S Venugopal LS 30A Lecture 3 (Fall 2016) Nov 4, 2016

Midterm Exam

Version A

Last Name: _____

First Name: _____

Student ID: _____

Signature: _____

Lab Sections:

3A (M 8:00A-9:50A)	3B (M 10:00A-11:50A)
3C (M 12:00P-1:50P)	3D (M 2:00P-3:50P)
3E (M 4:00P-5:50P)	3F (R 10:00A-11:50A)
3G (R 12:00P-1:50P)	3H (R 2:00P-3:50P)
3I (R 4:00P-5:50P)	3J (F 8:00A-9:50A)
3K (F 10:00A-11:50A)	3L (F 12:00P-1:50P)

Instructions: Do not open this exam until instructed to do so. You will have 1 hour and 50 minutes to complete the exam. Please print your name and student ID number above, and circle your lab section. You may not use books, notes, or any other material to help you. You may use a calculator. Please make sure **your cellphone is silenced** and stowed where you cannot see it. You may use any available space on the exam for scratch work, including the backs of the pages. If you need more scratch paper, please ask one of the proctors.

Please do not write below this line

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total	60	

1. [10 points] In 1665, the town of Eyam in England suffered an outbreak of bubonic plague. The town's records are sufficiently detailed to allow us to parametrize a model of plague dynamics. Use the following assumptions to write a model of this system:
   - Individuals are either susceptible (S), infected (I) or recovered (R).
   - Over the time scale of the model, births and deaths of healthy (susceptible or recovered) individuals are negligible and may be considered to be zero.
   - The per-capita infection rate of susceptible individuals was proportional to the number of infected individuals with a proportionality constant of 0.1203 per day.
   - Infected individuals recovered at a per-capita rate of 0.055 per day.
   - The per-capita death rate for infected individuals was 0.033 per day.
     Recovered individuals lost immunity at a per-capita rate of 0.0045 per day.

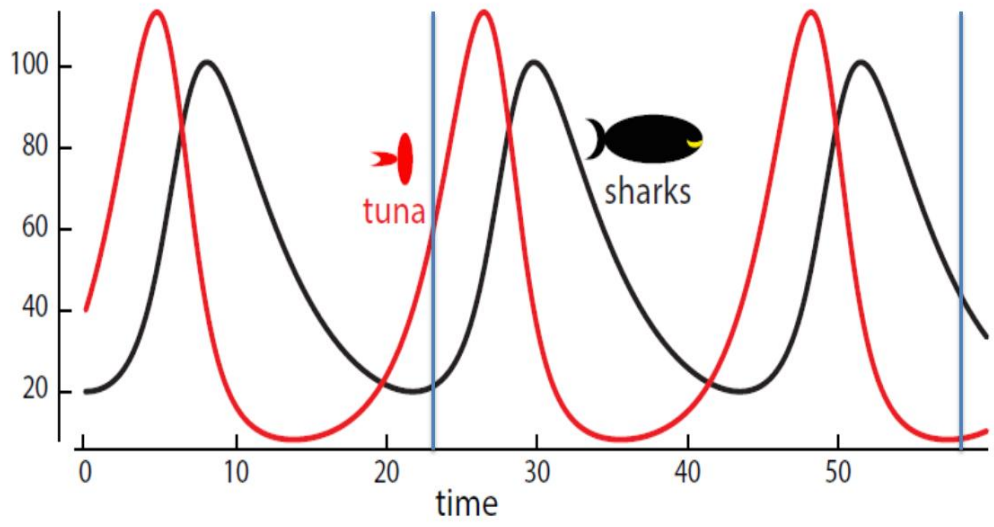
2. Say, you have brought in a cup of hot coffee to your midterm exam to help keep you awake as you had no sleep last night! But it turns out that Dr. V gave you a very lengthy exam and you had no time for even a sip!! The ambient room temperature is 25^0 Celsius.
   a. [1 point] Sketch a possible curve to show how the temperature of your coffee changed during the 2-hour midterm exam. Make sure you label your axes. Indicate the initial and final temperature on the graph.

   b. [3 points] Starting with average rate of change in coffee temperature between two times t_1 and t_2, explain the meaning of derivative at t_1.

c. [3 points] Using your graph above, describe the *geometric* meaning of derivative at t_1. Make sure you show this on your graph.

d. [3 points] If you were given the function $f(t) = 25+t^2$, explain how you would compute the derivative?

3. Briefly answer the following.
   e. [5 points] The following figure illustrates the time series for a Shark-Tuna predator-prey model. Briefly explain the dynamic processes occurring between the two vertical lines (blue).

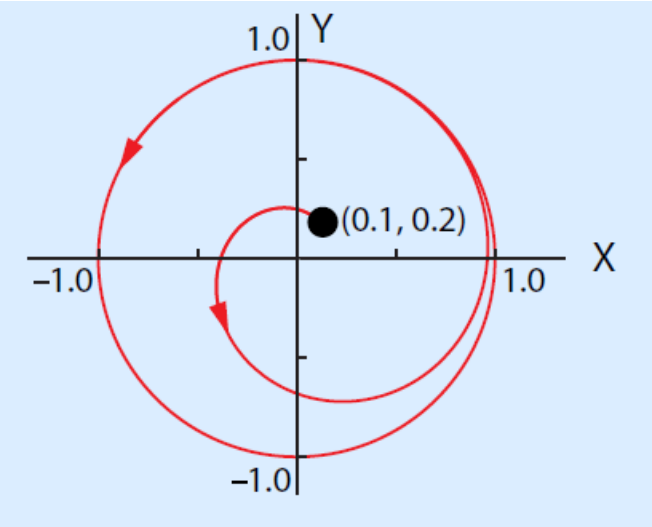


   f. [2.5 points] Draw a diagram to show the type of feedback (positive/negative) in this system. (No equations necessary)

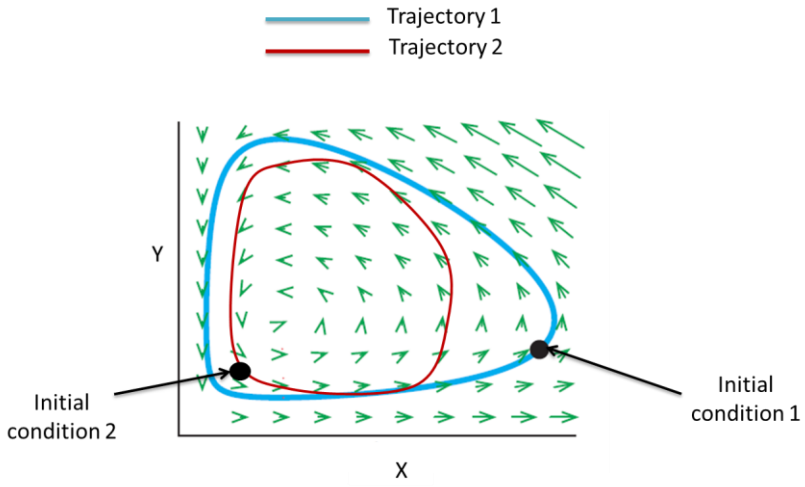
g. [2.5 points] Draw the state-space and sketch an approximate trajectory for the time series shown.

4. [10 points] Briefly explain the following.
   a. [4 points] Are the trajectories in the examples below valid? Explain why or why not?

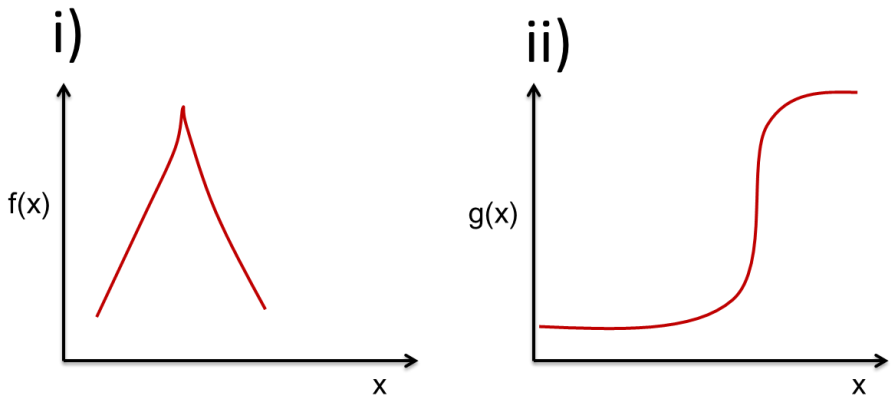
i)



ii)



b. [4 points] Briefly explain why the following functions are not differentiable.



c. [2 points] What makes a model deterministic?

5. [10 points] Romeo and Juliet are in a relationship. Romeo's love (or, if negative, hate) for Juliet is R and Juliet's love or hate for Romeo is J. The model describing this system is

$$R' = 0.5R + J$$
$$J' = R + 0.5J$$

a. [8 points] Sketch the vector field for this system (about 8 well-chosen change vectors should suffice).

b. [2 points] What are the possible long-term outcomes for this relationship?

6. [10 points] A disease is spreading in a population. A model for the number of susceptible individuals (S) and infected individuals (I) using the differential equations:

$$S' = 0.2I - 0.05SI$$
$$I' = -0.2I + 0.05SI$$

   a. [8 points] Suppose we start with 100 susceptible individuals and 5 infected ones. Use Euler's method with a step size of 0.1 weeks to determine the approximate numbers of susceptible and infected individuals at t = 0.2 weeks.

   b. [2 points] Briefly explain the advantages and disadvantages of using a very small step size in Euler's method.