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MUST DO BEFORE STARTING EXAM

UCLA ID: – – – – – – – – – – – TA Session: – – – – – –

LAST NAME (Please, PRINT): – – – – – –– (Put ID on your desk)

FIRST NAME: – – – – – – – – – – ––

(a) **WRITE AND MARK YOUR NAME AND ID ON THE SCANTRON, ON THE WORK PROBLEM SHEET AND ON THIS EXAM COVER.**

(b) **WRITE YOUR NAME ON ALL SIDES OF THE CHEAT SHEET, TOP RIGHT HAND CORNER.**

(c) **DO NOT DETACH ANY PAGES FROM THIS EXAM. EXAM MUST STAY STAPLED DURING THE WHOLE EXAM.**

(d) **PUT ALL YOUR BELONGINGS INSIDE YOUR BACKPACK UNDER THE CHAIR.**

(e) **ONLY ID, NUMBER 2 PENCIL AND PEN, ERASER, SCIENTIFIC CALCULATOR, SCANTRON, PROBLEM SHEET, EXAM AND CHEAT SHEET ALLOWED IN THE EXAM.**

(f) **PUT DOWN THE TABLES ON YOUR RIGHT AND LEFT. ALL ITEMS MUST BE ON YOUR DESK.**

Other important Instructions–Read. Points lost for not following directions.

- **Closed books, closed notes. Material covered is up to last day of lecture before the exam.**
- **Students without a cheat sheet must let the professor know that do not have one before the exam and sit on the front row of the exam room.**
- **You must be silent in the exam room throughout the whole time that you are in the room.**
- **Only scientific calculator allowed for computations. You may not use your phone or any other electronic device as calculator. Graphics calculators are not allowed. No exceptions.**
- **Phones and other electronic devices must be disconnected before you enter the classroom, placed inside your backpack and not accessed or turned on again until you are out of the room. While in the classroom, they must be in your backpack and your backpack on the floor under the chair. If you do not have a backpack, you must put the items on the front desk. Iwatches and other devices are not allowed either. Phones or devices in pockets will lead to big loss of points in the exam. It is not worth the risk.**
- **Answer for multiple choice questions will be marked in scantron AND the exam. Work will not be read in multiple choice;Failure to mark your name, ID or some answers will result in point deduction from the exam grade. This is the second time you do a test. So no more grace period on this.**

- **Answers for the work problems must be written on the sheet that we hand out with the scantron. Your name and ID must be written on that sheet.**
- **This sheet and the scantron, together with the cheat sheet will be inserted inside the exam before you turn in the exam.**
- **Left handed students must sit in a seat for left-handed students. The professor will tell students where to sit. Please, let the professor know that you are left handed once seated and she indicate where to move.**
- **ID must be ready to show BEFORE and at all times during the exam. NO ID, no exam.**
- **This midterm must show your individual work. Talking to others during the midterm, not adhering to the above, sharing information or breaking any other aspect of the student code of conduct at UCLA will not be tolerated and will be referred to the Dean of Students office. You can not exchange papers or information. All your things must be on the floor. You may not use the empty seats next to you to put things. Put the tables down. Honor code applies.**
- **Cheat sheet can have only formulas and definitions, no solved problems, no examples of any kind, no proofs, no numerical examples, no intermediate steps and no drawings or graphs of any kind.YOUR NAME MUST BE ON CHEAT SHEET AT ALL TIMES. Be ready to show your cheat sheet when the instructor requests it. The cheat sheet must be written all in English. Cheat sheets that do not comply will result in lower grade in the exam. You may have four pages, i.e., two sides of two 11 by 8 sheet, STAPLED. If you do not have a cheat sheet, you must tell the professor at the beginning of the exam and sit on the front row of the room.**
- **You may not speak to each other in the exam room. Wait until you are out of the room.**

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PART I. MULTIPLE CHOICE QUESTIONS. ONLY ONE ANSWER IS CORRECT. CHOICE MUST BE MARKED ON THE SCANTRON, AND ALSO HERE ON THE EXAM. ONLY THE SCANTRON WILL BE GRADED. You may use the space near the question for scratch work, but scratch work will not be read.

Problem 1. Suppose that the number of typographical errors on a single page in a book has a Poisson distribution with parameter $\lambda = 1/2$. The probability that there is at least one error on a randomly chosen page is, approximately

(a) 0.5

(b) 0.13

(c) 0.67

(d) 0.393

(e) 0.256

Solution 1. $P(X \geq 1) = 1 - P(X = 0) = 1 - e^{-1/2} \approx 0.393$

Problem 2. Let X be a random variable with density function Gamma($\alpha = 2, \lambda = 1$). One of the following is true. Which?

(a) E(3X +5) = 30

(b) E(3X + 5) = 25

(c) E(3X+15)= 17

(d) Var(5X) =50

(e) Var(5X+2)= 10

Solution 2. Var(5X) = 50

Problem 3. A random variable X has the following moment generating function

$$M_X(t) = \left(\frac{1}{2}\right)^{10} \left(e^t + 1\right)^{10}$$

X is

(a) A Poisson random variable with parameter $\lambda = 10$.

(b) A Binomial random variable with parameters $n = 2, p = \frac{1}{10}$

(c) A gamma random variable with parameter $\lambda = \frac{1}{2}$ and $\alpha = 10$.

(d) A uniform random variable with parameters $a = 0, b = 10$.

(e) A binomial random variable with parameter $n = 10$ and $p = 1/2$.

Solution 3. Binomial(n=10, p=1/2)

$$M_x(t) = (1/2e^t + 1/2)^{10} = (1/2e^t + 1 - 1/2)$$

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Problem 4. The service times $X_1, X_2, \ldots\ldots, X_{10}$, of 10 teller windows in a bank are independent and identically distributed random variables. Each X_i is exponential with an expected value of 3.2 minutes. In a given day, 10 independent customers arrive at 3:00 pm and each is serviced by a different window.

What is the joint probability that all 10 customers will be there at 3:05?

(a) 0.209

(b) 0.5

(c) 0.9

(d) 0

(e) 0.25

Solution 4. First find the probability of the event that one of them will still be there.

$\lambda = 1/3.2 = 0.3125; \qquad f(x) = \frac{1}{3.2} e^{-\frac{1}{3.2}x} \quad x > 0$

$P(X_i > 5) = 1 - P(X_i < 5) = 1 - (1 - e^{-\frac{1}{3.2}5}) = 0.2094$

Probability that all of them will be there is, by independence of events, the probability that the first will be there, times the probability that the second will be there, times the probability that the third will be there.... etc...

$P(X_1 > 5)P(X_2 > 5)\ldots\ldots.P(X_{10} > 5) = 0.2094^{10} \approx 0$

Problem 5. A large supermarket chain reports that 40% of all beer purchases are light. Consider the next 200 beer purchases made by independent individuals. Suppose Y is the number of those 200 beer purchases that are light. What is the probability that Y is larger than 90?

(a) 0.0001

(b) 0.9251

(c) 0.513

(d) 1

(e) 0.0749

Solution 5. Binomial(n=200, p=0.4)

Problem 6. When an emergency occurs, the response time (in hours) of the first police car is a random variable X with density

$$f(x) = 0.2e^{-0.2x}, \quad x \geq 0$$

The probability that the response time is larger than 6 hours is

(a) 0.698

(b) 0.301

(c) 0.5

(d) 0.2

(e) 0.871

Solution 6.

$$P(X > 6) = 1 - F(x = 6) = e^{-0.2(6)} = 0.301$$

Problem 7. The number of accidents that a person has in a given year is a Poisson random variable with expected value equal to λ. However, suppose that the value of λ changes from person to person, being equal to 2 for 60 percent of the population and 3 for the other 40 percent. If a person is chosen at random, what is the probability that he will have 3 accidents in a year?

(a) 0.00161

(b) 0.3518

(c) 0.001011

(d) 0.5

(e) 0.1978

Solution 7. $0.6 * (exp(-2)) * (2^3)/6 + 0.4 * (exp(-3)) * (3^3)/6 = 0.1978$

Problem 8. Consider the volumes of soda remaining in 100 cans of soda that are nearly empty. Let $X_1, ..., X_{100}$ denote the volumes (in ounces) of cans one through one hundred, respectively. Suppose that the volumes are independent, and that each $X_i, \quad i = 1, ..., 100$ is uniformly distributed between 0 and 2. Find the probability that the total amount of soda in all 100 cans is less than 90 ounces.

(a) 0.0418

(b) 0

(c) 1

(d) 0.2413

(e) 0.67

Solution 8.

$$f(x_i) = \frac{1}{2} \qquad 0 < x_i < 2, \qquad i = 1, ..., 100$$

$$E(X_i) = \frac{\alpha + \beta}{2} = 1, \qquad Var(X_i) = \frac{(\alpha - \beta)^2}{12} = \frac{1}{3}$$

$$Y = X_1 + X_2 + ... + X_{100}$$
$$E(Y) = 100E(X_i) = 100$$
$$Var(Y) = 100Var(X_i) = 100/3$$
$$Y \sim N\left(\mu_y = 100, \sigma_y^2 = 100/3\right)$$

$$P(Y < 90) = P\left(Z < \frac{90 - 100}{\sqrt{100/3}}\right) = P(z < -1.73) = 0.0418$$

Problem 9. The random variable X is a continuous variable with density function

$$f(x) = 0.75(1 - x^2), \qquad -1 \le X \le 1.$$

Consider 100 independent and identically distributed random variables with that density. That is, $X_i \sim f(x), \quad i = 1, 2, ..., 100$. What is the expected value of

$$\sum_{i=1}^{100} \left(\frac{1}{100}(x_i - 3)\right)$$

(a) 0

(b) -3

(c) 50

(d) $\frac{9}{100}$

(e) $\frac{2}{5}100$

Solution 9. $E(X) = 0$, can be found by solving $\int_{-1}^{1} xf(x)dx$, where f(x) is the density given above.
Then, $E\left(\sum_{i=1}^{100}\left(\frac{1}{100}(x_i - 3)\right)\right) = \sum_{i=1}^{100}\frac{1}{100}E(x_i) - 100(3/100) = -3$

Problem 10. Each time a pitcher delivers a fast ball, the speed is distributed between 90 and 100 miles per hour, with density 1/10. Assume that the speeds of pitches are independent.

Consider two fast balls thrown by the pitcher. Find the probability that at least one of them is 93 miles per hour or faster.

(a) 0.009

(b) 0.01

(c) 0.91

(d) 0.654

(e) 0.09

Solution 10. Let X and Y denote the speeds of the two pitches. Then

$$f(x) = 1/10 \qquad 90 \le x \le 100$$

$$f(y) = 1/10 \qquad 90 \le y \le 100$$

Since X and Y are independent,

$$\begin{aligned} P(X \ge 93 \ \texttt{or} \ \ Y \ge 93) &= 1 - P(X \le 93 \ \texttt{and} \ \ Y \le 93) \\ &= 1 - 9/100 \\ &= 91/100 \end{aligned}$$

Problem 11. You arrive at a bus stop at 10 oclock, knowing that the bus will arrive at some time uniformly distributed between 10 and 10:30. If at 10:15 the bus has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes?

(a) 0.666

(b) 0.444

(c) 0.333

(d) 0.1211

(e) 0.9

Solution 11.

$$X \sim unif[0,30]$$
$$P(X > 25|X > 15) = \frac{P(X>25)}{P(X>15)} = \frac{\int_{25}^{30} \frac{1}{30} dX}{\int_{15}^{30} \frac{1}{30} dX} = \frac{1}{3}$$

Problem 12. An analog signal received at a detector (measured in microvolts) is normally distributed with a mean of 100 and a variance of 256; What is the probability that the signal will be less than 120 microvolts given that it is larger than 110 microvolts.

(a) 0.6053

(b) 0.2211

(c) 0.732

(d) 0.8944

(e) 0.001

Solution 12. $P(X < 120 \mid X > 110) = \dfrac{P(X < 120 \text{and} X > 110)}{P(X > 110)} = \dfrac{P(110 < x < 120)}{1 - P(X < 110)} = \dfrac{P(0.625 < z < 1.25)}{1 - P(z < 0.625)} = \dfrac{0.8944 - 0.7324}{1 - 0.7324} = 0.60538$

Problem 13. Daily sales records for a car dealership show that it will sell 0, 1, 2, or 3 cars, with probabilities as listed

Number of cars (X)	0	1	2	3
Probability (P(X))	0.5	0.3	0.15	0.05

Consider 100 random days. What is the probability that the total number of cars sold in those 100 days is less than 87?

(a) 0.02

(b) 0.45

(c) 0.353

(d) 0.6368

(e) 0.9115

Solution 13. $\mu_x = 1 * 0.3 + 2 * 0.15 + 3 * 0.05 = 0.75$
$\sigma^2 = (1 * 0.3 + 4 * 0.15 + 9 * 0.05) - 0.75^2 = 0.7875$
$P(z < 1.3522) = P(z < 1.3522) = 0.9115$

Problem 14. The number of bacteria colonies of a certain type in samples of polluted water has a Poisson with a mean of two per cubic centimeter.

If four 1-cubic centimeter samples of this water are independently selected, find the probability that at least one sample will contain one or more bacteria colonies.

(a) 0.8646

(b) 0.999

(c) 0.00033

(d) 0.05

(e) 0.321

Solution 14. Let X= Number of bacteria per cubic centimeter $X \sim$ Poisson($\lambda = 2$)

$P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0) = 1 - \frac{2^0 e^{-2}}{0!} = 1 - e^{-2} = 0.8646$

Y = Number of cubic centimeter samples (out of 4) that will satisfy the event $X \geq 1$. This $Y \sim Binomial(n = 4, p = 1 - e^{-2})$.

$P(Y \geq 1) = 1 - P(Y = 0) = 1 - \binom{4}{0}(1 - e^{-2})^0(1 - (1 - e^{-2}))^4 = 1 - 0.00033 = 0.99966$

Notice how we first use the Poisson to find the probability of the event A(1 or more bacterial colonies in 1 cubic centimeter; and then we have 4 cubic centimeters and try to find the probability that the number of them (Y) that satisfy A is 1 or more.).

Problem 15. A professor teaches a class on introductory statistics with 200 students. He reports the following data on the final exam:

Mean = 60, Median = 60, SD = 25, minimum = 30, maximum = 90

What is the best answer?

(a) He can use the normal model to estimate the letter grades (A, B, etc.) because the mean and the median are equal to each other.

(b) In order to decide whether he can use the normal distribution to estimate the letter grades (A, B, etc.), we need to see the histogram of the original data.

(c) He cannot use the normal distribution to estimate the letter grades (A, B, etc) because the distribution that results from the above data does not fit the normal model.

(d) He can use the normal distribution to estimate the letter grades (A, B, etc.) because his distribution has symmetry and the standard deviation is less than the mean.

(e) He can use the normal distribution to estimate the letter grades (A, B, etc.) because his distribution has symmetry and the standard deviation is less than the mean.

Solution 15. He cannot use the normal distribution to estimate the letter grades (A, B, etc) because the distribution that results from the above data does not fit the normal model.

Problem 16. Consider the density function

$$f(x) = c, \qquad a \leq x \leq b,$$

for which you will have to figure out what c is. Once you figure out what c is determine what the following function:

$$\frac{x-a}{b-a}$$

is

(a) the moment generating function of X.

(b) the cumulative distribution of X

(c) the third moment of X

(d) the expected value of X

(e) the variance of X

Solution 16. the cumulative distribution of X.

Problem 17. A random variable X has cumulative distribution function

$$F(x) = \frac{1}{26}(2x^2 + x - 10) \qquad 2 \leq x \leq 4$$

Find the probability that X is between 2.8 and 4.

(a) 0.4231

(b) 0.2291

(c) 0.0023

(d) 0.0384

(e) 0.6739

Solution 17. F(4)= 1.
$1 - (1/26) * (2 * (2.8^2) + 2.8 - 10) = 1 - 0.3261 = 0.6739$

Problem 18. A continuous random variable has the following probability density function

$$f(x) = \frac{x}{2} \quad 0 \leq x \leq 2$$

The 50th percentile is

(a) 1.0

(b) 5/16

(c) 0.034

(d) 0.5

(e) 1.414

Solution 18. The 50th percentile is the median.
$\int_0^c \frac{x}{2} dx = 0.5.$
$\int_0^c \frac{x}{2} dx = \frac{x^2}{4} \Big\|_0^c = \frac{c^2}{4} = 1/2$
Thus $c^2 = 2$. So $c = \sqrt{2} = 1.414$.

Problem 19. Wires manufactured for a certain computer system are specified to have a resistance of between 0.12 and 0.14 Ohms. The actual measured resistances of the wires produced by Company A have a normal probability distribution, with expected value 0.13 Ohms and standard deviation of 0.005 Ohms. If four independent such wires are used in a single system and all are selected from company A, what is the probability that all four will meet specifications? Show work.

(a) 0.5122

(b) 0.9544

(c) 0.8297

(d) 0.0456

(e) 3.6

Solution 19. $P(0.12 < X < 0.14) = 0.9544$ Y Binomial (n=4, p=0.9544). P(Y= 4)= 0.8297

Problem 20. Which of the following is true if X is exponential and t and s are two constants?

(a)
$$P(X > t + s \mid x > t) = P(X > t + s)P(X > t)$$

(b)
$$P(X > t + s \mid x > t) = \frac{P(X > s)}{P(X > t)} = (1 - F(s))(1 - F(t))$$

(c)
$$P(X > t + s \mid x > t) = \frac{P(X > t)}{P(X > s)} = \frac{F(t)}{F(s)}$$

(d)
$$P(X > t + s \mid x > t) = P(X > s)P(X > t) = (1 - F(s))(1 - F(t))$$

(e)
$$P(X > t + s \mid x > t) = 1 - F(s)$$

Solution 20.

$$P(X > t + s \mid x > t) = 1 - F(s)$$

(a) Have you marked your name and ID in the scantron?

(b) Have you entered all the answers in the scantron and this exam booklet?

(c) Have you written your name and id in your cheat sheet?

(d) Have you written your name and id on the first page of the exam?

If you have check marked all of the above, insert cheat sheet and scantron within the pages of the exam. Close the exam and wait until we collect it from you. Remain seated until every student has turned in the exam. Do not grab your phone or talk until you are out of the exam room.

PART II. MUST SHOW WORK. You will answer this question in the sheet provided separately when we give you the scantron. The sheet must have your name and ID on it and inserted between the pages of the exam before you turn in your exam.

Problem 21. Let X be the cosine of the angle at which electrons are emitted in muon decay. X is a random variable with the following density function.

$$f(x) = \frac{1 + \alpha X}{2} \qquad -1 \leq x \leq 1$$

α is a constant, a parameter, that must have value between -1 and 1.

(a) Find the Expected cosine of the angle and the variance. Show work. (Notice that the answer will depend on α since that is a constant parameter).

(b) Let $x_1, x_2, ..., x_n$ be n independent and identically distributed random variables with the density given above each. Consider now the following functions (i) $y = \frac{3\sum_{i=1}^{n} x_i}{n}$; (ii) $w = \frac{\sum_{i=1}^{n} x_i}{n}$. Compute the expected value and variance of these two functions separately. Which one has the smallest standard deviation? Show work.

(c) Prove that the expected value of $x_1 + x_2 = 2E(x)$.

Solution 21. Show must show work that leads to the following solutions.

(a) $\mu_x = \mu = \frac{\alpha}{3}$ and $\sigma_x^2 = \frac{3-\alpha^2}{9}$.

(b) $E(Y) = 3E(\bar{x}) = 3\mu = 3\frac{\alpha}{3} = \alpha$

$Var(Y) = 9Var(\bar{x}) = 9\frac{\sigma^2}{n} = \frac{3-\alpha^2}{n}$

$\sigma_y = \sqrt{\frac{3-\alpha^2}{n}}$

$E(W) = E(\bar{x}) = \mu = \frac{\alpha}{3}$

$Var(W) = Var(\bar{X}) = \frac{\sigma^2}{n} = \frac{3-\alpha^2}{9n}$

$\sigma_w = \sqrt{\frac{3-\alpha^2}{9n}}$

So the standard deviation of W is smaller than the standard deviation of Y.

(c) Will be done in lecture .