CCLE

Winter 2021 - LIFESCI30B-1 - KERANEN

Started on	Monday, 8 February 2021, 6:30 PM PST
State	Finished
Completed on	Monday, 8 February 2021, 8:06 PM PST
Time taken	1 hour 35 mins
Grade	**73.50** out of 80.00 (**92**%)

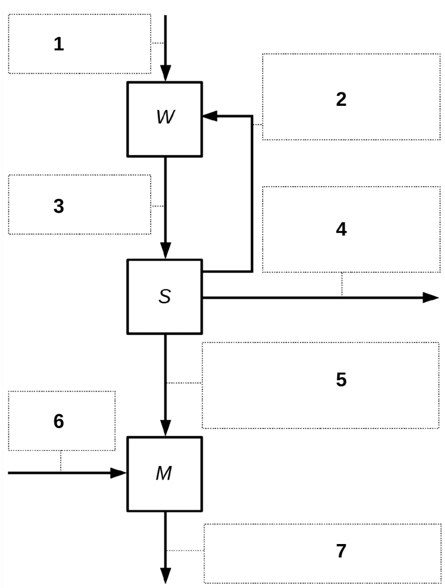
Some of you are already familiar with my favorite company, LSC Foods and Fertilizers, which has three categories of employees: *entry-level workers* (W), *staff supervisors* (S), and *managers* (M). The dynamics of the employee population of LSC Foods and Fertilizers can be modeled by the following assumptions. All rates are per-year rates.

For your convenience, you should begin by writing down the expression that belongs in each numbered box in the flow diagram below.

You will use this information to answer the first 5 questions.

Do not include a negative sign in front of any of your expressions (e.g. write "6W", not "-6W").

- Every employee is either an entry-level worker, a staff supervisor, or a manager.
- Every year, the company hires a fixed number a of entry-level workers.
- Every year, the company hires one new manager (from outside).
- The probability that any one entry-level worker is promoted to a staff supervisor is b.
- The supervisors help each other perform their duties more effectively. Thus, the probability that any one supervisor is promoted to a manager is an increasing sigmoid of the number of supervisors, with a half-saturation density of 50 and a maximum per-capita rate of 0.1. However, due to the excessive and arcane bureaucracy of the company, promotion decisions are made with a time delay of 2 years.
- Every year, some supervisors are demoted back to entry-level workers. The per-capita rate at which supervisors are demoted is a decreasing sigmoid of the number of entry-level workers, with a half-saturation density of 100 and a maximum per-capita rate of 0.2. Again, due to excessive bureaucracy, demotion decisions are made with a time delay of 1 year.
- Every year, some supervisors leave the company due to lack of prospects for a promotion. The per-capita rate at which supervisors leave is a non-sigmoid saturating function of the number of managers, with a half-saturation density of 10 and a maximum per-capita rate of 0.3. There is no time delay here.
- In any one 2-year period, at most one of the following can happen to any one supervisor: they are promoted, they are demoted, or they quit. (If you are not sure what the point of this assumption is, don't worry about. This is just for people who like to think too hard.)
- Every year, d managers retire.



Correct

3.00 points out of 3.00

Select the expression that belongs in **box 2**.

- A. $0.2\left(\frac{100^n}{100^n + W(t-1)^n}\right) S(t)$, where $n > 1$
- B. $0.2\left(\frac{h^n}{h^n + W(t-1)^n}\right) S(t-1)$, where $n > 1$
- C. $0.2\left(\frac{100^n}{100^n + W(t-1)^n}\right) W(t-1)$, where $n > 1$
- D. $0.2\left(\frac{100^n}{100^n + W(t-1)^n}\right) S(t-1)$, where $n \geq 1$
- E. $0.1\left(\frac{S(t-2)^m}{50^m + S(t-2)^m}\right)$, where $m > 1$
- F. $0.2\left(\frac{100^n}{100^n + W(t-1)^n}\right) S(t-1)$, where $n > 1$ ✔
- G. $0.1\left(\frac{S(t-2)^m}{50^m + S(t-2)^m}\right) S(t)$, where $m > 1$
- H. $0.1\left(\frac{S(t-2)^m}{50^m + S(t-2)^m}\right) S(t-2)$, where $m > 1$

The correct answer is:
$0.2\left(\frac{100^n}{100^n + W(t-1)^n}\right) S(t-1)$, where $n > 1$

Question **2**

Correct

3.00 points out of 3.00

Select the expression that belongs in **box 3**.

- A. $bW(t)$ ✔
- B. b
- C. $bW(t)^2$

The correct answer is:
$bW(t)$

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Correct

3.00 points out of 3.00

Select the expression that belongs in **box 4**.

A. $0.3\left(\frac{M(t)}{10+M(t)}\right)M(t)$

B. $0.3\left(\frac{S(t)}{10+S(t)}\right)S(t)$

C. $0.3\left(\frac{M(t)}{10+M(t)}\right)S(t)$ ✔

D. $0.3\left(\frac{S(t)}{10+S(t)}\right)M(t)$

The correct answer is:
$0.3\left(\frac{M(t)}{10+M(t)}\right)S(t)$

Question **4**

Correct

3.00 points out of 3.00

Select the expression that belongs in **box 5**.

A. $0.1\left(\frac{S(t-2)^m}{50^m+S(t-2)^m}\right)$, where $m > 1$

B. $0.2\left(\frac{100^n}{100^n+W(t-1)^n}\right)S(t-1)$, where $n \geq 1$

C. $0.2\left(\frac{h^n}{h^n+W(t-1)^n}\right)S(t-1)$, where $n > 1$

D. $0.1\left(\frac{S(t-2)^m}{50^m+S(t-2)^m}\right)S(t-2)$, where $m > 1$ ✔

E. $0.2\left(\frac{100^n}{100^n+W(t-1)^n}\right)S(t-1)$, where $n > 1$

F. $0.2\left(\frac{100^n}{100^n+W(t-1)^n}\right)S(t)$, where $n > 1$

G. $0.1\left(\frac{S(t-2)^m}{50^m+S(t-2)^m}\right)S(t)$, where $m > 1$

H. $0.2\left(\frac{100^n}{100^n+W(t-1)^n}\right)W(t-1)$, where $n > 1$

The correct answer is:
$0.1\left(\frac{S(t-2)^m}{50^m+S(t-2)^m}\right)S(t-2)$, where $m > 1$

Correct

3.00 points out of 3.00

Suppose that

- there are currently **100 supervisors** and **10 managers**, and
- that the overall number of employees **does not change** this year.

Which one of the following is true?

(*Hint*. What are the inflows and outflows from the employee population?)

A. The number of workers hired is the same as the number of managers who leave.

B. The number of workers hired is less than the number of managers who leave.

C. The number of workers hired is greater than the number of managers who leave. ✔

The correct answer is: The number of workers hired is greater than the number of managers who leave.

Partially correct

7.00 points out of 9.00

Female *dragons* go through 3 life stages: *egg* (E), *young* (Y), and ancient *wyrm* (W).

Below, all eggs, young adults, and wyrms are female.

Each year, on average,

- every young female lays 1 egg;
- every ancient female lays 0.1 eggs;
- the incubation period of an egg is 20 years and hence, on average 5% of the eggs hatch;
- 15% of the eggs are destroyed by "brave" adventurers;
- after hatching, dragons remain young for 100 years and hence, on average, 1% of the young ones become ancient wyrms;
- through magic, every 1 in 10 young females is converted back into an egg;
- through magic, every 1 in 5 ancient wyrms is converted back into a young one;
- young and ancient dragons never die.

E_{N+1} =

0.8 ✔ $E_N +$

1.1 ✔ $Y_N +$

0.1 ✔ W_N

Y_{N+1} =

0.05 ✔ $E_N +$

0.98 ✖ $Y_N +$

0.2 ✔ W_N

W_{N+1} =

0 ✔ $E_N +$

0.01 ✔ $Y_N +$

0.7 ✖ W_N

Correct

2.00 points out of 2.00

A single-variable time-delay differential equation cannot oscillate.

(*Hint*. What examples do you know of this kind of differential equation?)

Select one:

- True
- False ✔

The correct answer is 'False'.

Question **8**

Correct

2.00 points out of 2.00

If a model oscillates, then its state space has a limit cycle attractor.

(*Hint*. What are some of the models that oscillate?)

Select one:

- True
- False ✔

The correct answer is 'False'.

Question **9**

Correct

2.00 points out of 2.00

A trajectory that approaches a limit cycle attractor will eventually repeat itself exactly.

(*Hint*. An LCA is an *attractor*.)

Select one:

- True
- False ✔

The correct answer is 'False'.

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Correct

2.00 points out of 2.00

A single-variable differential equation **without** time delay cannot oscillate.

(*Hint*. What would a phase portrait here look like?)

Select one:

- True ✔
- False

The correct answer is 'True'.

Question **11**

Partially correct

3.50 points out of 7.00

Select all statements that are correct.

(*Please note*: there is a penalty for choosing any statement that is incorrect.)

- A. A Hopf bifurcation will involve the creation or destruction of an LCA.
- B. This choice was deleted after the attempt was started. ✖
- C. If there is only one equilibrium point inside an LCA in a model with 2 state variables, it must be an unstable spiral. ✔
- D. A Hopf bifurcation can only happen when **both** negative feedback and time delay in the system change.

The correct answers are:
A Hopf bifurcation will involve the creation or destruction of an LCA.,

If there is only one equilibrium point inside an LCA in a model with 2 state variables, it must be an unstable spiral.

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Partially correct

14.00 points out of 15.00

Grandpa Rick has done goofed! He's forgotten the value at time $t = 1.3$ of a super-important state variable $D(t)$. Luckily, Rick knows that

- for all $t \leq 1$, $D(t) = 5t$.
- for all $t \geq 1$, the dynamics of $D(t)$ can be modeled with

$$D'(t) = 4D(t - 0.2) - 4D^2(t - 0.1)$$

Use Euler's method with a step size of 0.1 to approximate $D(1.3)$ and help Rick ~~destroy~~ save the universe!

Rick *loves* precision, so you should not round your numbers.

Please pay careful attention to the label of each column below!

Please note: The column labels may not line up perfectly with the columns on your device. Their order, however, will be correct.

t	$D(t-0.2)$	$D(t-0.1)$	$D(t)$	$D'(t)$	$D(t+0.1)$
1.0	4 ✔	4.5 ✔	5 ✔	-65 ✔	-1.5 ✔
1.1	4.5 ✔	5 ✔	-1.5 ✔	-100 ✘	-3.4 ✘
1.2	5 ✔				

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✖

-3.4

✖

-101

✖

-13.5

✖

Question 13

Correct

6.00 points out of 6.00

Suppose that

$$\mathbf{U} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}, \mathbf{V} = \begin{pmatrix} -3 \\ 3 \end{pmatrix}$$

Find real numbers a and b such that

$$a \cdot \mathbf{U} + b \cdot \mathbf{V} = \begin{pmatrix} 11 \\ 17 \end{pmatrix}$$

a =

4

✔ , b =

3

✔

Correct

4.00 points out of 4.00

Suppose that $f : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear function such that

$$f(\begin{pmatrix} 22 \\ 34 \end{pmatrix}) = \begin{pmatrix} 6 \\ 8 \end{pmatrix},$$

and also suppose that

$$f(\mathbf{U}) = \begin{pmatrix} 6 \\ c \end{pmatrix}, f(\mathbf{V}) = \begin{pmatrix} d \\ -4 \end{pmatrix}.$$

By using your answer to Question 13 above, find c and d.

c =

4

✔ , d =

-7

✔

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Correct

16.00 points out of 16.00

Suppose that $f : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear function such that

$$f(\begin{pmatrix} 1 \\ 0 \end{pmatrix}) = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \text{ and } f(\begin{pmatrix} 1 \\ 1 \end{pmatrix}) = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$$

and suppose that g is defined by

$$g(\begin{pmatrix} X \\ Y \end{pmatrix}) = \begin{pmatrix} 3X + 4Y \\ 5X + 6Y \end{pmatrix}$$

Find the matrix of the function f:

4 ✔

1 ✔

4 ✔

1 ✔

Find the matrix of the function g:

3 ✔

4 ✔

5 ✔

6 ✔

Find the matrix of the function $f \circ g$:

17 ✔

22 ✔

17 ✔

22 ✔

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28

7

44

11

◀ Midterm Stage 3

Jump to...

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