19F-LIFESCI30A-2 MIDTERM

ZOE GLEASON

TOTAL POINTS

57 / 60

QUESTION 1

1 Chemical Reactions 10 / 10

✓ **- 0 pts Correct**

- 1 pts You must multiply by the number of molecules lost in the collision. In the top reaction, 2 molecules each of H and C are lost, so the terms with 0.02 must be multiplied by 2.

- 1 pts You must multiply the terms by the rates at which the reaction happens, given by the numbers on top of the arrows.

- 1 pts Inflows should be positive and outflows negative. The left hand side of each reaction gives outflows to both H and C, while the right hand side tells you that both reactions are inflows to C (at least one of your terms has the wrong signal)

- 2 pts The rate of formation of C molecules (right hand side) is given by collisions (left hand side)

- 1 pts Molecules colliding means you must multiply the stocks of each molecule, giving you H^2*C^2 for the first equation (times the appropriate rates)

- 1 pts zMolecules colliding means you must multiply the stocks of each molecule, giving you H*C for the second equation (times the appropriate rates)

- 5 pts You must have one differential equation per type of molecule (ie H' and C').

- 10 pts You should have: one change equation per molecule (H' and C'). Inflows are molecules being formed, outflows are molecules being consumed. Parameters are given by the reaction rates (0.02 and 0.06) and the amount of molecules lost/gained. Collisions means you multiply each stock, so for example H+H+C+C gives you H^2 and C^2

- 1 pts On the second reaction there is no net change to the amount of C. Your equation does not reflect that

- 3 pts Only one equation per type of molecule, including flows from both reactions

- 1 pts Coefficient of H2C2 in the Cprime should be 0.02

QUESTION 2

2 Nerds, Hipsters, Comic Books 8.5 / 10

- 0 pts Correct

- 1 pts Incorrect rate of nerds moving into town.

Did not make the rate of nerds moving in proportional to N *and/or* C. OR incorrectly multiplied 0.03NC with N. (should be 0.03NC)
(assumption 3)

- 1 pts \-0.04C, -0.04N^2, 0.04

for nerds moving out of town. (assumption 4)

- 0.5 pts incorrect term for rate at which new comic book stores are opened.

Most likely incorrectly multiplied 0.01N by C (assumption 1)

- 1.5 pts Incorrect term for rate of comic book stores going out of business (should be 0.02H^2C^2) (assumption 2)

✓ **- 0.5 pts Partially incorrect term for rate of comic book stores going out of business (should be 0.02H^2C^2).**

has correct per-store rate of comic book stores going out of business (-0.2H^2C) ...but did not multiply by C to get the total rate of comic book stores going out of business. (assumption 2)

✓ **- 1 pts Incorrect term for hipsters becoming nerds. Should be 0.06HC (gets credit if you have +0.06HC in either N' or -0.06HC H') (assumption 6)**

- **1 pts** did not match the terms in outflow from H' to inflow in N'. Can get points as long as the term that you used for assumption 6 matches in both N' and C'. (assumption 6)
- **1 pts** incorrect term for the rate of hipsters moving into town. May have used (1-C) for inverse of 1/C or multiplied by H to get 0.05*H/C. Should be 0.05/C. (assumption 5)
- **1 pts** some incorrect constants (see circled)
- **1 pts** has extra unnecessary terms/missing terms (see boxed)
- **3 pts** did not write down a complete set of change equations (merged all equations, etc)
- **2 pts** switching state variables

QUESTION 3

Company Dynamics 10 pts

3.1 Flow Chart 4 / 4

✓ **- 0 pts Correct**
- **0.5 pts** Partially correct labels
- **1 pts** Incorrect stocks
- **1 pts** Incorrect labels
- **2 pts** Incorrect arrow
- **0.5 pts** Incorrect / Missing arrows
- **1 pts** Flowchart structure
- **4 pts** Not attempted
- **1 pts** Incorrect / Missing arrows

3.2 Equations 4.5 / 5

- **0 pts** Correct
- **0.5 pts** Incorrect W' > a
- **0.5 pts** Incorrect W' > mS/W
- **0.5 pts** Incorrect W' > bW
- **0.5 pts** Incorrect S' > bW

✓ **- 0.5 pts Incorrect S' > mS/W**
- **0.5 pts** Incorrect S' > kM
- **0.5 pts** Incorrect S' > cS^2
- **0.5 pts** Incorrect M' > cS^2
- **0.5 pts** Incorrect M' > 1
- **0.5 pts** Incorrect M' > d
- **5 pts** Equations not written

3.3 Equilibrium 0 / 1

✓ **- 0 pts Correct or partially correct**

✓ **- 1 pts Incorrect or not attempted**

QUESTION 4

Vector Field 10 pts

4.1 Definition 2 / 2

✓ **+ 1 pts Vector field should be a function with domain and codomain.**

✓ **+ 0.5 pts Domain is the state space.**

✓ **+ 0.5 pts Codomain is the tangent space.**

4.2 Numbers 4 / 4

✓ **+ 0.5 pts Vector for (2,2) is (-0.6, 3.6)**

✓ **+ 0.5 pts Vector for (-2,2) is (2.6, -0.4)**

✓ **+ 0.5 pts Vector for (-2,-2) is (-2.6, 3.6)**

✓ **+ 0.5 pts Vector for (2,-2) is (0.6, -0.4)**

✓ **+ 0.5 pts Vector for (1,0) is (0, 0.4)**

✓ **+ 0.5 pts Vector for (0,1) is (0.5, 0)**

✓ **+ 0.5 pts Vector for (-1,0) is (0, 0.4)**

✓ **+ 0.5 pts Vector for (0,-1) is (-0.5, 0)**

+ **0 pts** No correct process

4.3 Graph 4 / 4

✓ **+ 0.5 pts Vector for (2,2)**

✓ **+ 0.5 pts Vector for (-2,2)**

✓ **+ 0.5 pts Vector for (-2,-2)**

✓ **+ 0.5 pts Vector for (2,-2)**

✓ **+ 0.5 pts Vector for (1,0)**

✓ **+ 0.5 pts Vector for (0,1)**

✓ **+ 0.5 pts Vector for (-1,0)**

✓ **+ 0.5 pts Vector for (0,-1)**

+ **0 pts** Click here to replace this description.

QUESTION 5

Euler's Method 10 pts

5.1 Definition 2 / 2

✓ **- 0 pts Correct**
- **0.5 pts** Incorrect symbols used
- **1 pts** Missing symbols in the formula

- **2 pts** Incorrect/missing answer

5.2 Calculation **8 / 8**

✓ **- 0 pts Correct**

- **1 pts** Incorrect change for S1: 29.5
- **0.5 pts** Incorrect S1 but correct S1': 16.8
- **1 pts** Incorrect change for S2: 101.136
- **0.5 pts** Incorrect S2 but correct S2': 57.254
- **1 pts** Incorrect change for T1: 15
- **0.5 pts** Incorrect T1 but correct T1': 306
- **1 pts** Incorrect change for T2: -20.808
- **0.5 pts** Incorrect T2 but correct T2': 297.6768
- **0.5 pts** Incorrect Final S: 57
- **0.5 pts** Incorrect Final T: 298
- **8 pts** No answer
- **1 pts** S2 incorrect due to incorrect S1
- **1 pts** T2 incorrect due to incorrect T1

QUESTION 6

Differentiation 10 pts

6.1 Definition of Derivative **1 / 1**

✓ **- 0 pts Correct**

- **0.5 pts** either limit part or difference part is incorrect.
- **0 pts** evaluate at a
- **1 pts** both limit part and difference part are incorrect

6.2 Connecting Average and Instantaneous **1 / 1**

✓ **- 0 pts Correct**

- **1 pts** no explanation
- **1 pts** no explanation of relationship between average and instantaneous

6.3 Tangent Line **2 / 2**

✓ **- 0 pts Correct**

- **0.5 pts** no independent variable, t
- **2 pts** no response
- **1 pts** incorrect tangent line formula
- **1 pts** slope is right but other parts are missing
- **0.5 pts** no X=
- **0.5 pts** what's m and what's a
- **0 pts** +X(a) should be + g(a)
- **0.5 pts** it's (t-a) not (a-t)
- **0.5 pts** your slope should be evaluated at t = a

6.4 Differentiation from Definition **3 / 3**

✓ **- 0 pts Correct**

- **2 pts** incorrect substitution/expansion of t+deltat
- **1 pts** incorrect cancellation of deltat
- **3 pts** no response
- **1 pts** evaluate at t=2
- **1 pts** evaluate at delta_t = 0
- **2 pts** use definition instead of approximation to find derivative

6.5 Linear Approximation **3 / 3**

✓ **- 0 pts Correct**

- **1 pts** incorrect linear approximation formula
- **2 pts** must use linear approximation
- **3 pts** no response
- **1 pts** incorrect variable substitution
- **1 pts** incorrect final answer

LS 30A: MATHEMATICS FOR LIFE SCIENTISTS
FALL 2019 - LECTURES 2 and 3
Jukka Keränen

MIDTERM EXAMINATION

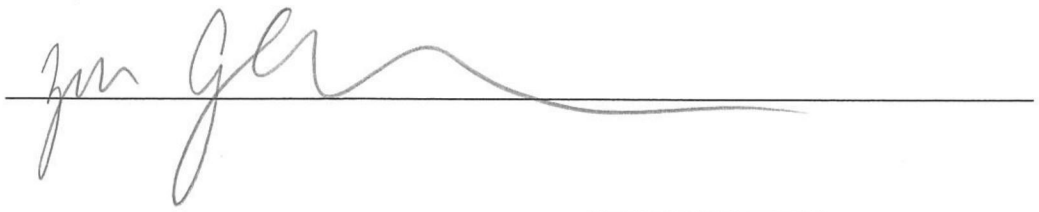
Your Name Zoe Gleason

Last Six Digits of Your Student ID number

3	0	8	2	8	1

Your TA Section 2N

By signing below, you confirm that you did not cheat on this exam. No exam booklet without a signature will be graded.



INSTRUCTIONS

- Please do **not** open this booklet until you are told to do so.
- In addition to basic writing instruments, you are allowed to use a **non-programmable** calculator.
- Your cell phone must be **turned off completely** and stowed away where you cannot see it.
- No books or notes.
- If you have a question at any time during the exam, please raise your hand.
- You will receive points only for work written on the numbered pages. Please use the reverse side as scratch paper.
- Make sure to write legibly. **Illegible work will not be graded.**
- Make sure to show all your work and justify your answers fully.
- **Please write the last six digits of your student ID number at the top of each page.**

SCORE

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

TOTAL _____

The Last **Six** Digits of Your Student ID number

3	0	8	2	8	1

1. Consider a solution with two different types of molecules in it, call them C and H. There are two types of chemical reactions taking place in the solution, described by the following reaction equations:

$$H + H + C + C \xrightarrow{0.02} C$$
$$H + C \xrightarrow{0.06} C$$

Recall that a reaction equation is to be interpreted as follows:

- In order for the reaction to take place, the molecules indicated on the left-hand side must collide with each other.
- Upon each such collision, there is a fixed probability, indicated by the number on top of the arrow, that the reaction will actually take place.
- All the molecules indicated on the left-hand side are consumed in the reaction.
- The reaction produces as output the molecules indicated on the right-hand side.

(10 pts) Write down the change equations for H and C.

$H' = (-2)(0.02H^2C^2) + (-1)(0.06HC)$

$H' = -2(0.02H^2C^2) - 0.06HC$

$C' = (-2+1)(0.02H^2C^2) + (-1+1)(0.06HC)$

$C' = -0.02H^2C^2 + 0(0.06HC)$

$H' = -2(0.02H^2C^2) - 0.06HC$

$C' = -0.02H^2C^2$

The Last **Six** Digits of Your Student ID number | 3 | 0 | 8 | 2 | 8 | 1 |

2. (10 pts) In a certain town, there lives a population of N nerds. The nerds patronize comic book stores, of which there are C many in this town. The nerds are harassed by mean hipsters, of which there are H. Write down a differential equation model of the nerd-hipster-comic book store system, by using the following assumptions. All rates are per-year rates.

- Since comic book stores rely on nerds for business, the rate at which new comic book stores are opened is proportional to the number of nerds, with a proportionality constant of 0.01.
- Pairs of hipsters amuse themselves by making fun of comic book stores, negatively affecting the image of the stores. So, comic book stores go out of business at a per store rate proportional to the likelihood of two hipsters running into each other in front of a store, with a proportionality constant of 0.02.
- Nerds enjoy living in a town with many other nerds and many comic book stores, so nerds move into town at a rate proportional to the number of nerds times the number of comic book stores, with a proportionality constant of 0.03.
- Nerds move out of town due to being hired by Google at a per capita rate of 0.04 per year.
- Hipsters like to live in a town with few comic book stores, so hipsters move into town at a rate proportional to the inverse of the number of comic book stores, with a proportionality constant of 0.05.
- Every once in a while, a hipster walks into a comic book store by chance, likes what they find in there, and becomes a nerd. The probability of a hipster liking what they find in a comic book store is 0.06.

- $C' = +0.01N$
- $\frac{C'}{C} = -0.02H^2 \rightarrow C' = -0.02H^2C$
- $N' = +0.03NC$
- $\frac{N'}{N} = -0.04 \rightarrow N' = -0.04N$
- $H' = +0.05\left(\frac{1}{C}\right)$
- $H' = -0.06H$

$N' = +0.06H$

$C' = 0.01N - 0.02H^2C$

$N' = 0.03NC - 0.04N + 0.06H$

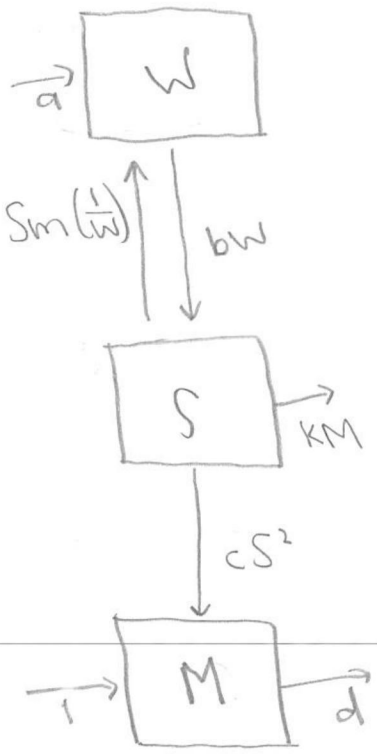
$H' = 0.05\left(\frac{1}{C}\right) - 0.06H$

The Last **Six** Digits of Your Student ID number

3	0	8	2	8	1

3. *a*) (4 pts) A certain company, LSC Foods and Fertilizers, has three categories of employees: entry-level workers W, staff supervisors S, and managers M. Draw a flow chart of the employee population of this company, by using the following assumptions. All rates are per-year rates.

- Every employee is either an entry-level worker, a staff supervisor, or a manager.
- Every year, the company hires a fixed number of new entry-level workers, a.
- Every year, the company hires one new manager (from outside).
- The probability that any one entry-level worker is promoted to a staff supervisor is b.
- The supervisors help each other perform their duties more effectively. Thus, the probability that any one supervisor is promoted to a manager is proportional to the number of supervisors, with a proportionality constant of c. $S' = cS^2$
- Every year, some supervisors are demoted back to entry-level workers. The per-capita rate at which supervisors are demoted is proportional to the inverse of the number of entry-level workers, with a proportionality constant of m. $\frac{S'}{S} = m\left(\frac{1}{W}\right)$
- Every year, some supervisors leave the company due to lack of prospects for a promotion. The number of supervisors who leave is proportional to the number of managers, with a proportionality constant of k. $S' = KM$
- Every year, d managers retire.



b) (5 pts) Write down a differential equation model of the employee population of LSC Foods and Fertilizers, by using the assumptions on the previous page.

$$W' = a + Sm\left(\frac{1}{W}\right) - bW$$

$$S' = bW - kM - cS^2$$

$$M' = l + cS^2 - d$$

c) (1 pts) By using the notation from the first two parts of this problem, write down an equation representing the condition that **the combined number of entry-level workers and staff supervisors is neither growing nor declining.**

$$a + Sm\left(\frac{1}{W}\right) - bW = bW - kM - cS^2$$

$$0 = a + Sm\left(\frac{1}{W}\right) - bW - bW + kM + cS^2$$

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4. *a*) (2 pts) By using the concepts we have learned in this course, state the definition of a *vector field.*

(*Hint.* Your answer should include the word "function".)

A vector field is a function, defined by the change equations of a model, from the state space into the tangent space of that model.

b) (4 pts) Let R be Romeo's love for Juliet (or hate if negative), and let J be Juliet's love for her Romeo (or hate if negative). Assume that the Romeo-Juliet system is modeled by

$$\begin{cases} R' = 0.5J - 0.4RJ \\ J' = 0.5RJ + 0.4R^2 \end{cases}$$

Compute the vectors from the vector field of this model at the following eight points (R, J):

(R,J)

$(2,2), (-2,2), (-2,-2), (2,-2)$

and

(R,J)

$(1,0), (0,1), (-1,0), (0,-1)$

$(R,J) \to (R',J')$

$(2,2) \to (-0.6, 3.6)$

$(-2,2) \to (2.6, -0.4)$

$(-2,-2) \to (-2.6, 3.6)$

$(2,-2) \to (0.6, -0.4)$

$(R,J) \to (R',J')$

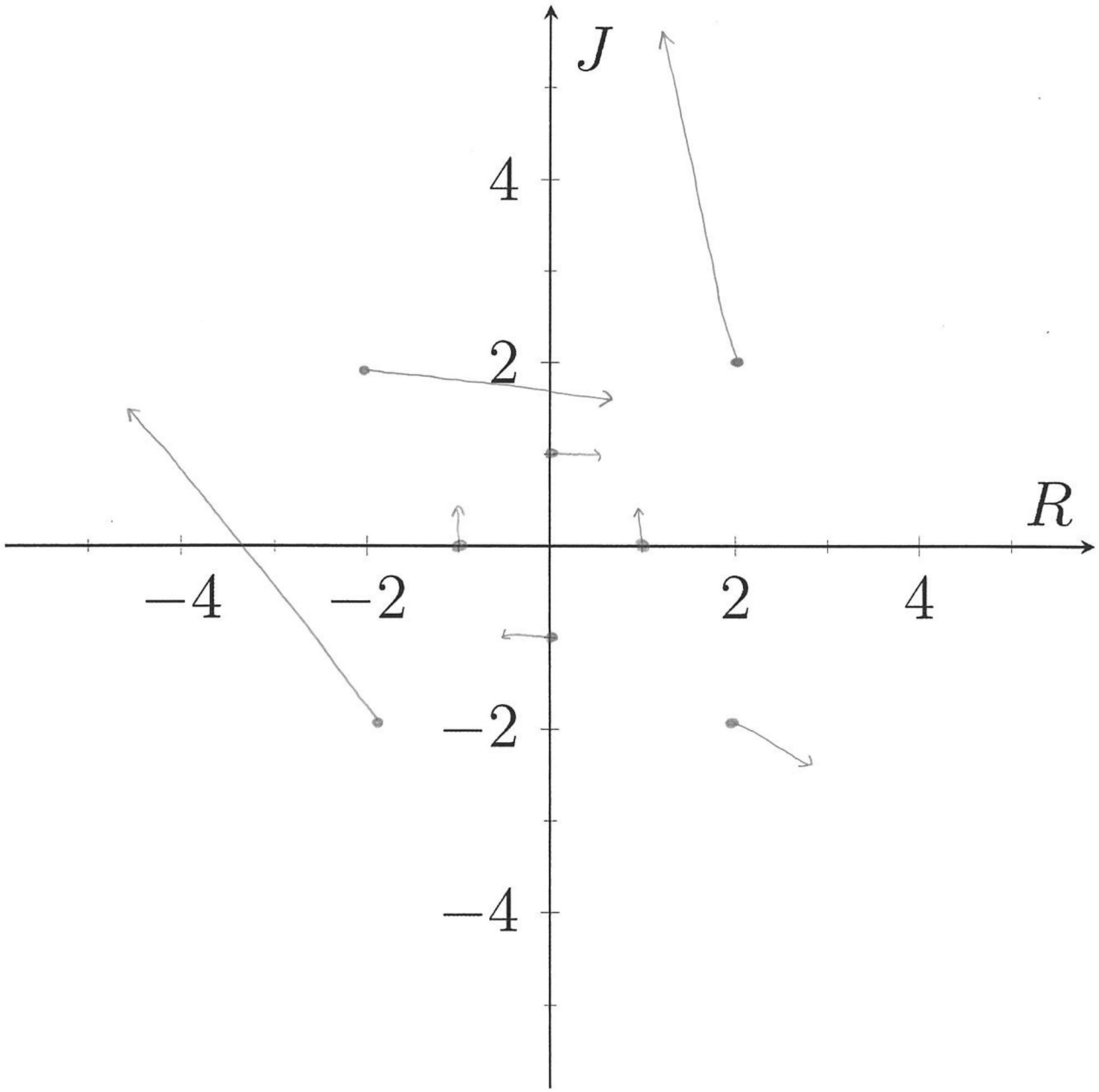
$(1,0) \to (0, 0.4)$

$(0,1) \to (0.5, 0)$

$(-1,0) \to (0, 0.4)$

$(0,-1) \to (-0.5, 0)$

c) (4 pts) Draw the vectors from part *b*) in the diagram below.



The Last **Six** Digits of Your Student ID number

3	0	8	2	8	1

5. *a*) (2 pts) State the formula for Euler's method in the case there are two state variables, call them S and T, by using the following notation:

S_n = the approximate value of S at step n in the method

T_n = the approximate value of T at step n in the method

$$S_{n+1} = S_n + S'(S_n, T_n) \cdot \Delta t$$

$$T_{n+1} = T_n + T'(S_n, T_n) \cdot \Delta t$$

b) (8 pts) LSC Foods and Fertilizers has hired you to predict the future of shark and tuna populations. Let's model the number of sharks (S) and tuna (T) by using the differential equations

$$S' = 0.02ST - 0.1S$$

$$T' = 0.1T - 0.01ST$$

Suppose we start with 5 sharks and 300 tuna. Use Euler's method with a step size of 0.4 years to determine the approximate numbers of sharks and tuna at $t = 0.8$ years. Do not round the numbers during your calculation. Round your final answer to whole numbers.

$S_0 = 5 \qquad T_0 = 300 \qquad \Delta t = 0.4$

$S_{0.4} = 5 + 29.5 \cdot 0.4 = 16.8$

$T_{0.4} = 300 + 15 \cdot 0.4 = 306$

$S_{0.8} = 16.8 + 101.136 \cdot 0.4 = 57.2544 = 57$

$T_{0.8} = 306 + -20.808 \cdot 0.4 = 297.6768 = 298$ (57, 298).

The Last **Six** Digits of Your Student ID number

3	0	8	2	8	1

6. *a*) (1 pt) State the definition of the **derivative of** $g = g(t)$ **with respect to** t **at** $t = a$:

$$\left.\frac{dg}{dt}\right|_{t=a} = \lim_{\Delta t \to 0} \frac{g(a+\Delta t) - g(a)}{\Delta t}$$

b) (1 pts) By referencing the definition you gave in part *a*), explain the relationship between the **instantaneous rate of change** of $g(t)$ at $t = a$ and **average rate of change** of g over the interval from $t = a$ to $t = b$, where $b - a$ is close to zero.

The average rate of change is given by $\frac{g(b)-g(a)}{b-a}$, where $b-a=\Delta t$ and $b=a+\Delta t$. As such, the limit of the average rate of change as Δt approaches 0 yields the instantaneous rate of change. Without this, the average rate of change approximates the instantaneous rate of change.

c) (2 pts) Write down an equation for the tangent line to the graph $X = g(t)$ at $(a, g(a))$. Your answer should be of the form $X =$ something; aside from that, you are free to state the equation in any form you prefer.

$$X = \left.\frac{dg}{dt}\right|_{t=a} (t-a) + g(a)$$

d) (3 pts) By using **only** the definition you stated in part *a*) of this problem, verify that for the function

$$g(t) = t^2 + 5t,$$

we have

$$\frac{dg}{dt}\Big|_{t=2} = 9.$$

$$g'(t) = \lim_{\Delta t \to 0} \frac{(t+\Delta t)^2 + 5(t+\Delta t) - t^2 - 5t}{\Delta t}$$

$$\lim_{\Delta t \to 0} \frac{t^2 + 2t\Delta t + \Delta t^2 + 5t + 5\Delta t - t^2 - 5t}{\Delta t}$$

$$\lim_{\Delta t \to 0} \frac{2t\Delta t + \Delta t^2 + 5\Delta t}{\Delta t}$$

$$\lim_{\Delta t \to 0} 2t + \Delta t + 5 = 2t + 5 \quad \text{where } t = 2$$

$$2(2) + 5 = 9 \checkmark$$

e) (3 pts) Let $g(t)$ be the function from part *d*), namely $g(t) = t^2 + 5t$. By using your answer to part *c*) and the fact that

$$\frac{dg}{dt}\Big|_{t=2} = 9,$$

find an approximation for $g(2.1)$. (You **cannot** answer this simply by evaluating $g(t)$ for $t = 2.1$.)

$g(2) = 2^2 + 5(2) = 14$ $\qquad 2.1 - 2.0 = 0.1$

$g(2.1) = 14 + 9 \cdot 0.1 = 14.9$