin{pmatrix}-1 & -2\\ 5 & 9\end{pmatrix} - \begin{pmatrix}4+\sqrt{15} & 0\\ 0 & 4+\sqrt{15}\end{pmatrix}\right] v = 0$$

$$\begin{pmatrix}-5-\sqrt{15} & -2\\ 5 & 5-\sqrt{15}\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = 0$$

second row gives equation

$$5x + (5-\sqrt{15})y = 0$$

$$x = \frac{-5+\sqrt{15}}{5} y$$

$$v = \begin{pmatrix}\frac{-5+\sqrt{15}}{5} y\\ y\end{pmatrix}$$

$$\boxed{v_1 = \begin{pmatrix}-5+\sqrt{15}\\ 5\end{pmatrix}}$$

$\lambda_2 = 4 - \sqrt{15}$

$Mv = \lambda v$

$$\begin{bmatrix}-1 & -2\\ 5 & 9\end{bmatrix}\begin{pmatrix}x\\y\end{pmatrix} = (4-\sqrt{15})\begin{pmatrix}x\\y\end{pmatrix}$$

$$\begin{cases}-x - 2y = (4-\sqrt{15})x\\ 5x + 9y = (4-\sqrt{15})y\end{cases}$$

$$\begin{cases}(-5+\sqrt{15})x - 2y = 0\\ 5x + (5+\sqrt{15})y = 0\end{cases}$$

using second row equation

$$5x + (5+\sqrt{15})y = 0$$

$$x = \frac{-5-\sqrt{15}}{5} y$$

$$v = \begin{pmatrix}\frac{-5-\sqrt{15}}{5} y\\ y\end{pmatrix}$$

$$\boxed{v_2 = \begin{pmatrix}-5-\sqrt{15}\\ 5\end{pmatrix}}$$

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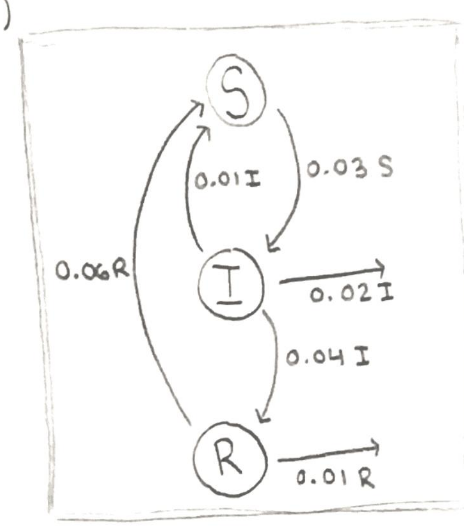
√ 4. **Linear Discrete-Time Models I** (10 pts)

susceptible individuals (S) infected individuals (I) immune individuals (R)

a)



- 0.03 of susceptible become infected
- 0.05(0.8) of infected become immune
  0.05(0.2) of infected become susceptible
- 0.02 of infected die :(
- 0.06 of immune become susceptible
- 0.01 of immune die :(

$$\begin{pmatrix} S_{N+1} \\ I_{N+1} \\ R_{N+1} \end{pmatrix} = \begin{pmatrix} 0.97 & 0.01 & 0.06 \\ 0.03 & 0.93 & 0 \\ 0 & 0.04 & 0.93 \end{pmatrix} \begin{pmatrix} S_N \\ I_N \\ R_N \end{pmatrix}$$

b)

$$\boxed{\begin{pmatrix} 0.97 & 0.01 & 0.06 \\ 0.03 & 0.93 & 0 \\ 0 & 0.04 & 0.93 \end{pmatrix}}$$

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5. Linear Discrete-Time Models II

a) (2 pts)

the stability of the equilibrium at (0,0) is stable
be cause both of the eigenvalues are less than one.

b) (2 pts)

$\lambda_u = 0.8$ has eigenvector $u = \binom{1}{1}$

$\lambda_v = 0.5$ has eigenvector $V = \binom{-1}{2}$

(0,0) being stable means that trajectories will flow towards the equilibrium point and nearby initial conditions will follow those trajectories towards the point (0,0).

we know $\vec{X}_0 = \alpha \cdot \vec{U} + \beta \cdot \vec{V}$ and $X_N = \alpha \, \lambda_{\vec{U}}^N \cdot \vec{U} + \beta \cdot \lambda_{\vec{V}}^N \cdot \vec{V}$

with $\lambda_{\vec{U}} = 0.8 < 1$ and $\lambda_{\vec{V}} = 0.5 < 1$ the more years from now (N) the smaller and smaller those fractions will exponentially become leading to (0,0) and verifying our answer that (0,0) is a stabe equilibrium.

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Linear Discrete-Time Models II (continued)

c) (3 pts)

$$X_0 = 2U + 3V$$

I refuse to analyze the trajectory of this initial population vector because we can't have a negative population.

$$\vec{X}_0 = 2\begin{pmatrix}1\\1\end{pmatrix} + 3\begin{pmatrix}-1\\2\end{pmatrix}$$
$$= \begin{pmatrix}2\\2\end{pmatrix} + \begin{pmatrix}-3\\6\end{pmatrix}$$
$$= \begin{pmatrix}-1\\8\end{pmatrix}$$

d) (3 pts)

$M_1 = -1$ and $U = \begin{pmatrix}1\\1\end{pmatrix}$

$M_2 = 2$ and $V = \begin{pmatrix}-1\\1\end{pmatrix}$

$$M^N \vec{X}_0 = ?$$

$$X_0 = \alpha\vec{U} + \beta\vec{V}$$
$$= \alpha\begin{pmatrix}1\\1\end{pmatrix} + \beta\begin{pmatrix}-1\\1\end{pmatrix}$$
$$= \begin{pmatrix}\alpha\\\alpha\end{pmatrix} + \begin{pmatrix}-\beta\\\beta\end{pmatrix}$$
$$= \begin{pmatrix}\alpha-\beta\\\alpha+\beta\end{pmatrix}$$

Since we are dealing with populations, values must be positive so

$$\begin{cases}\alpha - \beta \geq 0\\ \alpha + \beta \geq 0\end{cases}$$

we can add these inequalities to get

$$2\alpha \geq 0$$

so

$$\boxed{\alpha \geq 0}$$

now use Hint 2 for applying M

$$M(\alpha\vec{U} + \beta\vec{V})$$
$$= \alpha M_1\vec{U} + \beta M_2\vec{V}$$
$$= \alpha(-1)\begin{pmatrix}1\\1\end{pmatrix} + \beta(2)\begin{pmatrix}-1\\1\end{pmatrix}$$
$$= \begin{pmatrix}-\alpha\\-\alpha\end{pmatrix} + \begin{pmatrix}-2\beta\\2\beta\end{pmatrix}$$
$$= \begin{pmatrix}-\alpha-2\beta\\-\alpha+2\beta\end{pmatrix}$$

$$\begin{cases}-\alpha - 2\beta \geq 0\\ -\alpha + 2\beta \geq 0\end{cases}$$

add inequalities

$$-2\alpha \geq 0$$

so

$$\boxed{\alpha \leq 0}$$

since both $\alpha \geq 0$ and $\alpha \leq 0$ are true,

$$\boxed{\alpha = 0}$$

we can plug this in to figure out what it means for β

$$\begin{cases}0 - \beta \geq 0\\ 0 + \beta \geq 0\end{cases}$$

$$\begin{cases}-\beta \geq 0\\ +\beta \geq 0\end{cases}$$

since both these must be true, the only option is

$$\boxed{\beta = 0}$$

6. **Linear Differential Equation Models** (10 pts)

$X' = 7X - 5Y \qquad Y' = 10X - 8Y$

matrix : $\begin{bmatrix} 7 & -5 \\ 10 & -8 \end{bmatrix}$

$\begin{pmatrix} 7-\lambda & -5 \\ 10 & -8-\lambda \end{pmatrix} = \emptyset = (7-\lambda)(-8-\lambda) - (-5)(10)$

$\lambda^2 + \lambda - 56 + 50 = \lambda^2 + \lambda - 6 = (\lambda+3)(\lambda-2) = \emptyset$

eigenvalues: $\lambda = -3 \quad \lambda = 2$

one is positive and one is negative so (0, 0) is a saddle point

$7x - 5y = 0$
$10x - 8y = 0$

$7x = 5y$
$x = 5/7\, y$

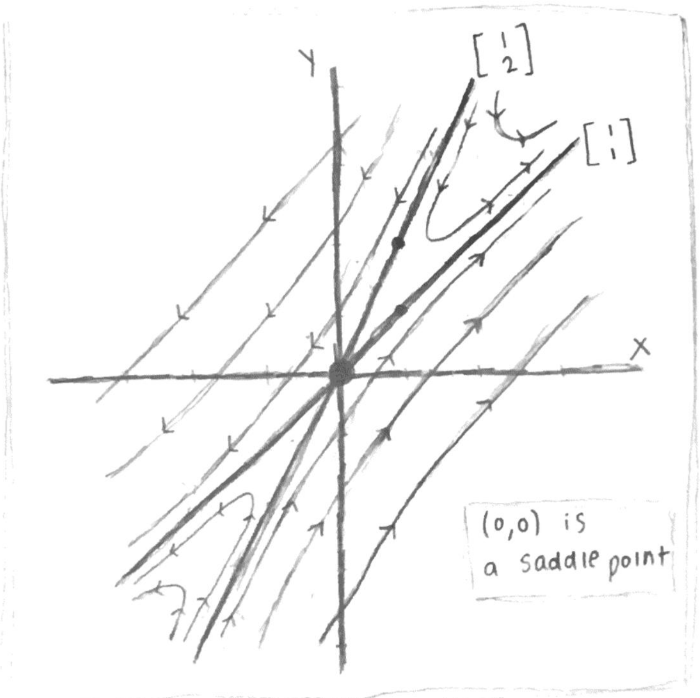
$10(5/7\, y) - 8y = 0$

$\frac{50}{7} y - 8y = 0$

only equilibrium point is (0, 0)

$MX = \lambda X$

plug in $\lambda = -3$	plug in $\lambda = 2$
$7X - 5Y = -3X$	$7x - 5Y = 2X$
$10X - 8Y = -3Y$	$10X - 8Y = 2Y$
$10x - 5Y = 0$	$10x - 10y = 0$
$10x = 5Y$	$10x = 10y$
$Y = 2X$	$x = y$
$(X, 2X)$	(x, y)
$(1, 2)$ ←eigenvectors→	$(1, 1)$



Linear Differential Equation Models (continued)

all work on the first linear differential equation page

7. Tangent Plane and Linear Approximation

a) (2 pts)

$f(x,y) = x^3 - 2x^2y + 3y^3$

at the point $(1, 2, f(1,2))$

$f(1,2) = (1)^3 - 2(1)^2(2) + (3)(2)^3$

$= 1 - 4 + 24$

$\boxed{f(1,2) = 21}$

b) (3 pts)

$\left.\frac{\partial f}{\partial x}\right|_{(1,2)} = 3x^2 - 4xy\Big|_{(1,2)} = 3(1)^2 - 4(1)(2) = 3 - 8 = -5$

$\left.\frac{\partial f}{\partial y}\right|_{(1,2)} = -2x^2 + 9y^2\Big|_{(1,2)} = -2(1)^2 + 9(2)^2 = -2 + 36 = 34$

$$\boxed{\left.\frac{\partial f}{\partial x}\right|_{(1,2)} = -5 \qquad \left.\frac{\partial f}{\partial y}\right|_{(1,2)} = 34}$$

Tangent Plane and Linear Approximation (continued)

c) (3 pts)

$$Z - Z_0 = \frac{\partial f}{\partial x}\Big|_{(x_0, y_0)} \cdot (x - x_0) + \frac{\partial f}{\partial y}\Big|_{(x_0, y_0)} \cdot (y - y_0)$$

$$Z - 21 = (-5)(x-1) + 34(y-2)$$

$$Z - 21 = -5x + 5 + 34y - 68$$

$$z = -5x + 34y - 42$$

$$\boxed{Z = -5x + 34y - 42}$$

d) (2 pts)

use the above part and estimate $f(1.02, 1.98)$

$$f(1.02, 1.98) = -5(1.02) + 34(1.98) - 42$$

$$= -5.1 + 67.32 - 42$$

$$\boxed{f(1.02, 1.98) = 20.22}$$

8. **The Jacobian**

a) (4 pts)

$D' = 24D - 2D^2 - 3DM$

$M' = 15M - M^2 - 3DM$

$24D - 2D^2 - 3DM = 0$

$D(24 - 2D - 3M) = 0$

1a $D = 0$

1b $24 - 2D - 3M = 0$

$15M - M^2 - 3DM = 0$

$M(15 - M - 3D) = 0$

$M = 0$ 2a

$15 - M - 3D = 0$ 2b

1a and 2a

$(0,0)$

1a and 2b

$D = 0$

$15 - M - 3(0) = 0$

$15 = M$

$(0,15)$

1b and 2a

$M = 0$

$24 - 2D - 3(0) = 0$

$24 = 2D$

$D = 12$

$(12,0)$

1b and 2b

$24 - 2D - 3M = 0$

$15 - M - 3D = 0$

$15 - 3D = M$

$24 - 2D - 3(15 - 3D) = 0$

$24 - 2D - 45 + 9D = 0$

$-21 + 7D = 0$

$7D = 21$

$D = 3$

$15 - 3(3) = M$

$M = 6$

$(3,6)$

equilibrium points are

$(0,0)$; $(0,15)$; $(12,0)$; $(3,6)$

The Jacobian (continued)

b) (4 pts) $(0,0)$ $(0,15)$ $(12,0)$ $(3,6)$

Jacobian: $\begin{bmatrix} \partial D'/\partial D & \partial D'/\partial M \\ \partial M'/\partial D & \partial M'/\partial M \end{bmatrix} = \begin{bmatrix} 24-4D-3M & -3D \\ -3M & 15-2M-3D \end{bmatrix}$

equilibrium point $(0,0)$

$J(0,0) = \begin{bmatrix} 24 & 0 \\ 0 & 15 \end{bmatrix}$

diagonal, so eigenvalues are

$\lambda = 24$ and $\lambda = 15$

both positive so

$\boxed{(0,0) \text{ is unstable}}$

equilibrium point $(0,15)$

$J(0,15) = \begin{bmatrix} 24-4(0)-3(15) & -3(0) \\ -3(15) & 15-2(15)-3(0) \end{bmatrix} = \begin{bmatrix} -21 & 0 \\ -45 & -15 \end{bmatrix}$

eigenvalues are

$\lambda = -21$ and $\lambda = -15$

both negative so

$\boxed{(0,15) \text{ is stable}}$

equilibrium point $(12,0)$

$J(12,0) = \begin{bmatrix} 24-4(12) & -3(12) \\ -3(0) & 15-3(12) \end{bmatrix} = \begin{bmatrix} -24 & -36 \\ 0 & -21 \end{bmatrix}$

eigenvalues are

$\lambda = -24$ $\lambda = -21$

both negative so

$\boxed{(12,0) \text{ is stable}}$

equilibrium point $(3,6)$

$J(3,6) = \begin{bmatrix} 24-4(3)-3(6) & -3(3) \\ -3(6) & 15-2(6)-3(3) \end{bmatrix} = \begin{bmatrix} -6 & -9 \\ -18 & -6 \end{bmatrix}$

$(-6-\lambda)(-6-\lambda) - (-18)(-9) = 0$

$\lambda^2 + 12\lambda + 36 - 162 = 0$

$\lambda^2 + 12\lambda - 126 = 0$

$\lambda = -18.73$ $\lambda = 6.73$

one is negative and one is positive so

$\boxed{(3,6) \text{ is a saddle point}}$

The Jacobian (continued)

c) (2 pts)

The two species cannot coexist in the long run.
The only two stable points are no deer and 15 moose and 12 deer and no moose.
The point where it was possible they could exist at 3 deer and 6 moose is unstable and therefore could not happen.

The Last **Six** Digits of Your Student ID number | 4 | 1 | 6 | 4 | 5 | 4 |

9. 1-D Optimization (Extra Credit 4 pts)

$g(x) = x^4 + x^3 - 4x^2 - 6x + 6$

$g'(x) = 4x^3 + 3x^2 - 8x - 6$

$0 = (4x+3)(x^2-2)$

$4x + 3 = 0 \qquad x^2 - 2 = 0$

$4x = -3 \qquad x^2 = 2$

$\boxed{x = -3/4 \qquad x = \sqrt{2} \qquad x = -\sqrt{2}}$

$g''(x) = 12x^2 + 6x - 8$

$g''(-3/4) = 12(-3/4)^2 + 6(-3/4) - 8$

$= 12(9/16) + 6(-3/4) - 8$

$= 108/16 - 18/4 - 8$

$= 108/16 - 72/16 - 128/16$

$= -92/16 = -23/4$

negative so,

$\boxed{x = -3/4 \text{ is a local max}}$

$g''(\sqrt{2}) = 12(\sqrt{2})^2 + 6(\sqrt{2}) - 8$

$= 12(2) + 6\sqrt{2} - 8$

$= 24 + 6\sqrt{2} - 8$

$= 16 + 6\sqrt{2}$

positive so

$\boxed{x = \sqrt{2} \text{ is a local min}}$

$g''(-\sqrt{2}) = 12(-\sqrt{2})^2 + 6(-\sqrt{2}) - 8$

$= 12(2) - 6\sqrt{2} - 8$

$= 24 - 6\sqrt{2} - 8$

$= 16 - 6\sqrt{2}$

≈ 7.52 positive so

$\boxed{x = -\sqrt{2} \text{ is a local min}}$

10. **2-D Optimization (Extra Credit 4 pts)**

$$f(x,y) = (x^2 + y^2)e^{-x}$$
$$= e^{-x}x^2 + e^{-x}y^2$$

$$\frac{\partial f}{\partial x} = \frac{d}{dx}(e^{-x}) \cdot x^2 + \frac{d}{dx}(x^2) \cdot e^{-x} + \frac{d}{dx}(e^{-x}) \cdot y^2$$
$$\frac{\partial f}{\partial x} = -(e^{-x})x^2 + (e^{-x})2x - (e^{-x})y^2$$
$$0 = e^{-x}(-x^2 + 2x - y^2)$$
$e^{-x} \neq 0$ ← no possible x can do this
$$-x^2 + 2x - y^2 = 0$$

$$\frac{\partial f}{\partial y} = 2y(e^{-x})$$
$$0 = 2y(e^{-x})$$
$$y = 0$$

$$-x^2 + 2x - 0^2 = 0$$
$$0 = x^2 - 2x$$
$$= x(x-2)$$
$$x = 0 \quad x = 2$$

critical points:
$(0,0)$ $(2,0)$

$$\nabla f = (-(e^{-x})x^2 + (e^{-x})2x - (e^{-x})y^2,\ 2y(e^{-x}))$$

Jacobian: $\begin{bmatrix} \partial x'/\partial x & \partial x'/\partial y \\ \partial y'/\partial x & \partial y'/\partial y \end{bmatrix}$

$$\frac{\partial x'}{\partial x} = (e^{-x})x^2 - 2x(e^{-x}) - 2x(e^{-x}) + 2(e^{-x}) + (e^{-x})y^2$$
$$= (e^{-x})x^2 - 4x(e^{-x}) + 2(e^{-x}) + y^2(e^{-x})$$
$$\frac{\partial x'}{\partial y} = -2y(e^{-x})$$
$$\frac{\partial y'}{\partial x} = -2y(e^{-x})$$
$$\frac{\partial y'}{\partial y} = 2(e^{-x})$$

Continued on next page

2-D Optimization (continued)

Jacobian: $\begin{bmatrix} e^{-x}(x^2-4x+2+y^2) & -2y(e^{-x}) \\ -2y(e^{-x}) & 2(e^{-x}) \end{bmatrix}$

determing stabilities:

(0,0)

$\begin{bmatrix} e^{-0}(0^2-4(0)+2+0^2) & -2(0)(e^{-0}) \\ -2(0)(e^{-x}) & 2(e^{-0}) \end{bmatrix}$

$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$\lambda_1 = 2 \quad \lambda_2 = 2$

both are positive so

$\boxed{(0,0) \text{ is a local min}}$

(2,0)

$\begin{bmatrix} e^{-2}(2^2-4(2)+2+0^2) & -2(0)(e^{-2}) \\ -2(0)(e^{-2}) & 2(e^{-2}) \end{bmatrix}$

$\begin{bmatrix} -2(e^{-2}) & 0 \\ 0 & 2(e^{-2}) \end{bmatrix}$

$\lambda_1 = -2e^{-2}$ or -0.271

$\lambda_2 = 2e^{-2}$ or 0.271

one is positive and one is negative

$\boxed{(2,0) \text{ is a saddle point}}$