LS 30A: MATHEMATICS FOR LIFE SCIENTISTS

FALL 2021 - LECTURES 1 and 2

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MIDTERM EXAMINATION – STAGE 2

- Once you have accessed this file, you have **2 hrs 5 mins** to submit it on **Gradescope**.
- You must submit your work by **6:00 PM on Saturday, October 30**.
- You can answer on your own sheet of paper (just as you would with homework).
- When you submit, be sure to **properly select each question** in Gradescope.
- You may use your study guide and all the materials posted on our Canvas site.
- You may (and should) use a calculator (like the one in your computer or phone).
- You may **not** discuss the content of this exam with anyone.
- You may **not** use the Internet while working on this exam.
- During the exam period (6:00 PM on Oct 29 through 6:00 PM on Nov 2), you should **ask any questions about this exam on Campuswire by selecting "Post to instructors and TAs"**. We will make your question public to create an FAQ if multiple people have the same question.
- By taking this exam, you agree to follow the UCLA Student Conduct Code, in particular the articles related to academic integrity.
- Please **sign your name** on the first page of your submission. By signing, you confirm that you did not cheat on this exam. No submission without a signature will be graded.

FRIENDS, GOOD LUCK!

YOU'VE GOT THIS!

1. (20 pts) Let's model the flow of visitors at a shopping mall. Our states variables will be:

S = the number of **people shopping**,

R = the number of **people eating at the restaurants**,

M = the number of **people at the movies**.

We will be using the following assumptions. All rates are hourly rates.

- All people who enter the mall start by shopping. Each hour, a people start shopping.
- After shopping, people will either go to the restaurants or to the movies.
- People don't like to wait in line at the restaurants. So, shoppers go to the restaurants at a per capita rate that is proportional to the inverse of the number of people already at the restaurants, with a constant of proportionality b.
- If the restaurants are packed, shoppers may decide to go to the movies first. Shoppers go to the movies at a per capita rate that is proportional to the number of people at the restaurants, with a constant of proportionality c.
- People finish eating at a constant per capita rate d.
- Of the people who finish eating, fraction e will go to the movies, while the rest leave the mall.
- After the movies, people go to the restaurants at a per capita rate f, and leave the mall entirely at a per capita rate g.

a) (7 pts) Draw a box-and-arrow diagram (in other words, a flow chart) of the visitor population. Make sure to indicate clearly all the flows as well as the rates of the flows.

b) (13 pts) By using our assumptions, write down the change equation for each state variable.

2. (30 pts) Consider a system in which three species of animals called A, B and C interact with each other. This system can be modeled with the following equations:

$$A' = 0.1A - 0.01A^2 - 0.5AB$$
$$B' = 0.1AB + 0.2BC - 0.1B$$
$$C' = 0.5C\left(1 - \frac{C}{200}\right) - 0.4BC$$

a) (7 pts) Explain what the terms $-0.5AB$ and $0.1AB$ concretely represent in terms of the species.

b) (9 pts) Which species prey(s) on which one(s)? Make sure to explain your reasoning.

Again, the model is

$$A' = 0.1A - 0.01A^2 - 0.5AB$$
$$B' = 0.1AB + 0.2BC - 0.1B$$
$$C' = 0.5C\left(1 - \frac{C}{200}\right) - 0.4BC$$

c) (7 pts) Is logistic growth assumed for any of these species? If so, which one(s)? Explain.

d) (7 pts) If due to a change in the environment more food became available to species C, how would the model change?

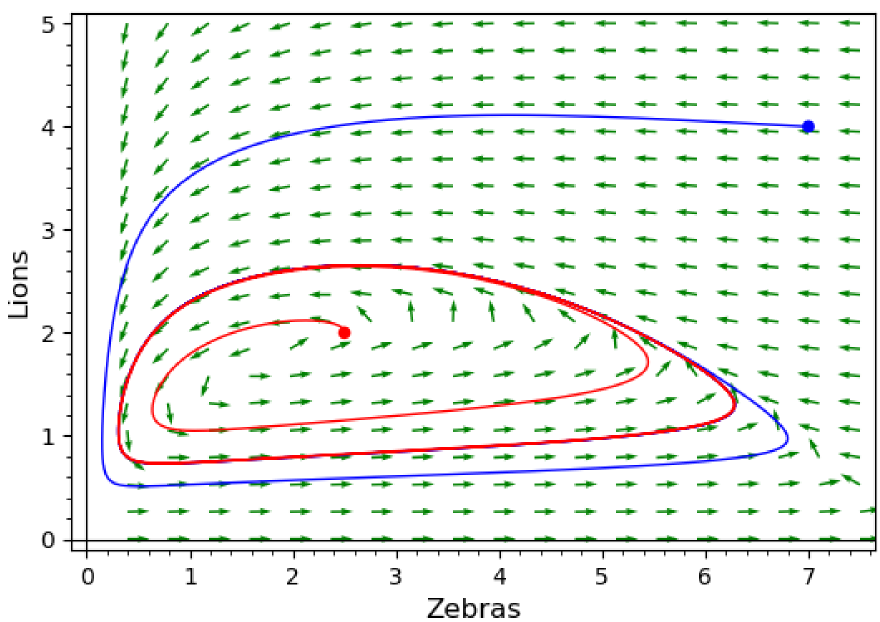
3. (30 pts) In Etosha National Park, Namibia, **zebras** (Z) are preyed upon by **lions** (L). This system can be modeled with the following equations:

$$\begin{aligned} Z' &= Z\left(1 - \frac{Z}{8}\right) - 1.2\frac{Z}{1+Z}L \\ L' &= 0.1L\left(1 - \frac{L}{Z}\right) \end{aligned}$$

where both L and Z have units of **thousand individuals**.

The picture below shows the vector field and two trajectories. The initial point of each trajectory is indicated by a dot. Note that the two trajectories get so close to each other that it looks like the blue trajectory gets below the red trajectory but they are actually next to each other.

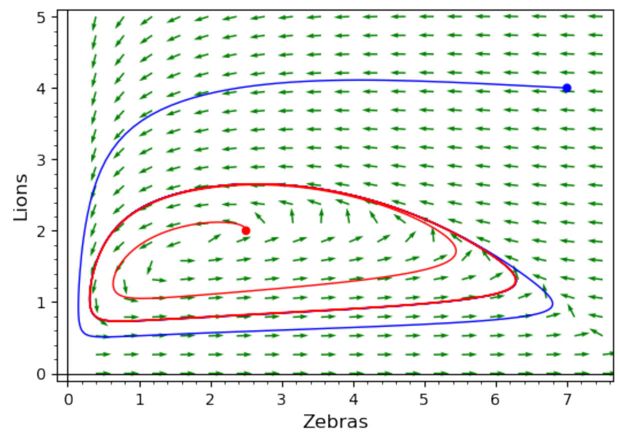
*Note that to make the vector field more readable, the vectors are **not** drawn to scale.*



a) (7 pts) Compute the change vectors for the points $(Z, L) = (2, 4)$ and $(Z, L) = (5, 1)$.
Do not round the numbers in your answer.

Again, the model and trajectories are:

$$Z' = Z\left(1 - \frac{Z}{8}\right) - 1.2\frac{Z}{1+Z}L$$
$$L' = 0.1L\left(1 - \frac{L}{Z}\right)$$

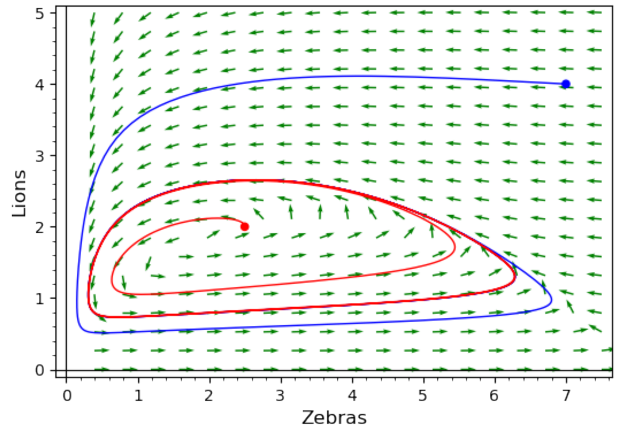


b) (7 pts) In what way are the vectors you computed in part a) consistent or not consistent with the trajectories drawn in the picture?

c) (7 pts) If you start at the blue dot at $(7, 4)$, will both populations survive in the long run or will one of them go extinct in Etosha? Please provide an explanation.

Again, the model and trajectories are:

$$Z' = Z\left(1 - \frac{Z}{8}\right) - 1.2\frac{Z}{1+Z}L$$
$$L' = 0.1L\left(1 - \frac{L}{Z}\right)$$



d) (9 pts) Sketch the time series associated with the red trajectory; this is the trajectory that starts at the point $(Z, L) = (2.5, 2)$.

4. (20 pts) Let's think about what we have learned about derivatives so far in the course.

a) (10 pts) We have seen that the definition of the derivative of $X(t)$ at $t = a$ is given by

$$X'(a) = \lim_{\Delta t \to 0} \frac{X(a + \Delta t) - X(a)}{\Delta t}.$$

Explain which part of this formula shows the **average rate of change** of $X(t)$, the **instantaneous rate of change** of $X(t)$, and the **slope of a secant line** to the graph of $X(t)$.

b) (10 pts) By using **only** the definition in part a) of this problem, verify that for the function

$$X(t) = 3t^2 + 4t + 3.14,$$

we have

$$X'(7) = 46.$$

5. (20 pts) You are studying the populations of **deer** (D) and **moose** (M) on an island. Based on your fieldwork, you have determined that, currently, the deer population is 10 individuals, and the moose population is 20 individuals. In your research log, you denote the current time as $t = 0.1$ years. Due to reasons, you have to leave the island and must now resort to mathematical modeling to predict the future course of the dear and moose populations. You will use the following model:

$$D' = 4D - 0.1DM - 0.1D^2$$
$$M' = 2M - 0.1DM - 0.2M^2$$

where the units in each equation are **individuals/year**.

Apply Euler's method with **time step** $\Delta t = 0.2$ years and **initial condition** $(D, M) = (10, 20)$ at time $t = 0.1$ years to predict the population counts at time $t = 0.5$ years.

Do not round numbers during your calculation.

Do not round the numbers in your final answer.