READ THIS AND NEXT PAGE TO KNOW REQUIREMENTS AND SAVE TIME THE DAY OF THE EXAM

- **MIDTERM 1 ON TUESDAY, JULY 16th,** and has two parts: 10:00-11:30 (regular exam with Multiple Choice showing no work + short answer questions showing work.); 11:35-11:50 simulation (required). Please, bring #2 pen for scantron. Multiple choice questions part will be answered on both the exam and a scantron (required). The scantron requires use of #2 pencil. Failure to write your multiple choice answers on the scantron will result in 0 points in the MC part. Read the instructions given inside the exam carefully.
- **ROOM: BROAD, 2160E** (Notice change of room). Please, wait outside the room until you are asked to enter (unless otherwise indicated). Students that are left handed will be seated in a left-handed seat. The tables of your neighboring chairs must be down during the exam.
- **EXAM COVERS:** all lecture notes posted in ccle, the two quizzes, the three homeworks and the work done during the TA sessions.
- **DURATION OF EXAM:** First part, 1 hour and 30 minutes (notice small modification of syllabus) - Then after regular exam, required simulation, part of the midterm. No TA session on July 16th. You must stay in the exam room during the duration of the exam. No breaks.
- Only scientific calculators will be allowed. You may have a cheat sheet with only formulas and definitions in it. No graphs, no drawings, no solved problems, no proofs, no intermediate steps and no numerical examples allowed. A sample of an appropriate and inappropriate cheat sheet will be posted. Read more details on the next page.
- **Having solved problems, numerical examples, intermediate steps of any result in your cheat sheet will result in deduction of points from your exam: 1 pt for each item. The cheat sheet will be checked during the exam.**
- ID is required at all times during the exam. Honor code applies as indicated in the syllabus.
- Pre-midterm EXTRA office hours (only this week) on Monday July 15: Gary Evans, 12:30-3:00 PM; Quian Xiao, 9:30 AM-12:30 PM. Prof. Sanchez, 8:00-9:00 AM and 3:00-3:50 PM.
- If there is a typo in the practice exam, I will probably catch it when preparing the key, but I will appreciate it if you mail me (jsanchez@stat.ucla.edu) if you catch one. I may not answer during the weekend, but I will make sure I fix it in the answer key.

READ: More detailed instructions on the next page. READ now, to save time the day of the exam. Failure to follow directions will results in points lost in the exam.

Further Instructions

- ID must be ready to show at all times during the exam.
- From the moment the exam starts until the moment you are out of the exam room, you can not talk to each other, you can not exchange papers or information, and you can not use your phone or other electronic devices. All your things must be on the floor. You may not use the empty seats next to you to put things.
- Only scientific calculator, cheat sheet and pen or pencil and exam and scantron on your desk. Everything else (disconnected) in your backpack and your backpack on the floor.
- Cheat sheet can have only formulas and definitions, no solved problems, no examples of any kind, no proofs.YOUR NAME ON CHEAT SHEET AT ALL TIMES. Be ready to show your cheat sheet when the instructor requests it. The cheat sheet must be written all in English. Cheat sheet must be turned in with exam. Cheats sheats that do not comply will result in lower grade in the exam.
- You will be given the table of distributions that is posted next to this exam. Only the discrete distributions in that table are needed for the exam. Do not detach from the exam.
- In questions where you show work, you must use the same notation we have used in lecture and the textbook.

NAME: –
UCLA ID: –

NOTE: This practice exam is just an indication of the format of the exam. The questions asked in the exam will be different and the length of the exam will be different (and appropriate for the time allowed).

KEY WILL BE POSTED SEPARATELY, SUNDAY AFTERNOON).
THE EXAM IS USELESS TO YOU IF YOU DO NOT TRY TO SOLVE THE QUESTIONS ON YOUR OWN. GRADE YOURSELF WITH THE KEY AFTER YOU HAVE STUDIED AND DONE THIS ON YOUR OWN. THEN IF YOU STILL HAVE QUESTIONS AFTER YOU HAVE STUDIED AND GRADED YOURSELF, USE OUR OFFICE HOURS ON MONDAY. Studying for the exam using only this exam is not recommended. The questions in the exam will be different.

EXAMPLES OF MULTIPLE CHOICE QUESTIONS. ONLY ONE ANSWER IS CORRECT. CHOICE MUST BE MARKED ON THE SCANTRON, AND ALSO HERE ON THE EXAM. ONLY THE SCANTRON WILL BE GRADED. NO MARKS ON SCANTRON OR MORE THAN ONE MARK WILL RESULT IN 0 POINTS FOR THE QUESTION NOT MARKED, EVEN IF IT IS MARKED ON THE EXAM.

Problem 1. A system consists of five components in parallel. The system works if at least one of the components works. If each component works with probability 0.97, what is the probability that the system works? Mark your answer here and in the scantron.

(a) 1.05

(b) 0.03

(c) 1

(d) 0

(e) 0.45

Solution 1. Prob(at least one works)= 1-Prob(none works). $1 - 0.03^5 = 1$

Problem 2. The probability of being able to access the internet using broadband at each attempt in a certain area is 0.7. A user keeps trying until access is achieved. What is the probability that more than four attempts will be required before access is attained? Mark your answer here and in the scantron.

(a) 0.0081

(b) 0.9919

(c) 1.4285

(d) 0.7

(e) 0.654

Solution 2. X is a geometric random variable with parameter p =0.7.
$P(X > 4) = 1 - P(X \leq 4) = 1 - (P(1) + P(2) + P(3) + P(4))$
$= 1 - (0.7 + (0.3 * 0.7) + (0.3^2) * 0.7 + (0.3^3) * 0.7) = 0.0081$

Problem 3. When a computer software package is developed, testing procedures are often put in place to eliminate the bugs in the package. One common procedure is to try the package on a set of well known problems to see if any error occurs. This goes on for a fixed period of time with all the errors being noted. The testing then stops and the package is checked to determine and remove the bugs caused by the errors. The mean number of errors due to a particular bug occurring in a minute is 0.0001. What is the probability that no error will occur in 20 minutes? Mark your answer here and in the scantron.

(a) 0.0001

(b) 0.002

(c) 0.998

(d) 0.03

(e) 0.234

Solution 3. X = number of errors in a minute. That is Poisson($\lambda = 0.0001$). Let Y = number of errors in 20 minutes. That is Poisson($\lambda^{'} = 0.002$). Using the Poisson distribution formula,

$$P(X = 0) = e^{-0.002} = 0.998$$

Problem 4. If the outcome of an experiment is the order of finish in a race among 7 horses having post positions 1,2,3,4,5,6,7, then how many outcomes are there in the sample space? You don't need to enumerate them, just select how many from the choices given and show how you find it. Mark your answer here and in the scantron.

(A) 5040 (B) 823543 (C) 35 (D) 34000 (E) 15000

Solution 4. S, the sample space, consists of all 7! = 5040 permutations of (1,2,3,4,5,6,7) or 5040. For example, one possible outcome is (2,3,1,6,5,4,7) which means that the number 2 horse comes in first, the number 3 horse comes second, and so on.

Problem 5. Suppose a foreman must select one worker from a pool of four available workers (numbered 1, 2, 3, 4) for a special job. He selects the worker by mixing the four names and randomly selecting one. Let A denote the event that worker 1 or 2 is selected, let B denote the event that worker 1 or 3 is selected, and let C denote the event that worker 1 is selected. One of the following is true; which one? Mark your answer here and in the scantron.

(a) A and B are not independent

(b) A and C are independent

(c) A and C are not independent

(d) The probability of event A is 1/4 Notice there was typo (1/2) before

(e) The probability of event C is 1/2

Solution 5. Randomly selected means that the probability of selecting each of the workers is 1/4 .
P(A)=P(1 or 2) = 1/4 + 1/4 = 1/2
P(B)=P(1 or 3) = 1/4 +1/4 = 1/2
P(C) = 1/4; P(AB)=P(1)= 1/4
Are A and B independent? Yes, 1/4 = P(AB)=P(A)P(B) = 1/4
A and C are not independent: P(AC) = P(1)=1/4 ≠ P(A)P(C)= 1/2 x 1/4 = 1/8

Problem 6. A diagnostic test for a certain disease is said to be 90% accurate; that is, if a person has the disease, the test will detect it with probability 0.9. Moreover, if a person does not have the disease, the test will report that he or she doesn't have it with probability 0.9. Only 1% of the population has the disease in question. If the diagnostic test reports that a person chosen at random from the population has the disease, what is the probability that the person does, in fact, have the disease? (Use the following notation: D= disease present; +=test is positive; −= test is negative; D^c= disease is not present. Mark your answer here and in the scantron.

(a) 0.1

(b) 0.108

(c) 0.9

(d) 0.0833

(e) 0.783

Solution 6. $P(D) = 0.01 \rightarrow P(D^c) = 0.99$; $P(+ \mid D) = 0.9; P(- \mid D^c) = 0.9; \rightarrow P(+ \mid D^c) = 0.1$. All this implies that $P(+) = P(+ \mid D)P(D) + P(+ \mid D^c)P(D^c) = 0.9(0.01) + 0.1(0.99) = 0.108$ Want

$$P(D \mid +) = \frac{P(+ \mid D)P(D)}{P(D)} = \frac{0.9(0.01)}{0.108} = 0.0833$$

Problem 7. The probability law that gives legitimacy to using simulation to estimate probabilities of events is known as (Mark your answer here and in the scantron.)

(a) Central Limit Theorem

(b) Law of Large Numbers

(c) Law of union of events

(d) Bayes theorem

(e) Product rule

Solution 7. Law of Large Numbers says that if we repeat an experiment many times, the proportion of times an event of interest occurs in that many times gets closer and closer to the true probability. If we could repeat infinite times, we would know the true probability. (For example, when we say we prefer someone with experience for a job, we mean someone that has observed events many times and knows a little more about the probability of the event than someone that has observed the event a few times, i.e., without experience. The laws of probability are more embedded in peoples minds than we think.

Problem 8. Let X have Binomial distribution with parameters p=0.4 and n=10. One of the following is true. Which? (Mark your answer here and in the scantron).

(a) E(3X + 5) =30

(b) Var(3X + 5)= 35

(c) E(3X + 5) =17

(d) Var(5X) = 5

(e) Var(5X+2)= 7

Solution 8. Answer: $E(3X + 5) = 17$

$E(X) = 10 \times 0.4 = 4; \quad Var(X) = 10 \times 0.4 \times 0.6 = 2.4$

So $E(3X + 5) = 3E(X) + 5 = 3 \times 4 + 5 = 17$

$Var(3X + 5) = 9Var(X) = 9 \times 2.4 = 21.6$

Problem 9. A series of 3 jobs arrive at a computing centre with 3 processors. Assume that each of the jobs is equally likely to go through any of the processors. Find the probability that all processors are occupied. Mark your answer here and in the scantron.

(a) 2/9

(b) 5/9

(c) 1/3

(d) 4/7

(e) 5/4

Solution 9. Answer: 2/9

Why?

For all 3 processors to be busy when 3 jobs arrive, there must be one job in each processor. Thus the probability is $(3!)/3^3 = 2/9$

Problem 10. 20% of the applicants for a certain sales position are fluent in English and Spanish. Suppose that four jobs requiring fluency in English and Spanish are open. Find the probability that two unqualified applicants are interviewed before finding the fourth qualified applicant, if the applicants are interviewed sequentially and at random. Mark your answer here and in the scantron.

(a) 0.5

(b) 0.01

(c) 0.75

(d) 0.8

(e) 0.2

Solution 10. Answer: 0.01 is the closest Why?

Probability that it takes 6 trials to find 4 qualified candidates. Assume independent trials, with 0.2 probability of finding a qualified candidate in any one trial. Let Y denote the number of trials it takes to find 4 qualified applicants. Y can be assumed to have a negative binomial distribution

$P(Y=6) = \binom{5}{3}(0.2)^4(0.8)^2 = 10(0.2^4)(0.8^2) = 0.0124$

Problem 11. Consider a plant manufacturing chips of which 10% are expected to be defective. A sample of 5 chips is randomly selected, and the number of defectives X is observed. The probability of obtaining at most 2 defectives is (Mark your answer here and in the scantron).

(a) 0.057

(b) 0.1

(c) 0.5

(d) 0.001

(e) 0.991

Solution 11. Answer: 0.991 Why?

$P(X < 3) = \binom{5}{0}0.1^0(0.9)^5 + \binom{5}{1}0.1(0.9)^4 + \binom{5}{2}0.1^2(0.9)^3 = 0.991$

Problem 12. The number of failures per day in a certain plant has a Poisson distribution with parameter 4. Present maintenance facilities can repair 3 machines per day, otherwise a contractor is called out. On any given day, what is the probability of having machines repaired by a contractor? (Mark your answer here and in the scantron).

(a) 0.43

(b) 0.57

(c) 0.1

(d) 0.04

(e) 0.96

Solution 12. Answer: 0.57

Why ?
$P(X > 3) = 1 - P(X \leq 3) = 1 - [P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)] = 1 - 0.43 = 0.57$

Problem 13. A computer student can repeat an examination until it is passed, but it is allowed to attempt the examination at most four times. The probability that the student passes the exam in each attempt is 0.6. Let E be the event that the student passes. The probability of E is (mark your answer here and in the scantron)

(a) 0.44

(b) 0.97

(c) 0.04

(d) 0.6

(e) 0.78

Solution 13. 0.97 why?

- List all the outcomes in the sample space S. $S = \{P, P^cP, P^cP^cP, P^cP^cP^cP, P^cP^cP^cP^c\}$
- List the outcomes in the event E (" the student eventually passes") $E = \{P, P^cP, P^cP^cP, P^cP^cP^cP\}$
- Find the probability of E. Show work. $Prob(E) = 1 - P(E^c) = 1 - Prob(P^cP^cP^cP^c) = 1 - 0.4^4 = 0.9744$

Problem 14. According to government data, 30% of married American marry after 30. A study of married people chooses a random sample of 400 married Americans and asks each person in the sample the age at marriage. How many of those do you expect to have married after 30? (Mark your answer here and in the scantron.)

(a) 30

(b) 4000

(c) 130

(d) 310

(e) 120

Solution 14. If people are chosen at random, then they are probably independent of each other, and because the US is so large, drawing 400 people without replacement will not change the 30% much. So we can consider drawing 400 people at random from the large population as 400 Bernoulli trials with probability of success (marrying after 30) of 0.3.
Answer: 120 Why?
$E(X) = 0.3 * 400 = 120$ Binomial

Problem 15. It is known that 40 percent of the romantic dates that an adult has while single are not much fun. Two college students seeking a romantic partner decide to start going out independently on dates with other people chosen at random, to look for the fun date out there. Find the probability that both of them find their first fun date in the 5th date. Mark your answer here and in the scantron.

(a) 0.601

(b) 0.00256

(c) 0.0002359

(d) 0.341

(e) 0.03072

Solution 15. Answer: 0.0002359296
X= the number of dates it takes to find the first fun one for each of the friends.
X is geometric with probability of success p=0.6.
For each friend separately, $P(X = 5) = 0.4^4 0.6 = 0.01536$
For both friends, P(both find the first fun date at the fifth attempt is)
$0.01536^2 = 0.0002359296$

Problem 16. An oil exploration firm is to drill ten wells, each of which has probability 0.1 of successfully striking recoverable oil. It costs $10,000 to drill each well so there is a total fixed cost of $100,000. A successful well will bring in oil worth $500,000. The expected value and standard deviation of the firms gain are, respectively, Mark your answer here and in the scantron.

(a) $500000, $500000

(b) $ 500000, $ 0.9

(c) $ 450000, $474.34

(d) $400000, $474341.6

(e) $100000, $1001.34

Solution 16. Answer: $400000, $474341.6
E(X)= np = 10*0.1 =1 Var(X)= 10*0.1*0.9 =0.9
Gain=500,000X - 100,000; E(Gain) = 500000E(X) - 100000= 400000;
Var(Gain) =$500000^2(0.9)$; $Sd(Gain) = 500000 * \sqrt{(}0.9) = 474341.6$

EXAMPLES OF QUESTIONS WHERE YOU MUST SHOW WORK. WORK MUST BE WRITTEN HERE ON THE SPACE GIVEN IN THE EXAM. YOU MAY USE THE BLANK PAGE AT THE END OF THE EXAM FOR SCRATCH WORK, BUT SCRATCH WORK WILL NOT BE READ.

Problem 17. A warehouse contains ten printing machines, 5 of which are defective. A company randomly selects 3 of the machines for purchase. Let X denote the number of defectives in the 3 machines selected. Write a table with the probability distribution of the random variable X. Compute its expected value and standard deviation.

Solution 17.

X	P(X)
0	$\frac{\binom{5}{0}\binom{5}{3}}{\binom{10}{3}} = 0.08333$
1	$\frac{\binom{5}{1}\binom{5}{2}}{\binom{10}{3}} = 0.41666$
2	$\frac{\binom{5}{2}\binom{5}{1}}{\binom{10}{3}} = 0.41666$
3	$\frac{\binom{5}{2}\binom{5}{1}}{\binom{10}{3}} = 0.08333$

$\mu = E(x) = \sum_x XP(X) = 0(0.08333) + 1(0.41666) + 2(0.41666) + 3(0.08333)$=1.4999
$E(X^2) = 0(0.08333) + 1(0.41666) + 4(0.41666) + 9(0.08333) = 2.8332$
$\sigma^2 = Var(X) = E(X^2) - [E(X)]^2 = 2.8332 - (1.4999^2) = 0.58357$
$\sigma = SD = \sqrt{0.58357} = 0.7639$

Problem 18. Let $S = (0,1]$ and define $A_i = ((1/i), 1], \ i = 1, 2, ...$ Do these events form a partition of the sample space? Why? Explain.

Solution 18. No. Why?
The union is the sample space, $\bigcup_{i=1}^{\infty} A_i = (0, 1]$
but the intersection of any two events is not empty, it is 1.
$\bigcap_{i=1}^{\infty} A_i = 1$

Problem 19. If X was a Geometric random variable with parameter p =0.4, what would be the value of $E(3X^2 - 5X + 100)$.

Solution 19. $E(3X^2 - 5X + 100) = \sum_x (3X^2 - 5X + 100)P(X)$
$= \sum_x 3X^2 P(X) - \sum_x 5XP(X) + \sum_x 100P(X)$
$= 3\sum_x X^2 P(X) - 5\sum_x XP(X) + 100\sum_x P(X)$
$= 3[\sum (X - E(X))^2 P(X) + [E(X)]^2] - 5E(X) + 100$
$= 3[Var(X) + [E(X)]^2] - 5E(X) + 100$
Thus, $E(3X^2 - 5X + 100) = 3Var(X) + 3[E(X)]^2 - 5E(X) + 100$
$= 3\left[\frac{0.6}{0.4^2} + \left(4^2\right)\right] - 5(4) + 100 = 139.25.$

Problem 20. Suppose that a surveyor is trying to determine the areas of a rectangular field, in which the measured length X and the measured width Y are independent random variables that fluctuate widely about the true values, according to the following probability distributions

X	P(X)
8	1/4
10	1/4
11	1/2

Y	P(Y)
4	1/2
6	1/2

Write a table that indicates the joint probabilities P(X,Y) for all values of X and Y.

Solution 20. Since X and Y are independent, P(X,Y)= P(X)P(Y)

		X		
		8	10	11
Y	4	1/8	1/8	1/4
	6	1/8	1/8	1/4

Problem 21. Of 25 laptops available in a business, 10 are used by the accounting personnel (A), 5 are used by the management (M), and 15 are not used by anybody(N). Symbolically (e.g., $M \cap T, T \cap M, etc.$) denote the following sets and write the number of laptops in each set. Write your answer in the table.

Solution 21.

Event	Write event symbolically	Number of laptops
Used by management and accounting	MA=M	5
Only used by management	MA^c	Empty set, none
Used by management or accounting but not used by both	$MA^c \cup AM^c = MA^c$	5
Used by at least one of the groups	$M \cup A$	10

Problem 22. Suppose the probability that a US resident has traveled to Canada is 0.18, to Mexico is 0.09, and to both countries is 0.04. What is the probability that two US residents chosen at random have both (write your answers in table

Event	Probability of event
Traveled to Canada but not Mexico?	
Traveled at least one of the countries?	
Not traveled to either country?	

Solution 22. First find probability of the event for one of them, then find the joint probability for both using the product rule for independent events.

Table 1: Traveling to Canada and Mexico

Event	Probability of event
Traveled to Canada but not Mexico?	0.14^2
Traveled at least one of the countries?	0.23^2
Not traveled to either country?	0.77^2

Problem 23. The roll of a die that is not fair follows the following probability distribution:

X	1	2	3	4	5	6
P(X)	0.2	0.1	0.2	0.1	0.3	0.1

Experiment done: roll the die twice. Answer the following questions:

(a) Write the sample space for this experiment.

(b) Write the probabilities of all outcomes in the sample space.

(c) Let the random variable defined on that sample Space be $Y = X + W$ where X is the value of the first roll, and W is the value of the second roll. Write the probability distribution of Y in tabular form.

Solution 23. (a) Write the sample space for this experiment.

$$S = \begin{pmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{pmatrix}$$

(b) Write the probabilities of all outcomes in the sample space.

$$P(s_i \mid s_i \in S) = \begin{pmatrix} 0.2^2 & 0.2(0.1) & 0.2^2 & 0.2(0.1) & 0.2(0.3) & 0.2(0.1) \\ 0.1(0.2) & 0.1^2 & 0.1(0.2) & 0.1^2 & 0.1(0.3) & 0.1^2 \\ 0.2^2 & 0.2(0.1) & 0.2^2 & 0.2(0.1) & 0.2(0.3) & 0.2(0.1) \\ 0.1(0.2) & 0.1^2 & 0.1(0.2) & 0.1^2 & 0.1(0.3) & 0.1^2 \\ 0.3(0.2) & 0.3(0.1) & 0.3(0.2) & 0.3(0.1) & 0.3^2 & 0.3(0.1) \\ 0.1(0.2) & 0.1^2 & 0.1(0.2) & 0.1^2 & 0.1(0.3) & 0.1^2 \end{pmatrix}$$

(c) Let the random variable defined on that sample Space be $Y = X + W$ where X is the value of the first roll, and W is the value of the second roll. Write the probability distribution of Y in tabular form.

Probability mass function for Y

Y	P(Y)
2	$0.2^2 = 0.04$
3	$2(0.2)(0.1) = 0.04$
4	$2(0.2^2) + 0.1^2 = 0.09$
5	$4(0.2(0.1)) = 0.08$
6	$2(0.2)0.3 + 2(0.1^2) + 0.2^2 = 0.18$
7	$2(0.2)(0.1) + 2(0.1)(0.3) + 2(0.2)(0.1) = 0.14$
8	$2(0.1^2) + 2(0.2)(0.3) + 0.1^2 = 0.15$
9	$2(0.2)(0.1) + 2(0.1)(0.3) = 0.1$
10	$2(0.1^2) + 0.3^2 = 0.11$
11	$2(0.3)(0.1) = 0.06$
12	$0.1^2 = 0.01$

EXAMPLE OF SIMULATION QUESTION.

You will have to describe what probability model to use to do the simulation and the steps of the simulation. Here is an example.

Problem 24. A building has 6 floors. Three individuals take the elevator. We want to determine the probability that they get off in different floors. Write down the steps of a simulation to determine an answer to that question. Make sure you describe in detail the following steps:

- Probability model and what it represents.
- What one trial consists of
- What you keep track of in each trial.
- How many times you will repeat the simulation
- What you must compute with the data obtained to estimate the probability desired.
- A table with a simple example of several trials (e.g. 5) and your computation.

Solution 24. There are 6 distinct floors.

(a) PRobability model. A six sided fair die

(b) One trial: roll the die 3 times

(c) Keep track of: 1=each passenger gets off in different floor; 0 otherwise.

(d) Repeat many times

(e) Compute ratio = number of 1s / total number of trials.