19F-LIFESCI30A-2 FINAL

ZOE GLEASON

TOTAL POINTS

95.5 / 101

QUESTION 1

1 Unicorns and Dragons 10 / 10

✓ **- 0 pts** Correct

- 1 pts There should be one equation for each creature (D' and U')

- 2 pts Unicorn predation term should be -0.03*D*U

- 2 pts Per capita death rate of unicorns is 2*0.5 (full term is -2*0.5*U)

- 2 pts unicorn birth is 0.5*U

- 2 pts dragon birth rate is just the constant 0.02

- 2 pts Per capita dragon death is proportional to dragons, so should be -0.01*D^2

- 0.5 pts Incorrect constant (see circled)

- 1 pts Wrong sign (see arrow)

+ 1 pts Close answer for Unicorn death rate (had -2*0.5*U*U)

+ 1 pts Close answer for Unicorn predation rate (had -0.03*D or -0.03*U*D^2 or -0.03*D^2 or -0.03*D*U^2)

QUESTION 2

2 Chemical Reactions 10 / 10

✓ **- 0 pts** Correct

- 1 pts Inflows should be positive and outflows negative. At least one of your terms has the wrong signal.

- 1 pts Missing terms for Oxygen entering indicated by w term in X' and deoxygenated hemoglobin leaving indicated by the term -rD in D'

- 1 pts You must multiply the terms by the rates at which the reaction happens, given by f or g on top of the arrows (either missing or wrong)

- 1 pts Incorrect coefficients to account for the resulting H molecule

- 2 pts Missing/incorrect coefficient to account for the resulting four molecules of X

- 1 pts Incorrect molecules to account for the resulting D molecule

- 10 pts You should have: one change equation per molecule (X', D' and H'). Inflows are molecules being formed, outflows are molecules being consumed. Parameters are given by the reaction rates (f and g) and the amount of molecules lost/gained. Oxygen entering is indicated by w term in X' and deoxygenated hemoglobin leaving is indicated by the term -rD in D')

- 5 pts Missing one differential equation

QUESTION 3

3 Euler's Method 10 / 10

✓ **- 0 pts** Correct

- 1 pts incorrect X'(0) = -14.4

- 1 pts incorrect X(1) = 165.6 (if no earlier mistakes)

- 1 pts incorrect X'(1) = -10.86 (if no earlier mistakes)

- 1 pts incorrect X(2) = 154.73 (if no earlier mistakes)

- 1 pts incorrect X'(2) = -8.47 (if no earlier mistakes)

- 1 pts incorrect X(3) = 146.27 (if no earlier mistakes)

- 4 pts incorrect process/formula

- 3 pts incorrect value for dt

- 3 pts used t instead of dt in the formula "Xnext = Xold + X'old*dt"

- 2 pts final answer past t = 3

- 2 pts did not integrate all the way to t = 3

- 1 pts reported state of system incorrectly (see explanation)

- 1 pts excessive rounding during calculation (rounding introduces errors and those errors build up over time)

QUESTION 4

Differentiation 10 pts

4.1 Parts a) 5 / 5

✓ **- 0 pts** Correct

- 1 pts Incorrect/incomplete statement of rules

- 0.5 pts Did not indicate where each rule is being used

- 2 pts Did not apply/incorrectly applied product rule

- 2 pts Did not apply/incorrectly applied power/logarithmic rule

4.2 Part b) 3 / 4

- 0 pts Correct

- 1 pts Incorrect/Incomplete stating of the rule

- 1 pts Incorrect/Incomplete Log Rule

- 1 pts Incorrect/Incomplete Chain Rule

✓ **- 1 pts** Incorrect/Incomplete Power Rule

4.3 Part c) 1 / 1

✓ **- 0 pts** Correct

- 0.5 pts Did not do second chain rule correctly

- 0.5 pts Did not do first chain rule

QUESTION 5

Solar Power Rules 11 pts

5.1 Part a) 4 / 4

✓ **- 0 pts** Correct

- 1 pts Incorrect use of Reimann Sum

- 4 pts No answer

- 2 pts Sigma notation k values incorrect (starting from 0 but difference not taken care of in the summation)

5.2 Part b) 1 / 1

✓ **- 0 pts** Correct

- 0.5 pts Underestimation is not specified

- 0.5 pts Overestimation is not specified

- 0.5 pts Underestimation/Overestimation interchanegd

5.3 Part c) 4 / 4

✓ **- 0 pts** Correct

- 0 pts Click here to replace this description.

- 3 pts Definite integral specified

- 2 pts Error in definite integral

- 4 pts No answer

- 2 pts Error in definite integral limit

5.4 Part d) 1 / 1

+ 0 pts Incorrect

✓ **+ 1 pts** **Correct (mention of symmetry of graph/balancing out of over and underestimation)**

5.5 Part e) 0 / 1

✓ **+ 0 pts** **Incorrect**

+ 1 pts Correct -mention of downward concavity of graph leading to a larger area lost from rectangles underestimating than area gained from rectangles overestimating. However, number of rectangles on either side of vertex are the same (3 overestimating, 3 underestimating)

QUESTION 6

Linear Stability Analysis 10 pts

6.1 Part a) 4 / 4

✓ **- 0 pts** Correct

- 1 pts Missing analysis of one equilibrium point

- 0.5 pts Calculation mistake

- 4 pts Incorrect

- 0.5 pts Calculation mistake: $x = -2$ unstable

- 3 pts Used derivative of df/dX to determine stability and not linear stability analysis

- 3 pts Used integral of df/dX to determine stability and not linear stability analysis

- 4 pts No/unclear use of linear stability analysis

6.2 Part b) 2 / 2

✓ **- 0 pts** Correct

- 2 pts Did not use/incorrect application of method of test points

- 0.5 pts Stability not indicated. $x = -2$ is unstable

- **1 pts** Unsuitable test points
- **1 pts** Stability incorrectly assessed. x = -2 is unstable
- **0.5 pts** Calculation error

6.3 Part c) **2 / 2**

✓ - **0 pts Correct**
- **2 pts** No answer
- **2 pts** Vector field should be 1 dimensional
- **0.5 pts** Vector field incomplete
- **2 pts** Arrow directions incorrect. Correct: <-- (-2) --> (2) <-- (4) -->

6.4 Part d) **2 / 2**

✓ - **0 pts Correct**
- **2 pts** Does not identify that x = 3 is in the basin of attraction of the stable equilibrium x = 2, and that therefore the system will approach 2 in the long run
- **2 pts** No answer

QUESTION 7

7 Identifying EPs **9 / 10**

- **0 pts** Correct
- **0.5 pts** Incorrect (6,8) unstable spiral

✓ - **1 pts Incorrect (7.5, .5) saddle point**
- **0.5 pts** Saddle point, not saddle node
- **1 pts** Incorrect (12,11) saddle point
- **0.5 pts** Wrong point
- **0.5 pts** Incorrect (12,5) stable spiral
- **1 pts** Missing 4+ points on the graph
- **0.5 pts** Missing <4 points on the graph
- **1 pts** Incorrect (2,6) saddle point
- **0.5 pts** Incorrect (0,10) stable node
- **0.5 pts** Incorrect (0.5,0) unstable node
- **0.5 pts** Stable saddle point doesn't exist
- **0.5 pts** Just spiral
- **0.5 pts** Just node

QUESTION 8

Deer-Moose 10 pts

8.1 Part a) **2 / 2**

✓ - **0 pts Correct**
- **1 pts** Evaluate the differential equation
- **1 pts** Setting the change in quantities to 0
- **0.5 pts** Minor error

8.2 Part b) **2 / 2**

✓ - **0 pts Correct**
- **2 pts** incorrect
- **0.5 pts** if one of them is incorrect
- **1 pts** if two of them are incorrect
- **1.5 pts** if three of them are incorrect
- **0.5 pts** extra equillibrium points

8.3 Part c) **3 / 3**

✓ - **0 pts Correct**
- **0.5 pts** if one of the nullclines is incorrect
- **1 pts** if two nullclines are incorrect
- **1.5 pts** if three null-clines are incorrect
- **3 pts** incorrect
- **0.5 pts** if one/two equillibrium points are incorrect
- **1 pts** if more than two equillibrium points are incorrect
- **2 pts** if all the null-clines are incorrect
- **0.5 pts** if extra-equillibrium points are indicated
- **0.5 pts** if extra null-clines are indicated

\+ **0.5 pts** attempt

8.4 Part d) **2 / 2**

✓ - **0 pts Correct**
- **0.5 pts** if one of the eq. point's stability is incorrect
- **1 pts** if two of the eq. points's stability is incorrect
- **1.5 pts** if three of the equillibrium points's stability is incorrect
- **0.5 pts** extra equillibrium points/ not indicated in the diagram
- **2 pts** incorrect

\+ **0.5 pts** attempt
- **1 pts** analysis is incorrect/stability of eq. points not indicated
- **0.5 pts** change direction indicated but stability not mentioned

8.5 Part e) **1 / 1**

✓ **- 0 pts** Correct

- 0.5 pts Insufficient / incorrect explanation - not much emphasis on equilibrium points / stability

- 1 pts Incorrect / Not attempted

QUESTION 9

The Murky Lake 10 pts

9.1 Part a) **1.5 / 2**

- 0 pts Correct

✓ **- 0.5 pts** **one value of nutrient level is wrong**

- 1 pts two values of nutrient level are wrong

- 0.5 pts type of bifurcation is wrong (one)

- 1 pts type of bifurcation is wrong (two)

- 2 pts answer is totally wrong.

- 0.5 pts spare bifurcation is found

9.2 Part b) **2 / 2**

✓ **- 0 pts** Correct

- 1 pts value (around 0.3) is wrong but with some explanations

- 2 pts value is wrong without explanation

- 0.5 pts value estimation is too far (<0.2 or >0.5)

- 0.5 pts no explicit estimated value

9.3 Part c) **0 / 2**

- 0 pts Correct

- 1 pts value (around 1.0) is wrong but with some explanations

✓ **- 2 pts** **value is wrong without explanation**

9.4 Part d) **2 / 2**

✓ **- 0 pts** Correct

- 1 pts value (around 7) is wrong but with some explanations

- 2 pts value is wrong without explanations

9.5 Part e) **2 / 2**

✓ **- 0 pts** Correct

- 2 pts nutrient level should be reduced below 0.5

- 0.5 pts making nutrient level exactly 0.5 is not proper.

- 1 pts "decrease nutrient level" but the value is not right

- 1 pts make a part of sense

QUESTION 10

10 Let's Draw a Bifurcation Diagram **10 / 10**

Lines are not drawn correctly for X' = rX

- 0.5 pts Incorrect line for r = 1/2

- 0.5 pts Incorrect line for r = 1 (or else)

- 0.5 pts Incorrect line for r = 2 (or else)

- 0.5 pts Incorrect line for r = 4 (or else)

- 0.5 pts Incorrect line for r = 6 (or else)

- 0.5 pts Incorrect line for r = 8

Equilibrium points not correctly marked

- 0.5 pts Equilibrium points for r = 1/2 (1 EP)

- 0.5 pts Equilibrium points for r = 1 (1 EP, or else)

- 0.5 pts Equilibrium points for r = 2 (1 EP, or else)

- 0.5 pts Equilibrium points for r = 4 (3 EP, or else)

- 0.5 pts Equilibrium points for r = 6 (3 EP, or else)

- 0.5 pts Equilibrium points for r = 8 (1 EP)

- 1 pts Equilibrium points not clearly marked

Stability of EPs not correctly marked

- 0.5 pts r = 0.5, 1 EP, stable

- 0.5 pts r = 1, 1 EP, stable (or else)

- 0.5 pts r = 2, 1EP, stable (or else)

- 0.5 pts r = 4, 3EP, stable, unstable, stable (or else)

- 0.5 pts r = 6, 3EP, stable, unstable, stable (or else)

- 0.5 pts r = 8, 1EP, stable

- 1 pts Stability not clearly marked

- 1 pts Incorrect bifurcation diagram (wrong connection of all EPs)

✓ **- 0 pts** **Correct answer**

- 10 pts No correct progress

LS 30A: MATHEMATICS FOR LIFE SCIENTISTS
FALL 2019 - LECTURES 2 and 3
Jukka Keranen

FINAL EXAMINATION

Your Name Zoe Gleason

The Last **Six** Digits of Your Student ID number | 3 | 0 | 8 | 2 | 8 | 1 |

Your TA Section 2N, Q1

By signing below, you confirm that you did not cheat on this exam. No exam booklet without a signature will be graded.

INSTRUCTIONS

- Please do **not** open this booklet until you are told to do so.
- In addition to basic writing instruments, you are allowed to use a non-programmable calculator.
- Your cell phone must be **turned off completely** and stowed away where you cannot see it.
- No books or notes.
- If you have a question at any time during the exam, please raise your hand.
- You will receive points only for work written on the numbered pages. Please use the reverse side as scratch paper.
- Make sure to write legibly. **Illegible work will not be graded.**
- Make sure to show all your work and justify your answers fully.

SCORE

1. ______ 6. ______

2. ______ 7. ______

3. ______ 8. ______

4. ______ 9. ______

5. ______ 10. ______

TOTAL ______

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1. (10 pts) Unicorns, U, are preyed upon by dragons, D. Write a differential equation model of the unicorn and dragon populations, by using the following assumptions. In what follows, all rates are per year rates.

- The unicorn per capita birth rate is 0.5.
- The unicorn per capita death rate (independent of predation) is twice the unicorn per capita birth rate.
- Only dragons prey upon unicorns.
- The probability that any one unicorn gets eaten by a dragon is proportional to the number of dragons, with a proportionality constant of 0.03.
- Dragon mating takes place according to ancient rules, and is largely independent of things like the availability of food. Accordingly, the dragon birth rate can be modeled as a constant 0.002.
- Being essentially immortal and peerless in power, dragons only die in combat with other dragons. Thus, the dragon per capita death rate is proportional to the number of dragons, with proportionality constant 0.01.

- $U' = +0.5U$
- $U' = -U$
- $U' = -0.03UD$
- $D' = 0.002$
- $D' = -0.01D^2$

$U' = 0.5U - U - 0.03UD$

$D' = 0.002 - 0.01D^2$

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2. Oxygenation of hemoglobin is a common chemical reaction in red blood cells. In order to model this reaction, let's denote
   - X = the concentration of oxygen molecules in the cell,
   - D = the concentration of deoxygenated hemoglobin in the cell,
   - H = the concentration of oxygenated hemoglobin in the cell.

In the cell, the following processes are occurring:

- Deoxygenated hemoglobin combines with 4 oxygen molecules to form oxygenated hemoglobin, as depicted below:

$$D + 4X \xrightarrow{f} H;$$

  in particular, the rate constant of this process is f.

- Oxygenated hemoglobin can also split up into deoxygenated hemoglobin and 4 oxygen molecules, as depicted below:

$$H \xrightarrow{g} D + 4X;$$

  in particular, the rate constant of this process is g.

In addition,

- oxygen enters the cell at a constant rate of w, and
- deoxygenated hemoglobin leaves the cell at a per-molecule (like per capita) rate of r.

(10 pts) Write a differential equation model for X, D, and H, by using the assumptions above. All rates are per second.

- $D' = -fDX^4$
  $X' = -4fDX^4$
  $H' = +fDX^4$

$$D' = -fDX^4 + gH - rD$$
$$X' = -4fDX^4 + 4gH + w$$
$$H' = fDX^4 - gH$$

- $D' = +gH$
  $X' = +4gH$
  $H' = -gH$
- $X' = +w$
- $D' = -rD$

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3. (10 pts) The annual rate of change of a hunted unicorn population, X, is given by the differential equation

$$X' = 0.2X\left(1 - \frac{X}{200}\right) - 0.1X.$$

The current population is 180 unicorns. Use Euler's method with step size of 1 year to find the approximate population three years later. During your calculation, use values accurate to 2 decimal places. Round your final answer to whole numbers.

$X_0 = \underset{-14.4}{\underset{\downarrow}{180}}$ $\Delta t = 1$

$X_1 = 180 - 14.4 = 165.6$

$X_2 = 165.6 - 10.86 = 154.74$

$X_3 = 154.74 - 8.47 = 146.30 \approx$ $\boxed{146 \text{ unicorns}}$

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4. By using the differentiation rules we learned in this course, find $\frac{df}{dx}$ for each of the following functions $f(x)$. Indicate clearly which rules you are using at each step of your calculation.

(*Hint.* You will need to use, among other things, the fact that $\frac{d}{dx}(\ln x) = \frac{1}{x}$.)

a) (4 pts) $f(x) = (x^4 + x^3)(x^6 + \ln x)(\sqrt{x} + 3)$

product rule (entire derivative)

$$f'(x) = \underbrace{(4x^3 + 3x^2)}_{\text{power rule}}(x^6 + \ln x)(\sqrt{x} + 3) + \underbrace{\left(6x^5 + \frac{1}{x}\right)}_{\text{power rule and logarithmic rule}}(x^4 + x^3)(\sqrt{x} + 3)$$

$$+ \underbrace{\left(\frac{1}{2}x^{-1/2}\right)}_{\text{power rule}}(x^4 + x^3)(x^6 + \ln x)$$

$$f'(x) = (4x^3 + 3x^2)(x^6 + \ln x)(\sqrt{x} + 3) + \left(6x^5 + \frac{1}{x}\right)(x^4 + x^3)(\sqrt{x} + 3) + \left(\frac{1}{2}x^{-1/2}\right)(x^4 + x^3)(x^6 + \ln x)$$

b) (4 pts) $f(x) = \ln\left(\frac{1}{x^6 + x^4 + 3}\right)$

$$f'(x) = \underbrace{(x^6 + x^4 + 3)}_{\text{logarithmic rule}}\underbrace{(6x^5 + 4x^3)}_{\text{chain rule/power rule}}$$

$$f'(x) = (x^6 + x^4 + 3)(6x^5 + 4x^3)$$

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By using the differentiation rules we learned in this course, find $\left.\frac{df}{dx}\right|_{x=1}$ for the following function $f(x)$. Indicate clearly which rules you are using at each step of your calculation.

c) (2 pts) $f(x) = e^{x^{\ln x}}$

(You will need to use the fact that $\ln 1 = 0$.)

$$f'(x) = \underbrace{x^{\ln x}\; e^{x^{\ln x}}\; (\ln x\, x^{\ln x - 1})}$$

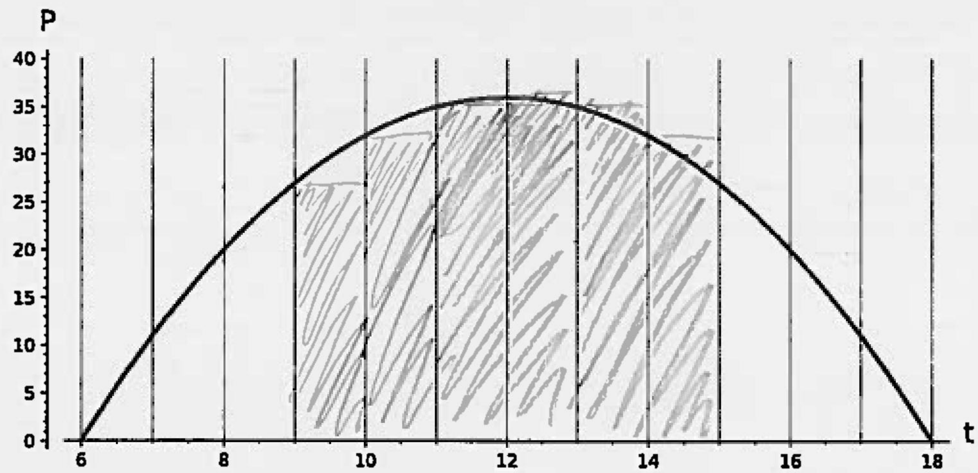
exponent rule chain rule/ power rule

$$f'(1) = 1^{0} \cdot e^{1} \cdot 0 \cdot 1^{-1} = 0$$

5. A **solar power plant** has a power output of $P(t)$ kW, where t represents time (measured in hours). Suppose that from 6:00 (6 AM) to 18:00 (6 PM), the power output $P(t)$ is given by the function

$$P(t) = -(t-6)(t-18),$$

whose graph is shown below:



a) (4 pts) By using time interval $\Delta t = 1$ hour, write down and evaluate the Riemann sum approximating the total energy produced by the power plant over the period from $t = 9$ (9 AM) to $t = 15$ (3 PM).[1]

(*Reminder.* In your answer, energy will have units of kWh, kilowatt hours.)

$$\sum_{k=0}^{k=5} -((k+9)-6)((k+9)-18) = 27 + 32 + 35 + 36 + 35 + 32$$

$$= \boxed{197 \text{ kWh}}$$

$\Delta t = 1$

$t=9$, $P=27$

$t=10$, $P=32$

$t=11$, $P=35$

$t=12$, $P=36$

$t=13$, $P=35$

$t=14$, $P=32$

b) (1 pts) In the diagram above, indicate the graphical meaning of the Riemann sum you calculated in part *a*).

Area shaded is the area calculated "under the curve"

[1] For us, "Riemann sum" always means "left Riemann sum". If you don't know what this remark means, you don't need to worry about it.

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c) (4 pts) Write down and evaluate the definite integral that represents the total energy produced by the power plant over the period from $t = 9$ (9 AM) to $t = 15$ (3 PM).

(*Hint.* You may wish to use the fact that $P(t) = -t^2 + 24t - 108$.)

$$\int_9^{15} -t^2 + 24t - 108 \, dt = -\frac{1}{3}t^3 + 12t^2 - 108t \Big|_9^{15}$$

$$-\frac{1}{3}(15)^3 + 12(15)^2 - 108(15) - (-\frac{1}{3}(9)^3 + 12(9)^2 - 108(9))$$

$$-45 + 243 = \boxed{198 \text{ kWh}}$$

d) (1 pts) The correct numerical answer to part *a*) is 197; the correct numerical answer to part *c*) is 198. By referencing the shape of the graph of $P(t)$, explain why these answers are so similar.

The answers are so similar because the Riemann sum is symmetrical, so the over and under estimations nearly equal out, creating a more accurate answer and becoming similar to the integral, which is very accurate.

e) (1 pt extra credit) By referencing the shape of the graph of $P(t)$, explain why the answer to part *a*) is a little bit smaller than the answer to part *c*).

The left handed sum uses a P value that is not matched on the other side, creating an underestimation in the sum that is not present in the integral

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6. You are studying a differential equation model, $X' = f(X)$, of a dynamical system. Suppose you know that

$$\frac{df}{dX} = 3X^2 - 8X - 4,$$

and that the equilibrium points are at

$$X = -2, X = 2, X = 4.$$

a) (4 pts) Determine the stability of each equilibrium point by using linear stability analysis. (*Note.* So far, you have only been given $\frac{df}{dX}$, not $f(X)$.)

$X=-2$ $\quad 3(-2)^2 - 8(-2) - 4 = 24$, unstable

$X=2$ $\quad 3(2)^2 - 8(2) - 4 = -8$, stable

$X=4$ $\quad 3(4)^2 - 8(4) - 4 = 12$, unstable

b) (2 pts) Suppose now that you know, *in addition*, that

$$f(X) = X^3 - 4X^2 - 4X + 16.$$

Choose suitable test points and verify that the stability of $X = -2$ is what you claimed it is in part *a*).

$X=-3$ $\quad f(-3) = -35$ $\quad f(-3) < 0$ $\qquad X=-2$ is unstable ✓

$X=-1$ $\quad f(-1) = 15$ $\quad f(-1) > 0$

c) (2 pts) Sketch the vector field with equilibrium points of this model. (*Note.* You only need to represent accurately the direction of the change vectors, not their length.)

d) (2 pts) Suppose you discover that the system is currently in the state $X = 3$. According to this model, how will the system's state change in the future?

The system's state will shift to the stable equilibrium point at $X=2$

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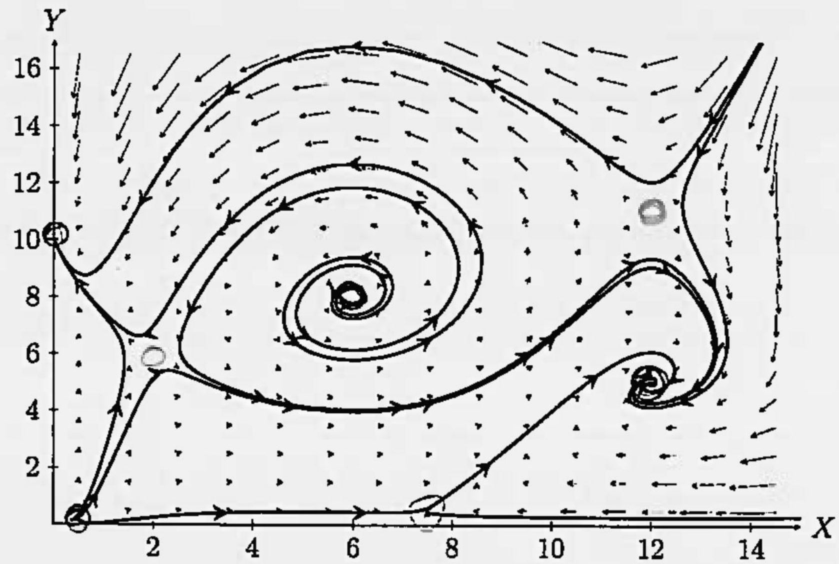
7. The diagram below shows the vector field and several trajectories of a 2-variable system of differential equations.



(10 pts) Indicate the location of all the **seven** equilibrium points in the diagram, then list them in the space below and indicate their type (in the classification of equilibrium points in 2 dimensions that we learned).

(0,0) unstable node

(10,0) stable node

(2,6) saddle point

(6,8) unstable spiral

(12,11) saddle point

(12,5) stable spiral

(7,0) unstable node

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8. Let D be the size of a population of deer, and M the size of a population of moose. Suppose that the population dynamics of the two species can be modeled by the following differential equations:

$$D' = 24D - 2D^2 - 3DM$$
$$M' = 15M - M^2 - 3DM$$

(D, M)

a) (2 pts) Find the nullclines of this model.

$D' = D(24 - 2D - 3M)$ 1a $D = 0$ 1b $M = -\frac{2}{3}D + 8$

$M' = M(15 - M - 3D)$ 2a $M = 0$ 2b $D = -\frac{1}{3}M + 5$

OR

$M = -3D + 15$

b) (2 pts) By using your answer to part *a*), find the equilibrium points of this model.

1a, 2a $\rightarrow (0,0)$

1a, 2b $\rightarrow (0,15)$

1b, 2a $\rightarrow (12,0)$

1b, 2b $\rightarrow (3,6)$

$-\frac{2}{3}D + 8 = -3D + 15$

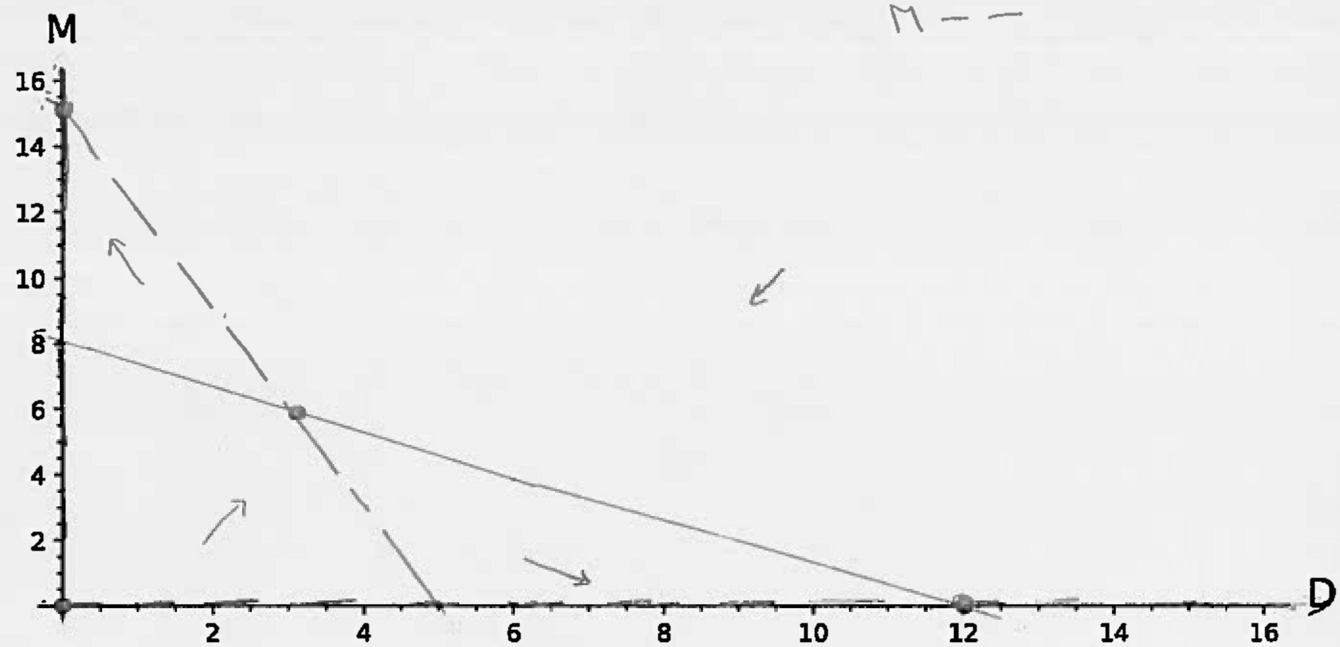
$\frac{7}{3}D = 7$

$D = 3$

$M = 6$

c) (3 pts) On the axes provided below, draw the nullclines and equilibrium points of this model. Make sure to indicate clearly which ones are the D-nullclines, which ones the M-nullclines.

D ——
M - -



d) (2 pts) By picking a test point from within each region demarcated by the nullclines, determine for each equilibrium point whether it is stable or unstable. Indicate the result of your analysis in the diagram.

(D, M)	(D', M')
(2, 2)	(32, 14)
(6, 1)	(54, -4)
(1, 10)	(-8, 20)
(10, 10)	(-260, -250)

(0, 0) unstable
(12, 0) stable
(0, 15) stable
(3, 6) unstable

e) (1 pt) Can the two species coexist in the long run? Justify your answer by referencing your answer to part *d*).

The two species cannot coexist because the system only reaches a stable equilibrium when one of the populations equals zero.

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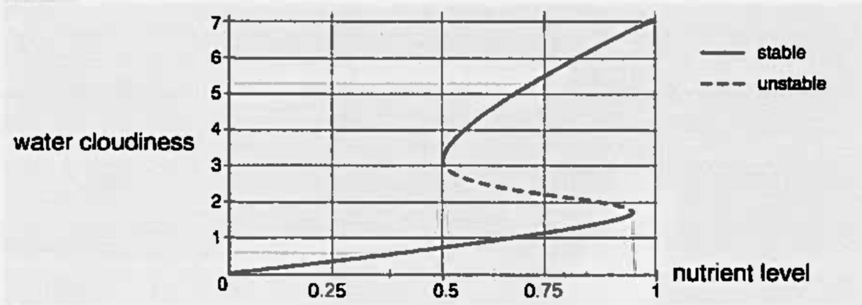
9. The diagram below shows a possible relationship between nutrient levels and water cloudiness ("murkiness") in a lake.



Let's assume that, from the point of view of the dynamics of this system, 1 year is a "long time".

a) (2 pts) List the bifurcations that occur in this diagram. For each one, state what type of bifurcation it is and at what nutrient level it occurs.

2 saddle nodes nutrient level = 0.5, 0.95

b) (2 pts) At the beginning of the year, the nutrient level is 0.2 and the water cloudiness is 7. At what level will the water cloudiness be at the end of the year?

The water cloudiness will be at about 0.3.

c) (2 pts) At the beginning of the year, the nutrient level is 0.4 and the cloudiness is 2. There is now a sudden increase in pollution, and the nutrient level quickly increases to 0.7. At what level will the water cloudiness be at the end of the year?

The water cloudiness will be at about 5.3.

d) (2 pts) At the beginning of the year, the nutrient level is 0.8 and the cloudiness is 1. There is now a further sudden increase in pollution, and the nutrient level quickly increases to 1. At what level will the water cloudiness be at the end of the year?

The water cloudiness will be at 7

e) (2 pts) You are now in the year-end situation of part *d*). Propose a water quality restoration policy that will return water cloudiness to below 1 with the least possible reduction in the nutrient level.

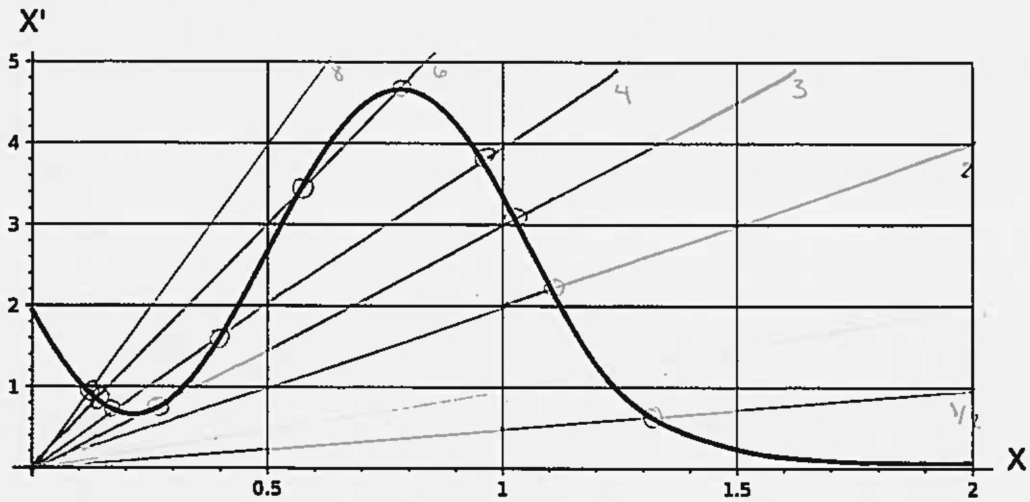
Reduce the nutrient level to below 0.5 so that the water cloudiness falls to the lower equilibrium, which is below one (until the nutrient level reaches about 0.7).

The Last **Six** Digits of Your Student ID number | 3 | 0 | 8 | 2 | 8 | 1 |

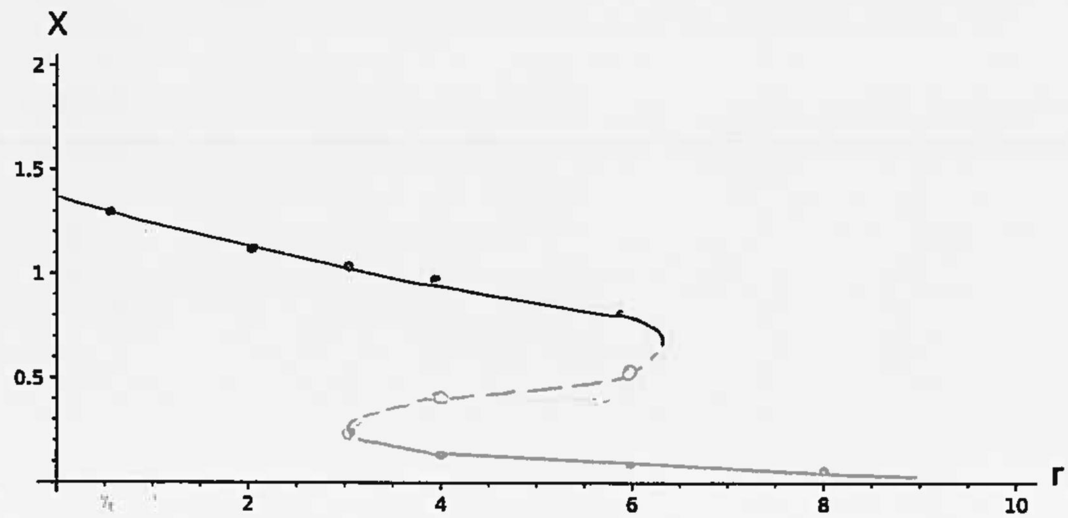
10. A hormone is produced at a rate that depends on the current level of that hormone in the body, according to the function whose graph is shown below. The hormone leaves the system at a rate proportional to the current hormone level, with proportionality constant r. In other words, if X is the hormone level, then X satisfies the differential equation

$$X' = [\text{Input}] - rX,$$

where [Input] is the function whose graph is shown below:



(10 points) On the axes below, draw a bifurcation diagram for this system, as the parameter r varies from 0 to 8.



- Be sure to indicate graphically the stability of each equilibrium point in your diagram.
- You should consider 6 different values for r, including $r = \frac{1}{2}$ and $r = 8$.
- It is not necessary to consider values of X greater than 2.
- The purpose of the grid is to help you find the r.

— • stable
-- o unstable