100%: AK $P(AK) = \frac{(4C1)(4C1)}{52C2}$ $P(AA) = \frac{(4C1)(4C1)}{52C2}$

50%: AA

30%: pair $P(pair) = \frac{13 \cdot (4C2)}{52C2}$

1. Suppose you know that your opponent would go all in before the flop with 100% probability if she had AK, she would go all in before the flop with 50% probability if she had AQ, and she would go all in before the flop with 30% probability if she had a pocket pair. With other hands, there is 0% probability that she would go all in before the flop. Given that she has gone all in before the flop, and given no other information about her cards or her opponents' cards, what is the probability that she has AK?

a. 33.8%. b. 41.2%. c. 50.0%. d. 62.2%. e. None of the above.

For the next two problems, suppose Minieri has Q♥ 2♥, his only opponent Lederer has 10♥ 8♥, and the flop is 2♣ 10♣ 9♣. Before the turn is revealed, the pot is \$100. The turn is 4♣, and Lederer goes all in for \$20. Minieri has more than \$20 and is deciding whether to call.

Queen or 5

2 10 9 4

2. Assuming Minieri wants to maximize his expected number of chips after this hand, and assuming knowledge of his cards, Lederer's cards, and the 4 cards on the board, should Minieri call?

a. Yes, because Minieri should call if his probability of winning the hand is at least 14.1%, and his probability of winning the hand is 14.3%.

b. Yes, because Minieri should call if his probability of winning the hand is at least 11.2%, and his probability of winning the hand is 11.4%.

c. No, because Minieri should only call if his probability of winning the hand is at least 11.8%, but his probability of winning the hand is only 11.4%.

d. No, because Minieri should only call if his probability of winning the hand is at least 14.3%, but his probability of winning the hand is only 11.4%.

e. None of the above.

$P(win) = \frac{5}{44} = 11.4\%$. $P(win) > \frac{20}{100+20+20} = 0.143$

3. Minieri calls, and the river is the 2♣, giving him 3 of a kind. How much equity did Minieri gain due to luck on the river?

a. \$101. b. \$112. c. \$118. d. \$124. e. None of the above.

$1/(\$140) - (5/44)(\$140) = \$124$

For the next two problems, let X = \$5 if you are dealt two hole cards of the same suit as each other on your next hand, and X = \$0 otherwise.

$P(\text{same suit}) = \frac{4 \cdot 13C2}{52C2} = 0.2353$

4. What is the expected value of X, in dollars?

a. 1.18. b. 2.20. c. 2.45. d. 2.89. e. None of the above. $E(X) = 5(0.2353)$

5. What is the variance of X, in dollars?

$Var(X) = E(X^2) - [E(X)]^2 = 4.50$ $E(X^2) = 5^2(0.2353)$

a. 2.75. b. 4.50. c. 6.28. d. 7.04. e. None of the above.

$P(AB) = P(A) \cdot P(B|A)$

6. Let A be the event that both your hole cards are kings, and let B be the event that both your hole cards are red cards. Are A and B independent?

a. No. b. Yes. c. Cannot be determined from the information given.

$P(A) = \frac{4C2}{52C2}$ $P(B|A) = \frac{2C2}{50C2}$

A = both are kings B = both are red

7. Suppose you play 100 hands of Texas Holdem. Let X = the number of hands where both your hole cards are clubs. What is the expected value of X?

a. 4.29. b. 4.75. c. 5.88. d. 6.20. e. None of the above.

$P(X) = \frac{13C2}{52C2} = 0.0588$ $\mu = np = 100(0.0588) = 5.88$

(Binomial)

Geometric

$\mu = 1/p = 1/.0588 = 17.0$

8. Let X = the number of hands until the first time you are dealt 2 cards that are both clubs. What is E(X)?
a. 15.1. b. 16.2. c. 17.0. d. 18.9. e. None of the above.

9. Suppose X and Y are uniform on (0,1) and X and Y are independent. What is cov(2X+Y, 3X+4Y)?
a. -0.404. b. 0.404. c. 0.518. d. 0.833. e. None of the above.

10. Suppose X is exponential with $\lambda = 2$, so E(X) = 1/2. What is $P(2 < X < 3)$? (e ~ 2.718282).
a. 0.593%. b. 1.58%. c. 7.29%. d. 8.03%. e. None of the above.

11. Suppose the odds against X are 3, or as it is sometimes written, 3:1. What is P(X)?
a. 20.0%. b. 25.0%. c. 33.3%. d. 50.0%. e. None of the above. odds against X = 3:1

$P(X) = \frac{1}{4}$

12. What is the probability that the suits of your 2 hole cards will be different from each other?
a. 50.0%. b. 76.5%. c. 81.5%. d. 84.5%. e. None of the above.

13. What is the probability that your 2 hole cards will form a pair, given that your 2 hole cards are of different suits?
a. 4.33%. b. 5.02%. c. 6.49%. d. 7.69%. e. None of the above.

$3/39$

14. What is the probability that the 3 cards on the flop are all different numbers? For instance, if the cards are 7♠ Q♥ Q♣, then we would say two of them are the same number (the two queens). Or if the cards are 7♠ 7♥ 7♣, then they are all the same number (all 7s). But if they are 7♠ Q♠ 4♠ then they are all different numbers.
a. 82.8%. b. 84.4%. c. 87.0%. d. 89.2%. e. None of the above.

$6X^2 + 3XY + 8XY + 4Y^2$

a) $\text{cov}(2X+Y, 3X+4Y) = E[(2X+Y)(3X+4Y)] - E(2X+Y)E(3X+4Y)$

$= E[6X^2 + 11XY + 4Y^2] - E(2X+Y)E(3X+4Y)$

$= E(6X^2) + E(11XY) + E(4Y^2) - [E(2X) + E(Y)][E(3X) + E(4Y)]$

$= 6E(X^2) + 11E(XY) + 4E(Y^2) - [2E(X) + E(Y)][3E(X) + 4E(Y)]$

$= 6E(X^2) + 11E(XY) + 4E(Y^2) - 6E(X)E(X) - 8E(X)E(Y) - 3E(X)E(Y) - 4E(Y)E(Y)$

$= 6E(X^2) + 11E(XY) + 4E(Y^2) - 6E(X)^2 - 11E(X)E(Y) - 4E(Y)^2$

$= 6E(X^2) - 6[E(X)]^2 + 4E(Y^2) - 4[E(Y)]^2$

$= 6\,\text{Var}(X) + 4\,\text{Var}(Y)$

$\text{cov}(2X+Y, 3X+4Y) = 10/(1/2) = 5.33$

X, Y = uniform on (0, 1)

$\text{Var}(X) = E(X^2) - [E(X)]^2$

$\text{Var}(X) = \frac{1}{3} - (\frac{1}{2})^2 = \frac{1}{12}$

$\text{Var}(Y) = 1/12$

$E(X) = \int_0^1 y\,dy = \frac{y^2}{2}\Big|_0^1 = 1/2$

$E(X^2) = \int_0^1 y^2\,dy = \frac{y^3}{3}\Big|_0^1 = 1/3$