Started on	Wednesday, 17 November 2021, 2:56 PM PST
State	Finished
Completed on	Wednesday, 17 November 2021, 2:56 PM PST
Time taken	23 secs
Grade	**0.00** out of 80.00 (**0**%)

Question **1**

Not answered

Not graded

By taking this exam, I agree to follow UCLA Student Code of Conduct and in particular I will abide by all the rules related to academic integrity.

I am aware that I am **not allowed** to communicate about the questions of this midterm while the midterm is open (in other words, until Tuesday Nov 10 at 6.30pm).

Please sign below by entering your full name.

Answer:

The correct answer is:

Information

In this course, we have often considered a simple model of predator-prey systems, the *Shark-Tuna model*: $S' = ST - S$, $T' = T - ST$.

Today, let's develop a better model. We will consider three species that form a food chain: *plankton* (P), *tuna* (T), and *sharks* (S). We will make the following assumptions; in what follows, all rates are **per year** rates.

- plankton grows exponentially, with a rate constant of 0.01;
- the tuna population undergoes logistic growth, with a per capita birth rate of 0.05 and carrying capacity that is proportional to the amount of plankton, with constant of proportionality of 0.5;
- each shark consumes tuna at a rate that is an increasing sigmoid of the amount of tuna, with a maximum rate of 10 (and exponent equal to 2);
- the shark population grows at a rate proportional to the number of tuna eaten, with a constant of proportionality of 0.2;
- each shark has a probability of 10% of dying in any one year.

Question **2**

Not answered

Points out of 6.00

The plankton change equation:

P' = [ ] [ ]

The tuna change equation:

T' = [ ] [ ] (1 – [ ] ÷ [ ] [ ]) – [ ] ([ ] · [ ] ÷ (1 + [ ] · [ ])) · [ ]

The shark change equation:

S' = [ ] ([ ] · [ ] ÷ (1 + [ ] · [ ])) · [ ] – [ ] [ ]

P T S

0.01 0.05 0.1 0.5 1 2 10 20 100

The correct answer is:
The plankton change equation:

P' = [0.01] [P]

The tuna change equation:

T' = [0.05] [T] (1 – [T] ÷ [0.5] [P]) – [10] ([T]· [T] ÷ (1 + [T]· [T])) · [S]

The shark change equation:

S' = [2] ([T]· [T] ÷ (1 + [T]· [T])) · [S] – [0.1] [S]

Information

Griffins (*G*) were noble creatures that roamed the ancient world. They were second in power only to *dragons* (*D*), which envied them greatly. Griffins and dragons would fight over *unclaimed territory* (*U*), seeking to lay claim to the riches of the land and the treasures of the deep.

While the age of griffins and dragons is long past, we know from old manuscripts that their struggle unfolded as follows:

- Whenever dragons and griffins met, they would engage in combat; either all the dragons or all the griffins would die, while the other side would survive.
- Solitary dragons and solitary griffins would sometimes encounter each other while exploring unclaimed territory. The more unclaimed territory there was, the less likely these encounters were. Three-quarters of the these encounters would result in the destruction of the griffin, leaving the dragon victorious; the other one-quarter would see the opposite outcome.
- Sometimes, however, two griffins would happen upon a solitary dragon. If one griffin was formidable enough, two of them together would be a force even a dragon would not be able to withstand. Thus, such encounters would always result in the death of the dragon. These encounters were rare, and the amount of unclaimed territory played no role here.
- Being of violent temper, dragons would also fight one another over unclaimed territory; the less territory remained unclaimed, the more likely were such battles. One of the two dragons would survive, while the other one would be destroyed.
- A typical female griffin mated and produced a single offspring once in every ten years. Female griffins made up exactly one-half of the entire population of griffins.
- Dragon mating, on the other hand, took place according to ancient rules and, consequently, the dragon birth rate was a constant, one new dragon being born every 100 years.
- Being essentially immortal and nearly peerless in power, neither dragons nor griffins would die in any way other than in combat with dragons or griffins.
- As the years wore on, dragons and griffins would claim previously unclaimed territory. Each dragon would claim 100 square miles every 20 years, while each griffin would claim 50 square miles a year.

In writing your change equations, please

- convert all rates to **per year rates**, and
- when writing a product of state variables, enter the state variables in **the alphabetical order.**

Question **3**

Not answered

Points out of 8.00

D' = [ ] − ([ ] [ ] · [ ] ÷ [ ]) − ([ ] · [ ] · [ ]) − ([ ] · [ ] ÷ [ ])

G' = ([ ] [ ]) − ([ ] [ ] · [ ] ÷ [ ])

U' = −([ ] [ ]) − ([ ] [ ])

D G U

0.01 0.05 0.1 0.25 0.75 1 5 10 50 100

The correct answer is:

D' = [0.01] − ([0.25] [D]· [G] ÷ [U]) − ([D]· [G]· [G]) − ([D]· [D] ÷ [U])

G' = ([0.05] [G]) − ([0.75] [D]· [G] ÷ [U])

U' = −([5] [D]) − ([50] [G])

Information

Consider the following model:

$$A' = A^2 - B$$
$$B' = AB - A$$

Let us use Euler's method to approximate the evolution of this model starting at (A,B)=(5,6) and t=0.3.

In the questions below, you will fill in some of the entries of the table below. For each question please double-check you are entering the correct entry.

Step	t	(A,B)	(A', B')	(A,B)+Δt(A',B')
0	0.3	(5,6)	(1)	(2)
1	(3)	(4)	(66.44, 88)	(5)
2	(6)	(22.088, 28.6)	(7)	(8)
3	0.9	(9)		

Question 4

Not answered

Points out of 2.00

Answer: (1)= (✖ , ✖)

Question 5

Not answered

Points out of 2.00

Answer: (2)= (✖ , ✖)

Question 6

Not answered

Points out of 1.00

Answer: (3)= (✖)

Question 7

Not answered

Points out of 2.00

Answer: (5)= (✖ , ✖)

Question 8

Not answered

Points out of 2.00

Answer: (9)= (✖ , ✖)

Information

Let's practice using our differentiation rules.

Question 9

Not answered

Points out of 3.00

By using the differentiation rules we learned, find derivative with respect to x of the following function:

$f(x) = \sin((x+1)(2x+2)(3x+3))$

- A. $18(x+1)^2 \sin((x+1)(2x+2)(3x+3))$
- B. $18(x+1)^2 \cos((x+1)(2x+2)(3x+3))$
- C. $108(x+1)^2 \cos((x+1)(2x+2)(3x+3))$
- D. $18(x+1)^3 \cos((x+1)(2x+2)(3x+3))$
- E. $6(x+1)^2 \sin((x+1)(2x+2)(3x+3))$

The correct answer is:
$18(x+1)^2 \cos((x+1)(2x+2)(3x+3))$

Information

The formula we learned for linear approximations was

$f(b) \approx f'(a)(b-a) + f(a)$

Let's use linear approximation to estimate $\sqrt{16.16}$.

Question 10

Not answered

Points out of 2.00

What values should we use for a and b ?

a = (✖)

b = (✖)

Question 11

Not answered

Points out of 1.00

What is the (exact) value of $f'(a)$ in this case?

$f'(a)$ = (✖)

Question **12**

Not answered

Points out of 1.00

What is the estimate for $\sqrt{16.16}$ we get from linear approximation?

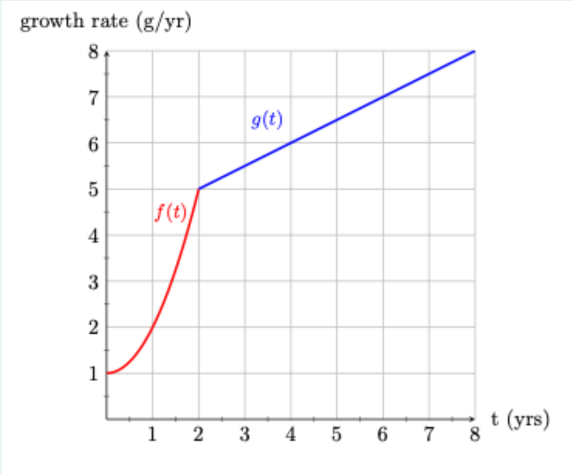
$\sqrt{16.16} \approx$ (✖)

Information

The model below shows the **growth rate** of a tumor in a patient. On the day the patient discovers they have a tumor, the tumor has a mass of **3 grams**.

For the first two years after discovering they have a tumor, the patient takes drug A. During that time, the growth rate is given by the function f(t). After two years, a new drug, drug B, becomes available and the patient switches to this new drug. The growth rate of their tumor is then given by the function g(t).

The graph of this model is shown below. The function f(t) is given by $f(t) = t^2 + 1$.



Question **13**

Not answered

Points out of 2.00

By how much does the tumor grow during the first two years?

Give your answer in grams with two decimal places.

Answer: ✖

The correct answer is: 4.67

Question **14**

Not answered

Points out of 2.00

By how many grams does the tumor grow during the first three years?

Give your answer in grams with two decimal places.

Answer: ✖

The correct answer is: 9.92

Question **15**

Not answered

Points out of 2.00

By how many grams does the tumor grows between the second and fourth year (ie. between t=2 and t=4)?

Give your answer in grams with two decimal places.

Answer: ✖

The correct answer is: 11

Question **16**

Not answered

Points out of 2.00

What is the size of the tumor after 5 years.

Give your answer in grams with two decimal places.

Answer: ✖

The correct answer is: 24.92

Information

The rate of change in soil microbe population is a product of four terms as given below:

$$f(X) = X' = (1 - 1.5X)(1 - 0.5X)X(1 - X)$$

Follow the instructions provided carefully and answer the following questions.

Question **17**

Not answered

Points out of 2.00

Find all the equilibria of this model.

- List them in ascending order by dragging the correct values into the blue boxes.
- **If necessary,** round it up to the 4th decimal place.
  Example formats: -1, 0.34, 1.5, 2.9899

0	1.5	-1.5	2	1.1	3	2.4667	0.6667	1	1.67	-1	3.8889

-0.25

Question 18

Not answered

Points out of 2.00

To perform linear stability analysis to classify the equilibria, provide the expression for df(X)/dX.

- Your answer must be a polynomial expression with increasing power of X (for example: $1+2X+3X^2+X^4$)
- Numerical coefficients of each term must be dragged into appropriate gray boxes provided.
- Each corresponding X power term must be dragged into the corresponding white boxes provided.

Note: a few people have reported having a technical challenge with this question. If this happens, upload a screenshot or handwritten answer on Gradescope.

df(X)/dX = [ ] + [ ] [ ] + [ ] [ ] + [ ] [ ]

3 | -3 | -2.75 | -6 | 8.25 | 0.5 | -1 | 1 | 2.75 | -0.5 | 0.75 | 2.75 | 3

X^-4 | X^2 | X^3 | X^-3 | X | X^-2 | X^4 | X^-1

The correct answer is:

To perform linear stability analysis to classify the equilibria, provide the expression for df(X)/dX.

- Your answer must be a polynomial expression with increasing power of X (for example: $1+2X+3X^2+X^4$)
- Numerical coefficients of each term must be dragged into appropriate gray boxes provided.
- Each corresponding X power term must be dragged into the corresponding white boxes provided.

Note: a few people have reported having a technical challenge with this question. If this happens, upload a screenshot or handwritten answer on Gradescope.

df(X)/dX = [1]+[-6][X] + [8.25][X^2] + [-3][X^3]

Question 19

Not answered

Points out of 2.00

Evaluate the slope of the tangent line to f(X) at each equilibrium point.

- Drag and drop the correct values of the slopes in the ascending order of the equilibrium points.
- If necessary, round it up to the 4th decimal place.
  Example format: -1, 0.34, 1.5, 2.9899

-1.25 | 0.4444 | 1.25 | -2 | -0.25 | -1 | -0.2222 | 0.2222 | 0.25 | 1.0 | -0.4444

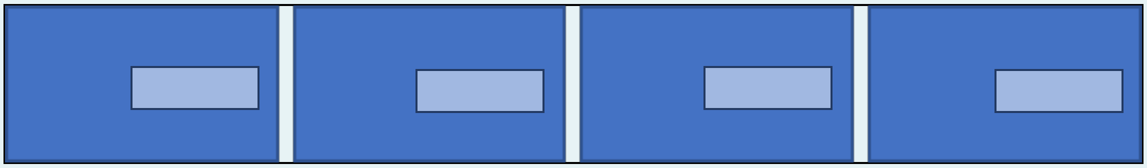
2 | -2.5 | 0 | -0.3 | -3.5

Question **20**

Not answered

Points out of 1.00

Classify the equilibria based on your linear stability analysis in the previous questions. Drag **the type** into the appropriate blue box in the ascending order of the values of the equilibria.



Unstable	Semi-Stable	Stable

Information

The guts and microbes again!! Two bacterial strains, X and Y are found in the gut system. The system of equations below describe their population growth despite within species crowding and interspecies competition.

$$X' = X(1 - X) - XY$$

$$Y' = 2Y\left(1 - \frac{Y}{2}\right) - 3XY$$

Question **21**

Not answered

Points out of 4.00

Find the equilibria algebraically. Drag and drop each correct answer into the text boxes provided. Your answer must be in the ascending order of values (with priority on the first entry).

Example:

(X*,Y*): (0,0), (1,5), (3,0), (3,5), (5,5) etc

____ , ____ , ____ , ____

(5,0)	(4,0)	(0.25,0.25)	(0,3)	(0,2)	(-0.5,1.5)	(1,0)	(0.1,0.1)
(-0.5,-0.5)	(0,5)	(2,0)	(0,0)	(1.5,-0.5)	(3,0)	(0.2,0.2)	(0.5,0.5)
(0,1)	(0,4)						

The correct answer is:

Find the equilibria algebraically. Drag and drop each correct answer into the text boxes provided. Your answer must be in the ascending order of values (with priority on the first entry).

Example:

(X*,Y*): (0,0), (1,5), (3,0), (3,5), (5,5) etc

[(0,0)], [(0,2)], [(0.5,0.5)], [(1,0)]

Question **22**

Not answered

Points out of 1.00

Which of the following is a nullcline of this model?

- a. X=1
- b. X=0
- c. X=Y
- d. X=0.5

The correct answer is:
X=0

Question **23**

Not answered

Points out of 1.00

Which of the following is a nullcline of this model?

- a. Y=1
- b. Y=X
- c. Y=0.5
- d. Y=0

The correct answer is:
Y=0

Question **24**

Not answered

Points out of 1.00

Which of the following is a X-nullcline of this model?

- a. Y=1-X
- b. Y=3X-2
- c. Y=X-1
- d. Y=2-3X

The correct answer is:
Y=1-X

Question **25**

Not answered

Points out of 1.00

Which of the following is a Y-nullcline of this model?

- a. Y=3X-2
- b. Y=1-X
- c. Y=2-3X
- d. Y=-(2-X)/3

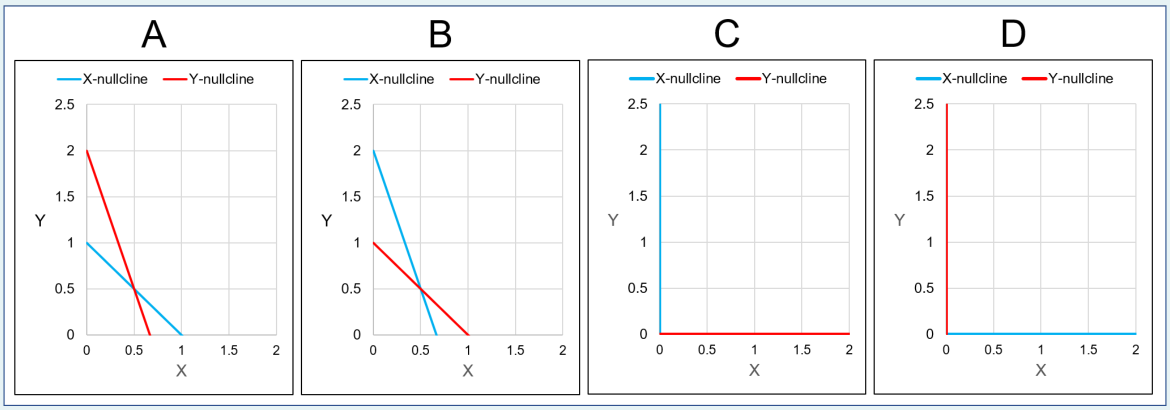
The correct answer is:
Y=2-3X

Question **26**

Not answered

Points out of 1.00

Which of the following is/are correct?



- a. A
- b. B
- c. C
- d. D
- e. None of the above

The correct answers are:
A,

C

Information

Let's study the cell fate determination in the **insect eye**:

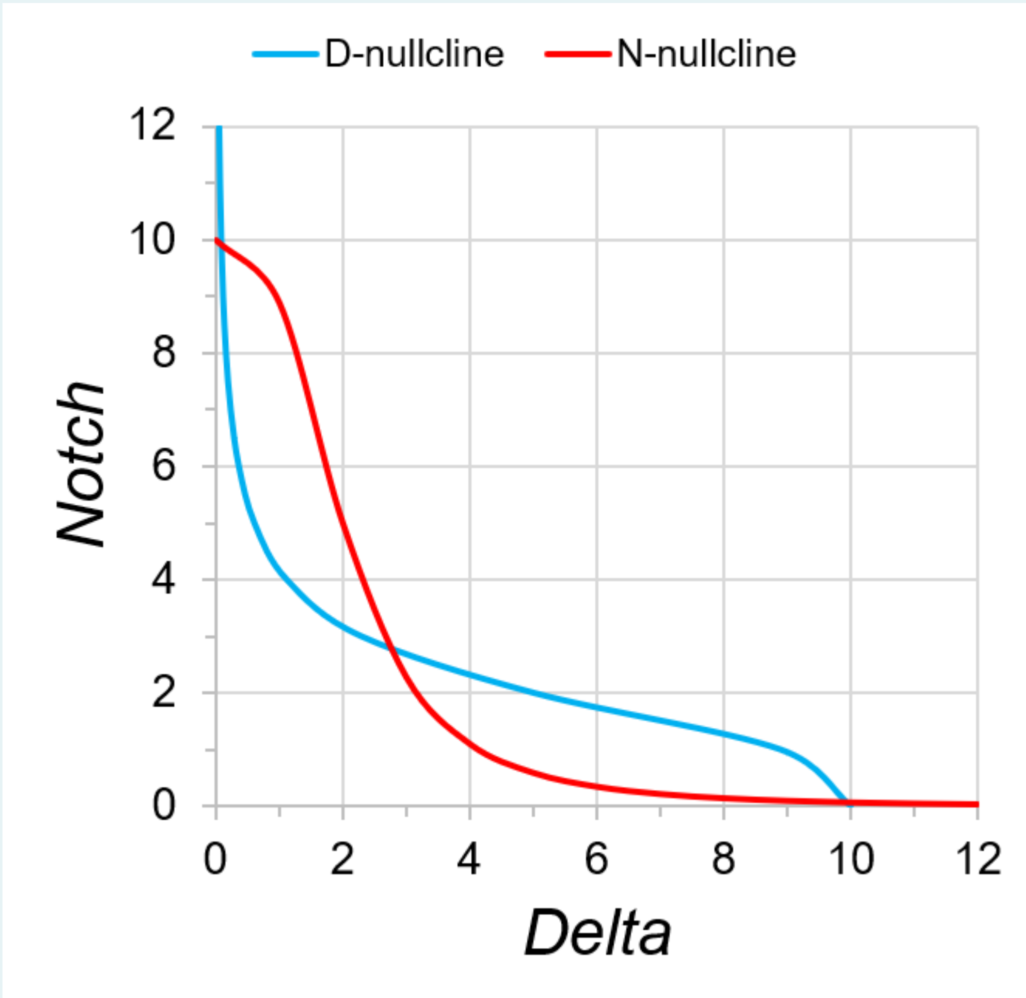
"An insect's compound eye is made up of many individual units packed together to form the surface of the eye. These units are hexagonal in shape and called **ommatidia** (singular ommatidium). Each eye can have more than a thousand ommatidia. Each ommatidia has several photoreceptors and these allow the compound eye to form a mosaic image."
(source: https://www.amentsoc.org/insects/glossary/terms/ommatidia)

Question **27**

Not answered

Points out of 1.00

During development, two types of photoreceptors R3 and R4 are formed in each ommatidium. The developmental decision of which type of photoreceptor is largely dependent on two genes, ***Delta (D)*** and ***Notch (N)***, which repress one another. If D is highly expressed, there will be more R3s, while if N is highly expressed, there will be more R4s. A two-variable system of equations for the Delta/Notch system is developed and the state-space with the nullclines is shown below:



Based on this picture, how many equilibria are in this system?

- a. 0
- b. 1
- c. 2
- d. 3
- e. 4

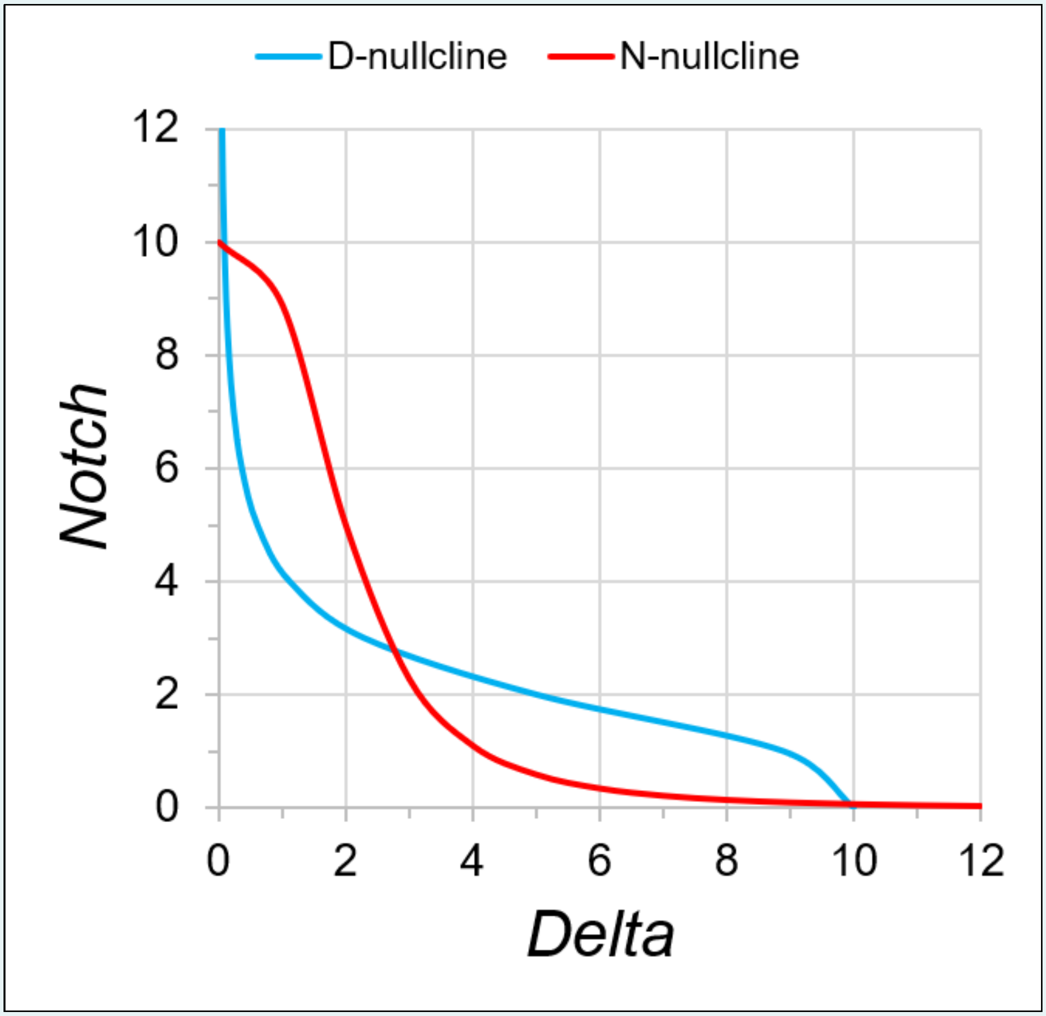
The correct answer is:
3

Question 28

Not answered

Points out of 2.00

For the Delta/Notch model, if you were to use the nullclines and test points approach to find the stability of equilibria, suppose four test-points are given to you. In the diagram below, drag and drop the 4 points into the appropriate segment of the state space partitioned by these nullclines:



(6,4) (2,2) (6,0.5) (0.5,6)

Question **29**

Not answered

Points out of 2.00

Match each test point (D,N) with its change vector (D',N'). The model equations are once again given below:

$$D' = \frac{2}{1 + \left(\frac{N}{2}\right)^3} - 0.2D$$

$$N' = \frac{2}{1 + \left(\frac{D}{2}\right)^3} - 0.2N$$

(6,4) Choose...

(0.5,6) Choose...

(6,0.5) Choose...

(2,2) Choose...

The correct answer is:
(6,4) → (-0.98,-0.73),

(0.5,6) → (-0.03,0.77),

(6,0.5) → (0.77,-0.03),

(2,2) → (0.6,0.6)

Question **30**

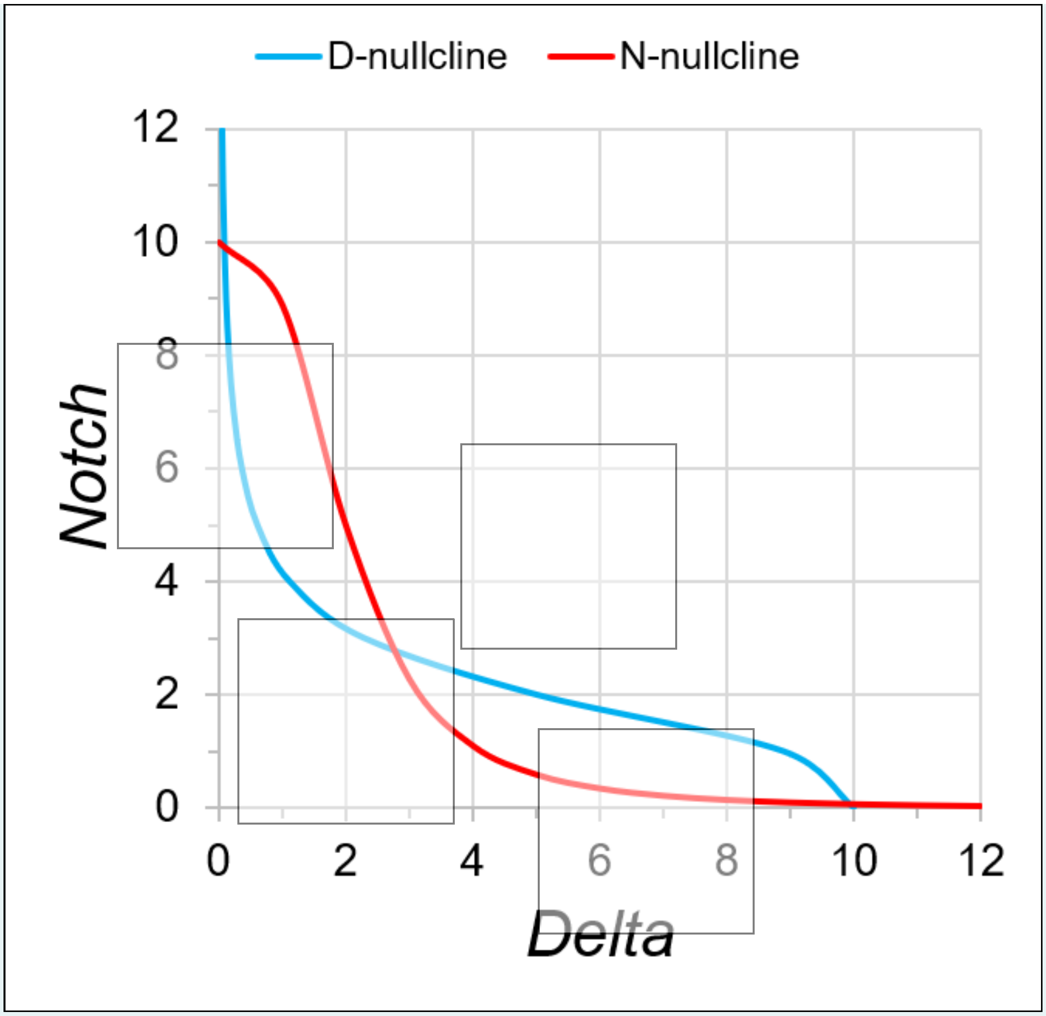
Not answered

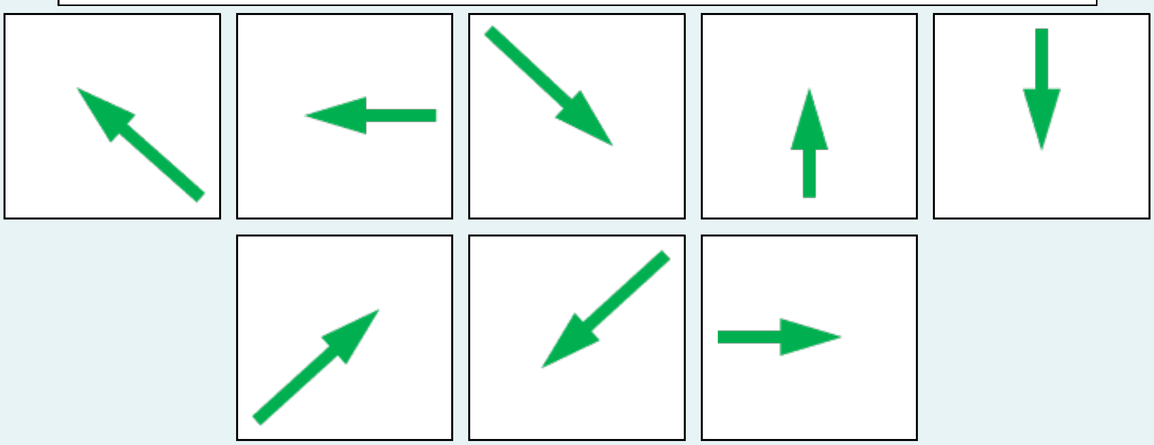
Points out of 2.00

Given the Delta/Notch model equations as below, compute the change vectors for the four test points, **(2,2)**, **(6,0.5)**, **(6,4)** and **(0.5,6)**. Determine the general direction of these arrows. Drag and drop the arrows given here into the appropriate positions in the state space for each of the four test points given. (Note: The size of arrows provided won't be the actual length of the vectors. Don't worry, just make sure that the general direction is correct).

$$D' = \frac{2}{1 + \left(\frac{N}{2}\right)^3} - 0.2D$$

$$N' = \frac{2}{1 + \left(\frac{D}{2}\right)^3} - 0.2N$$


D-nullcline
N-nullcline
Notch
Delta
0
2
4
6
8
10
12



Question **31**

Not answered

Points out of 1.00

Which of the following describes the behavior of the Delta/Notch system?

a. Bistable switch

b. Monostable switch

c. Allee effect

d. Logistic growth

The correct answer is:
Bistable switch

Information

In the 1990s, wolves started reappearing in the Western Alps. Since that time, wolves populations have been growing. Since wolves attack sheep, sheep farmers have been actively supporting regulations to allow hunters to kill wolves. The goal of this exercise is to see how many animals could be killed per year without leading to the extinction of wolves in the Western Alps.

Suppose that in the given area, the wolf population (W) follows the model

$$W' = W\left(1 - \frac{W}{100}\right) - c$$

where c is the number of wolves killed by hunters every year.

Question **32**

Not answered

Points out of 1.00

What is the name of this model when c=0?

Select one:

a. Holling-Tanner model

b. population model with logistic growth

c. population model with exponential growth

d. Lotka-Volterra model

e. population model with Allee effect

The correct answer is:
population model with logistic growth

Question **33**

Not answered

Points out of 0.50

What are the equilibrium points of this model when c=0?

Give the value of the *smallest* equilibrium point.

Answer: ✖

The correct answer is: 0

Question **34**

Not answered

Points out of 0.50

What is the stability of the equilibrium point you have computed in the previous question?

Select one:

- a. stable
- b. semi-stable
- c. unstable

The correct answer is:
unstable

Question **35**

Not answered

Points out of 0.50

What are the equilibrium points of this model when c=0?

Give the value of the *largest* equilibrium point.

Answer:

The correct answer is: 100

Question **36**

Not answered

Points out of 0.50

What is the stability of the equilibrium point you have computed in the previous question?

- a. stable
- b. unstable
- c. semi-stable

The correct answer is:
stable

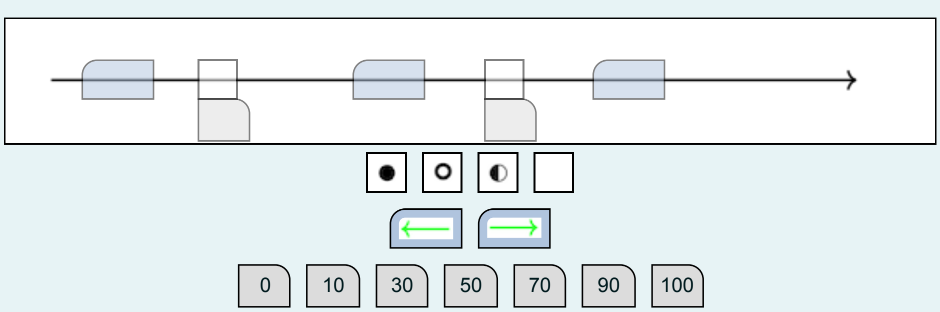
Question **37**

Not answered

Points out of 2.00

Sketch the phase portrait of this model for c=9, ie. for $W' = W\left(1 - \frac{W}{100}\right) - 9.$

Here, solid dots represent stable equilibrium points, hollow dots represent unstable equilibrium points, and half-solid dots represent semi-stable equilibrium points; you can also drag in the blank square to indicate that no equilibrium point is present.



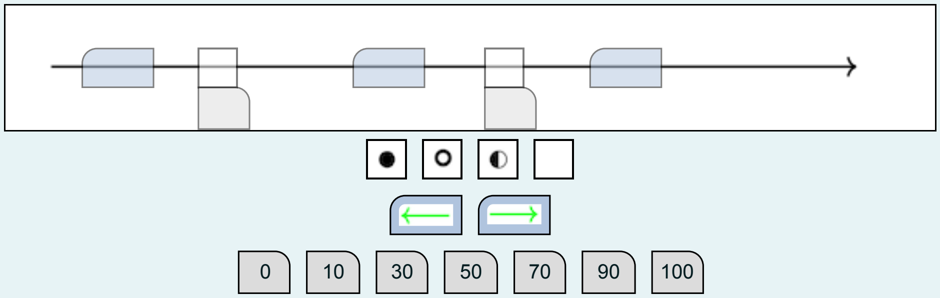
Question **38**

Not answered

Points out of 2.00

Sketch the phase portrait of this system for c=21, ie. for $W' = W\left(1 - \frac{W}{100}\right) - 21$.

Here, solid dots represent stable equilibrium points, hollow dots represent unstable equilibrium points, and half-solid dots represent semi-stable equilibrium points; you can also drag in the blank square to indicate that no equilibrium point is present.



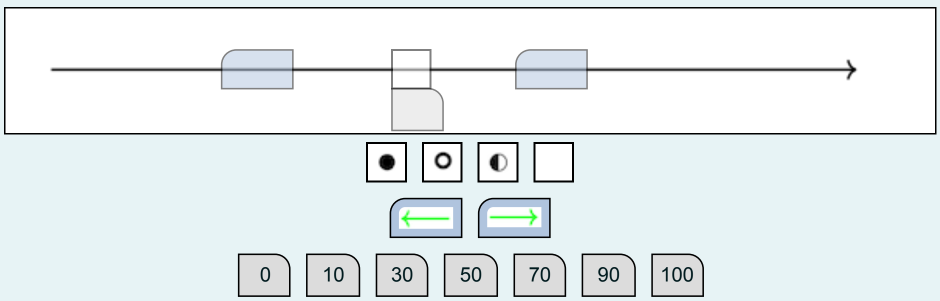
Question **39**

Not answered

Points out of 2.00

Sketch the phase portrait of this model for c=25, ie. for $W' = W\left(1 - \frac{W}{100}\right) - 25$.

Here, solid dots represent stable equilibrium points, hollow dots represent unstable equilibrium points, and half-solid dots represent semi-stable equilibrium points; you can also drag in the blank square to indicate that no equilibrium point is present.



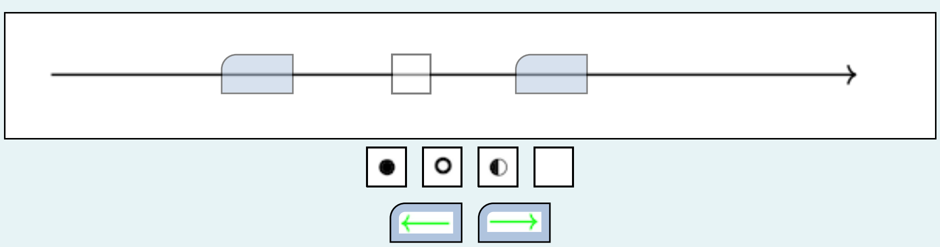
Question **40**

Not answered

Points out of 1.00

Sketch the phase portrait of this model for c=27, ie. for $W' = W\left(1 - \frac{W}{100}\right) - 27$.

Here, solid dots represent stable equilibrium points, hollow dots represent unstable equilibrium points, and half-solid dots represent semi-stable equilibrium points; you can also drag in the blank square to indicate that no equilibrium point is present.



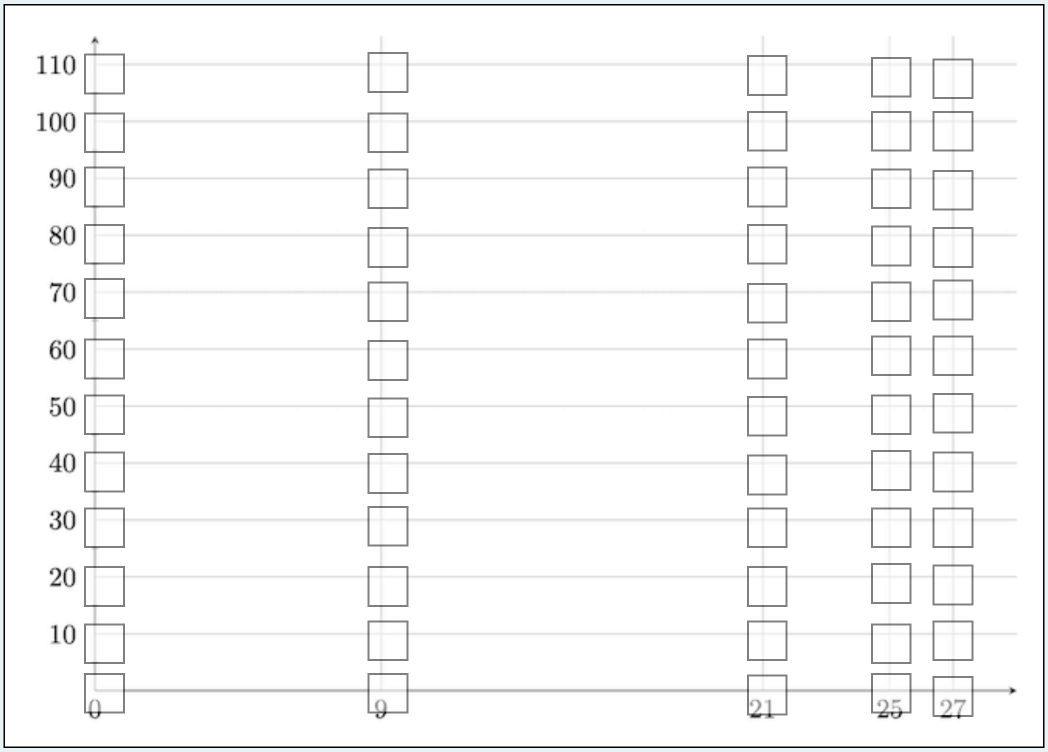
Question **41**

Not answered

Points out of 2.00

Complete the bifurcation diagram below by correctly placing stable equilibrium points (full dots), unstable equilibrium points (empty dots) and semi-stable equilibrium points (half-full dots). For the areas where there is no equilibrium point use the blank picture.

NOTE: some people have reported having technical difficulties with this question. INSERT FIRST THE EQUILIBRIUM POINTS AND TAKE A SCREENSHOT. In case there is a glitch, upload your screenshot (or handwritten answer) on Gradescope.



Question **42**

Not answered

Points out of 1.00

How many bifurcation(s) is/are there in this diagram?

Answer:

The correct answer is: 1

Question **43**

Not answered

Points out of 1.00

Give one value of c for which a bifurcation occurs.

Answer:

The correct answer is: 25

Question **44**

Not answered

Points out of 1.00

Suppose that 9 wolves are hunted and killed every year. You have determined that the wolf population currently has 20 individuals. In the long-run, how many individuals will the population tend to?

Answer: ✖

The correct answer is: 90

Question **45**

Not answered

Points out of 1.00

Suppose that 21 wolves are hunted and killed every year. You have determined that the wolf population currently has 20 individuals. In the long-run, how many individuals will the population tend to?

Answer: ✖

The correct answer is: 0

Question **46**

Not answered

Points out of 1.00

Suppose that 25 wolves are hunted and killed every year. You have determined that the wolf population currently has 80 individuals. In the long-run, how many individuals will the population tend to?

Answer: ✖

The correct answer is: 50

Question **47**

Not answered

Points out of 1.00

Suppose that 27 wolves are hunted and killed every year. You have determined that the wolf population currently has 80 individuals. In the long-run, how many individuals will the population tend to?

Answer: ✖

The correct answer is: 0

◄ Final Exam Stage3

Jump to...

Office Hours Dec 14 - 9am ►