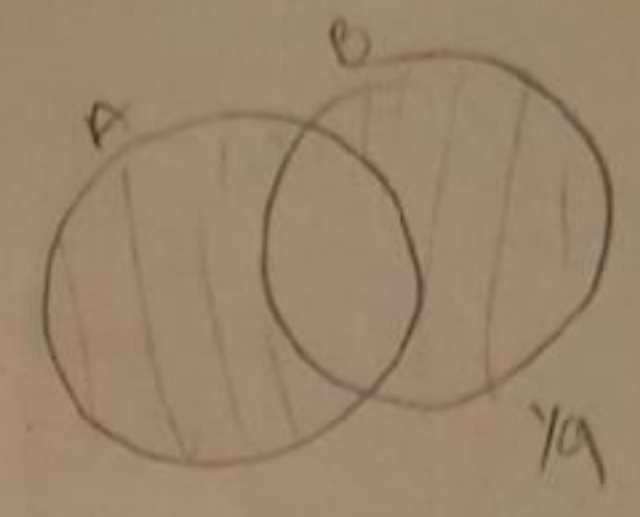


1 Multiple Choice Problems



$P(A) = P(B)$
 $\frac{8}{9} = 2P(A) - P(A)^2$
 $\frac{8}{9} = P(A)[2 - P(A)]$
 $1 - 2$

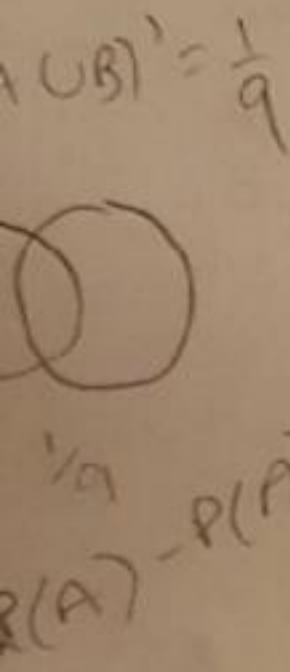
The problems in this section only has one correct answer among the choices a, b, c and d. You will get 1 credit if your choice is correct and 0 credit otherwise.

1. Suppose that A and B are two independent events.

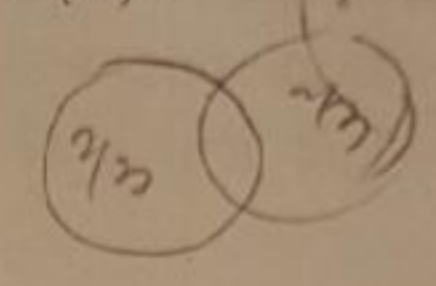
(1) If $P(A) = 0.4$ and $P(A \cup B) = 0.7$, what is $P(B'|A)$?
 (a) 0.3 (b) 0.2 (c) 0.6 (d) 0.5

$P(A \cup B) = P(A) + P(B) - P(A)P(B)$
 $0.7 = 0.4 + P(B) - 0.4P(B)$
 $0.3 = 0.6P(B)$
 $P(B) = 0.5$
 $P(B'|A) = P(B)$

(2) If $P(A' \cap B') = \frac{1}{9}$ and $P(A' \cap B) = P(A \cap B')$, what is $P(A)$?
 (a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{5}{9}$ (d) $\frac{1}{3}$



(3) If $P(A) = 0.6$ and $P(B) = 0.5$, what is $P(A \cap B)$?
 (a) 0.3 (b) 0.2 (c) 0.7 (d) 0.5



(4) If $P(A) = 0.4$ and $P(B) = 0.5$, what is $P(A \cup B)$?
 (a) 0.3 (b) 0.2 (c) 0.7 (d) 0.5

$P(A \cup B) = 0.4 + 0.5 - 0.4(0.5)$

(5) If $P(A) = 0.5$ and $P(B) = 0.5$, what is $P(A' \cup B')$?
 (a) 0.45 (b) 0.25 (c) 0.75 (d) 0.50

$P(A' \cup B') = 1 - P(A \cap B)$
 $1 - 0.5 \times 0.5 = 0.75$

2. There are 3 red balls and 7 blue balls in a bag. We take 4 balls randomly from the bag without replacement.

(6) What is the probability of getting 2 red balls?
 (a) $\frac{3}{10}$ (b) $\frac{1}{6}$ (c) $\frac{4}{7}$ (d) $\frac{3}{7}$

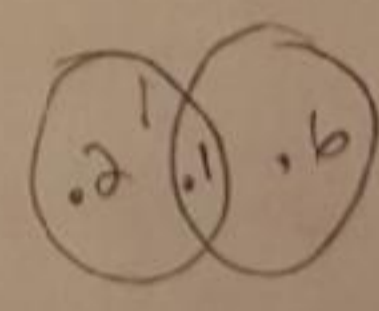
$\frac{3 \times 2 \times 7 \times 6}{10 \times 9 \times 8 \times 7}$
 $+\frac{3 \times 7 \times 6 \times 2}{10 \times 9 \times 8 \times 7}$

(7) What is the probability of getting 4 blue balls?
 (a) $\frac{3}{10}$ (b) $\frac{1}{8}$ (c) $\frac{4}{7}$ (d) $\frac{3}{7}$

$\frac{7 \times 6 \times 5 \times 4}{10 \times 9 \times 8 \times 7}$

3. Suppose that A, B are two events with $P(A) = 0.2$, $P(B) = 0.6$ and $P(A \cup B) = 0.7$. Then:

(8) the probability of A' equals:
 (a) 0.3 (b) 0.1 (c) 0.8 (d) 0.5



(9) the probability of $A \cap B'$ equals:
 (a) 0.3 (b) 0.1 (c) 0.8 (d) 0.5

(10) the probability of $A' \cap B'$ equals:
 (a) 0.3 (b) 0.1 (c) 0.9 (d) 0.6

(11) the probability of $A' \cup B'$ equals:
 (a) 0.3 (b) 0.1 (c) 0.9 (d) 0.6

4. There are 90 red balls and 10 blue balls in a bag.

$$\frac{8010}{9900}$$

(12) If we take two balls from the bag sequentially without replacement, what's the probability of getting 2 red balls?

$$\frac{90}{100} \cdot \frac{89}{99}$$

$$\frac{900}{9900} + \frac{900}{9900} +$$

- (a) 0.8091 (b) 0.0091 (c) 0.9901 (d) 0.9000

(13) If we take two balls from the bag sequentially without replacement, what's the probability of getting at least one red balls?

$$\frac{90}{100} \cdot \frac{10}{99} + \frac{10}{100} \cdot \frac{90}{99} + \frac{90}{100} \cdot \frac{89}{99}$$

- (a) 0.8091 (b) 0.0091 (c) 0.9909 (d) 0.9000

(14) If we take two balls from the bag sequentially with replacement, what's the probability of getting 2 red balls?

$$\frac{90}{100} \cdot \frac{90}{100}$$

- (a) 0.8100 (b) 0.5091 (c) 0.9871 (d) 0.9000

5. A and B are two events. We know that $P(A) = \frac{1}{4}$, $P(B|A) = \frac{1}{3}$ and $P(A|B) = \frac{1}{2}$.

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{1}{3}$$

(15) What is $P(B)$?
(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $\frac{1}{5}$

$$P(B \cap A) = \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$$

(16) What is $P(A \cup B)$?
(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $\frac{1}{5}$

$$P(A|B) = \frac{P(B \cap A)}{P(B)} = \frac{1}{2} \quad P(B) = \frac{1}{12} \cdot 2 = \frac{1}{6}$$

$$P(A \cup B) = \frac{1}{4} + \frac{1}{6} - \frac{1}{12}$$

6. Suppose that there are 5 dollar bills in a box: three 1 dollar bills, one 5 dollar bill and one 10 dollar bill. You are allowed to pick up two bills (without replacement) from the box randomly. Let X denote the money you get from this game.

$$f(x) = \frac{x}{5} \quad S_x = 3, 4, 10$$

(17) What's probability of getting at least 10 dollars?
(a) 0.5 (b) 0.4 (c) 1.0 (d) 0.7

$$P(X \geq 10)$$

$$X = 1, 5, 10$$

$$P(X=5, X=10) \quad P(X=10)$$

(18) What's the mean of X?
(a) 5.0 (b) 3.6 (c) 8.0 (d) 7.2

$$\frac{3}{5}(1) + \frac{1}{5}(5) + \frac{1}{5}(10)$$

$$\frac{3}{5} + \frac{5}{5} + \frac{10}{5}$$

(19) What's the variance of X?
(a) 12.64 (b) 5.75 (c) 25.60 (d) 18.96

$$\frac{3}{5} + \frac{25}{5} + \frac{100}{5} - 36 = 12.96$$

7. In an instant lottery game, the probability of drawing a winning ticket is 0.1. Suppose that you buy 10 tickets.

$$\frac{1}{5} \cdot \frac{3}{4} + \frac{3}{5} \cdot \frac{1}{4} + \frac{1}{5} \cdot \frac{1}{4} + \frac{1}{5} \cdot \frac{1}{4}$$

$$3 + 3 + 1 + 1 = \frac{8}{20} = \frac{4}{10}$$

$$\frac{1}{5} \cdot \frac{3}{5} + \frac{1}{5} \cdot \frac{1}{5} + \frac{1}{5} \cdot \frac{1}{4} + \frac{1}{5} \cdot \frac{1}{4}$$

(20) What's probability of having two winning tickets among the 10?
 (a) 0.7361 (b) 0.9298 (c) 0.1937 (d) 0.3487

(21) What's probability of having at least five winning tickets among the 10?
 (a) 0.9872 (b) 0.9984 (c) 0.0112 (d) 0.0016 $P(X \geq 5)$

(22) What's probability of having at most three winning tickets among the 10?
 (a) 0.9872 (b) 0.9984 (c) 0.0112 (d) 0.9298 $P(X \leq 3)$

8. Suppose that X is a random variable with support $S_x = \{1, 2, 3\}$ and probability mass function (p.m.f.):

$$f(x) = c(4 - x) \text{ for any } x \in S_x$$

where c is some constant.

(23) What's value of c ?
 (a) $\frac{1}{5}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{6}$

(24) What's the mean of X ?
 (a) $\frac{5}{3}$ (b) $\frac{3}{2}$ (c) $\frac{5}{9}$ (d) $\frac{5}{6}$

(25) What's the variance of X ?
 (a) $\frac{5}{3}$ (b) $\frac{3}{2}$ (c) $\frac{5}{9}$ (d) $\frac{5}{6}$

$$c(4-1) + c(4-2) + c(4-3) = 1$$

$$3c + 2c + 1c = 1$$

$$6c = 1$$

$$c = \frac{1}{6}$$

$$f(x) = \frac{1}{6}(4-x)$$

$$\frac{1}{6}(3)(1) + \frac{1}{6}(2)(2) + \frac{1}{6}(1)(3)$$

$$\frac{3}{6} + \frac{4}{6} + \frac{3}{6} = \frac{10}{6}$$

9. There are 10 keys and only one of them can open a lock. We do not know which key can open the lock, but we will try them one by one until we find the right key. We record the key(s) which we have already tried.

(26) What's the probability of finding the right key in the first trial?
 (a) 0.01 (b) 0.1 (c) 0.4 (d) 0.125

(27) What's the probability of finding the right key in the third trial?
 (a) 0.01 (b) 0.1 (c) 0.4 (d) 0.125

(28) If we know that the first two keys we tried are not the right key, what's the probability of finding the right key in the next/third trial?
 (a) 0.01 (b) 0.1 (c) 0.4 (d) 0.125

(29) Let X denote the number of times tried to find the right key. What's the mean of X ?
 (a) 2.5 (b) 5.25 (c) 5.5 (d) 5

(30) Let X denote the number of times tried to find the right key. What's the variance of X ?
 (a) 8.5 (b) 8.25 (c) 4.25 (d) 8

$$P(3^{rd} / \text{not } 1^{st} \& 2^{nd}) = \frac{P(3^{rd} \cap \text{not } 1^{st} \& 2^{nd})}{\text{not } 1^{st} \& 2^{nd}} = \frac{\frac{9}{10} \cdot \frac{8}{9} \cdot \frac{1}{8}}{\frac{9}{10} \cdot \frac{8}{9}} = \frac{1}{8}$$

$$1^2 + 4 + 9 + 16 + 25 + 36 + 49 + 64 = 385$$

$$385 - (5.5)^2 = 385 - 30.25 = 354.75$$