Stat 100 -Intro Probability
Midterm 1

J. Sanchez
UCLA Department of Statistics

DO NOT DETACH ANY PAGES FROM THIS EXAM. EXAM MUST STAY STAPLED DURING THE WHOLE EXAM.

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Instructions

- Closed books, closed notes.
- Answer for multiple choice questions will be marked in scantron AND the exam. Work will not be read in multiple choice;Failure to mark your name, ID or some answers will result in point deduction from the exam grade.
- Left handed students will sit in a seat for left-handed students. The professor will tell students where to sit. Please, let the professor know that you are left handed ahead of time.
- ID must be ready to show at all times during the exam. NO ID, no exam.
- This midterm must show your individual work. Talking to others during the midterm, not adhering to the above, sharing information or breaking any other aspect of the student code of conduct at UCLA will not be tolerated. You can not exchange papers or information, and you can not use your phone or other electronic devices. All your things must be on the floor. You may not use the empty seats next to you to put things. Close the tables. Honor code applies. You may not talk or use your electronic devices from the moment the professor starts distributing the midterm until they have been collected from everybody.
- Only scientific calculator and one side of a 8×11 cheat sheet, and pen or pencil and exam and scantron can be on your desk. Everything else (disconnected) in your backpack and your backpack on the floor. Close the tables of the chair near you.
- Cheat sheet can have only formulas and definitions, no solved problems, no examples of any kind, no proofs, no numerical examples, no intermediate steps and no drawings or graphs of any kind.YOUR NAME MUST BE ON CHEAT SHEET AT ALL TIMES. Be ready to show your cheat sheet when the instructor requests it. The cheat sheet must be written all in English. Cheat sheets that do not comply will result in lower grade in the exam.
- In questions where you show work, you must use the same notation we have used in lecture and the textbook.
- Phones and all other electronic devices must be disconnected and in your backpack. Your backpack must be on the floor.

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MULTIPLE CHOICE QUESTIONS. ONLY ONE ANSWER IS CORRECT. CHOICE MUST BE MARKED ON THE SCANTRON, AND ALSO HERE ON THE EXAM. ONLY THE SCANTRON WILL BE GRADED. NO MARKS ON SCANTRON OR MORE THAN ONE MARK WILL RESULT IN 0 POINTS FOR THE QUESTION NOT MARKED, EVEN IF IT IS MARKED ON THE EXAM. Work near the multiple choice question will not be read.

Problem 1. Of 25 laptops available in a business, 10 are used by the accounting personnel (A), 5 are used by tl A), management (M), and 15 are not used by anybody(N). Which of the following statements is true? is true true?

(a) The number of laptops used by management or accounting but not used by both is 5.

(b) The event $MA^c \cup AM^c$ has 10 laptops in it.

(c) MA^c contains 5 laptops.

(d) The number of laptops used by at least one of the groups is 5.

(e) MA is the empty set.

Problem 2. Police report that 78% of drivers stopped on suspicion of druiiaoniving are given a breath test, 36% a blood test, and 22% both tests. What is the probability that a randomly selected DWI suspect is given a blood test or a breath test, but not both?

(a) 0.7

(b) 0.3

(c) 0.08

(d) 0.22

(e) 0.12

Problem 3. Let $S = (0, 1]$ and define $A_i = ((1/i), 1],\ i = 1, 2, ...$ The union of these events is

(a) S, the whole sample space.

(b) The interval $[0, 0.5]$.

(c) The set with only 1 in it.

(d) The empty set.

(e) The interval $[0, 10]$.

Problem 4. A coin which lands heads with probability p is tossed repeatedly. Assuming independence of the tosses, the probability that the fifth head appears on the 12th toss is

(a) $\binom{9}{5} p^5 (1-p)^4$

(b) $(1-p)^6 p$

(c) $\sum_{k=0}^{5} \binom{8}{k} p^k (1-p)^{8-k} . \binom{5}{k} p^k (1-p)^{5-k}$

(d) $\binom{11}{4} p^4 (1-p)^7 p$

(e) 2/x

Problem 5. The Center for Disease Control says that about 30% of high school students smoke tobacco (down from a high of 38% in 1997). Suppose you randomly select high school students to survey them on their attitude towards scenes of smoking in the movies. What is the probability that the first smoker is the 6th person you choose?

(a) 0.000729

(b) 0.0504

(c) 0.000504

(d) 0.5

(e) 0.002

Problem 6. A series of 3 jobs arrive at a computing centre with 3 processors. Assume that each of the jobs is equally likely to go through any of the processors. Find the probability that all processors are occupied. Circle one, work will not be read.

(a) 5/9

(b) 1/3

(c) 4/7

(d) 1/10

(e) 2/9

Problem 7. The number of failures per day in a certain plant has a Poisson distribution with parameter expected value 4. Present maintenance facilities can repair 3 machines per day, otherwise a contractor is called out. On any given day, what is the probability of having machines repaired by a contractor?

(a) 0.75

(b) 0.43335

(c) 0.5665

(d) 0.101

(e) 0.96

Problem 8. The number of people arriving to an emergency room can be modeled by a Poisson process with a rate parameter λ of five per hour. How many people do you expect to arrive during a 45 minute period?

(a) 45

(b) 3.75

(c) 5

(d) 0.75

(e) 0.1

Problem 9. On a multiple-choice exam with three possible answers for each of the 5 questions, what is the probability that a student would get four or more correct answers just by guessing?

(a) 0

(b) 11/243

(c) 12/243

(d) $(1/3)^5$

(e) 2/3

Problem 10. An oil exploration firm is to drill ten wells, eacll on which has probability 0.1 of successfully striking recoverable oil. It costs \$10,000 to drill each well so there is a total fixed cost of \$100,000. A successful well will bring in oil worth \$500,000. The expected value and standard deviation of the firms gain are closest to, respectively, (Choose one, work will not be read). would

(a) \$500000, \$500000

(b) \$ 500000, \$ 0.9

(c) \$ 450000, \$474.34

(d) \$400000, \$474341.6

(e) \$100000, \$1001.34 $kx^9(1$

Problem 11. What is the constant k that makes the following function a valid density?

$$f(x) = kx^9(1-x)^2 \qquad 0 \le x \le 1$$

(a) 0.00151

(b) 2

(c) 660

(d) 210

(e) 0.32

Problem 12. A continuous random variable has the following probability density function

$$f(x) = \frac{x}{2} \quad 0 \leq x \leq 2$$

The 50th percentile is

(a) 1.0

(b) 5/16

(c) 0.034

(d) 0.5

(e) 1.414

$0 < x < 5$

Problem 13. The cumulative distribution fulction of a random variable X is

$$F(x) = \frac{x-2}{5} \qquad 0 < x < 5$$

. Which of the following could be the density function of this random variable?

(a) $f(x) = 2x, \quad 0 < x < 5$

(b) Uniform (a=0, b=2/5)

(c) Exponential with parameter $\lambda = \frac{2}{5}$

(d) Uniform(a=0,b=5)

(e) Poisson($\lambda = 3$)

Problem 14. By definition, if X is a random variable,

$$Var(X) = E(X - E(X))^2 = \sum_x (X - E(X))^2 P(X)$$

Which of the following does NOT equal the variance of X?

(a)
$$\sum_x (X^2 + (E(X))^2 - 2XE(X))P(X)$$

(b)
$$\sum_x X^2 P(X) + \sum_x (E(X))^2 P(X) - \sum_x 2XE(X)P(X)$$

(c)
$$\sum_x X^2 P(X) + \sum_x \mu^2 P(X) - \mu \sum_x 2XP(X)$$

(d)
$$E(\mu^2) + (E(X))^2 - 2(E(X))^2$$

(e)
$$E(X^2) - \mu^2$$

Problem 15. Suppose that the moment generating function of a random variable X is given by

$$M_x(t) = e^{3(e^t - 1)}.$$

The Probability that X is 0 is

(a) 0.3467

(b) 0.1456

(c) 0.0497

(d) 0.0001

(e) 1.237

Problem 16. Daily sales records for a car dealership show that it will sell 0, 1, 2, or 3 cars, with probabilities as listed

Number of cars (X)	0	1	2	3
Probability (P(X))	0.5	0.3	0.15	0.05

The expected value of X^3 is

(a) Impossible to compute without more information

(b) 0.75+1

(c) 2.85

(d) 5

(e) 3

(f) 0

TABLE 7.1: DISCRETE PROBABILITY DISTRIBUTION

	Probability mass function, $p(x)$	Moment generating function, $M(t)$	Mean	Variance
Binomial with parameters n, p; $0 \le p \le 1$	$\binom{n}{x} p^x (1-p)^{n-x}$ $x = 0, 1, \ldots, n$	$(pe^t + 1 - p)^n$	np	$np(1-p)$
Poisson with parameter $\lambda > 0$	$e^{-\lambda}\frac{\lambda^x}{x!}$ $x = 0, 1, 2, \ldots$	$\exp\{\lambda(e^t - 1)\}$	λ	λ
Geometric with parameter $0 \le p \le 1$	$p(1-p)^{x-1}$ $x = 1, 2, \ldots$	$\frac{pe^t}{1-(1-p)e^t}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Negative binomial with parameters r, p; $0 \le p \le 1$	$\binom{n-1}{r-1} p^r (1-p)^{n-r}$ $n = r, r+1, \ldots$	$\left[\frac{pe^t}{1-(1-p)e^t}\right]^r$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$

TABLE 7.2: CONTINUOUS PROBABILITY DISTRIBUTION

	Probability mass function, $f(x)$	Moment generating function, $M(t)$	Mean	Variance
Uniform over (a, b)	$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$	$\frac{e^{tb} - e^{ta}}{t(b-a)}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exponential with parameter $\lambda > 0$	$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \ge 0 \\ 0 & x < 0 \end{cases}$	$\frac{\lambda}{\lambda - t}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Gamma with parameters $(s, \lambda), \lambda > 0$	$f(x) = \begin{cases} \frac{\lambda e^{-\lambda x}(\lambda x)^{s-1}}{\Gamma(s)} & x \ge 0 \\ 0 & x < 0 \end{cases}$	$\left(\frac{\lambda}{\lambda - t}\right)^s$	$\frac{s}{\lambda}$	$\frac{s}{\lambda^2}$
Normal with parameters (μ, σ^2)	$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2} \quad -\infty < x < \infty$	$\exp\left\{\mu t + \frac{\sigma^2 t^2}{2}\right\}$	μ	σ^2
Chisquare with n degrees of freedom.	a gamma with parameters. $\lambda = 1/2$ $\alpha = \frac{n}{2}$	$(1-2t)^{n/2}$	n	$2n$

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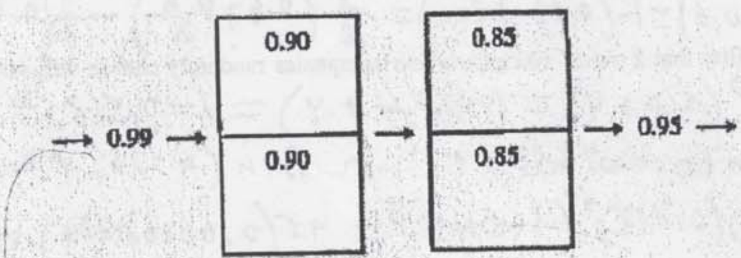
PROBLEMS WHERE YOU MUST SHOW WORK. YOU MUST ENTER YOUR NAME AND ID ABOVE.

PLEASE, WRITE YOUR ANSWER HERE AND ON THE SHEET HANDED OUT FOR THIS QUESTION. ONLY THE SHEET WILL BE GRADED.

Problem 17. The following system in Figure 17 operates only if there is a path of functional devices from left to right. The probability that each device functions is shown on the graph. Assume that the devices are independent. Calculate the reliability of the system. Show work.



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$$Rel = 0.9 \times (1-0.1^2) \times (1-0.15^2) \times 0.95 = 0.91014.$$

1pt

Problem 18. The distribution of the amount of gravel (in tons) sold by a particular construction supply company in a given week is a continuous random variable X with density function

$$f(x) = 1.5(1-x^2), \qquad 0 \le x \le 1$$

(a) How often does this company sell less than 0.4 tons per week? Show work.

$$P(X \le 0.4) = \int_0^{0.4} 1.5(1-x^2)\,dx = \frac{3}{2}x - \frac{3}{2}\frac{x^3}{3}\Big|_0^{0.4}$$

$$= \frac{3}{2} \times \frac{4}{10} - \frac{1}{2}\left(\frac{4}{10}\right)^3 = \frac{6}{10} - \frac{4}{125} = 0.568$$

1pt

(b) What is the variability (in tons per week) of the amount sold each week? Show work.

$$E(X^2) = \int_0^1 x^2\left(\frac{3}{2}\right)(1-x^2)dx = \frac{3}{2}\int_0^1 (x^2 - x^4)dx$$
$$= \frac{3}{2}\left(\frac{x^3}{3}\right) - \frac{3}{2}\left(\frac{x^5}{5}\right) = \frac{3}{2}\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{3}{2}\left(\frac{2}{15}\right) = \frac{1}{5}$$
$$E(x) = \frac{3}{2}\int_0^1 x(1-x^2)dx = \frac{3}{2}\int_0^1 (x - x^3)dx = \frac{3}{2}\left(\frac{x^2}{2}\right) - \frac{3}{2}\frac{x^4}{4} = \frac{3}{2}\left(\frac{1}{2}\right) - \frac{1}{4}\Big) = 0.375$$
$$Var(x) = 0.2 - 0.375^2 = 0.059375; \quad \boxed{\sigma = \sqrt{0.059375} = 0.2436 \text{ tons}}$$

1pt

(c) Write down the cumulative distribution function of the random variable X and use it to compute $P(0.2 \le X \le 0.6)$. Show work.

$$F(x) = \int_0^x \frac{3}{2}(1-t^2)dt = \frac{3}{2}\left(x - \frac{x^3}{3}\right), \quad 0 \le x \le 1.$$
$$P(0.2 \le X \le 0.6) = F(0.6) - F(0.2) = \frac{3}{2}\left(0.6 - \frac{0.6^3}{3}\right) - \frac{3}{2}\left(0.2 - \frac{0.2^3}{3}\right) = 0.496$$

1pt

(d) What is the probability that 2 out of 10 construction companies randomly chosen will sell more than 0.4 tons per week? $P(X > 0.4) = 1 - P(X \le 0.4) = 1 - 0.568 = 0.432 = p$

Y = # of construction co. that sell $> .4 \sim Bin(n=10, p=0.432)$

$$P(X=2) = \binom{10}{2}(0.432)^2(1-0.432)^8 = 45(0.002021874) = 0.0909 8$$

1pt

(e) How would you find the moment generating function of X? No need to give the final formula, only the setup and intermediate steps.

$$M_X(t) = \int_0^1 e^{tx}\frac{3}{2}(1-x^2)dx$$

up to here fine

$$= \frac{3}{2}\int_0^1 e^{tx}dx - \frac{3}{2}\int_0^1 \underbrace{e^{tx}}_{dv}\underbrace{x^2}_{u}dx$$
$$= \left(\frac{3}{2}\right)\left(\frac{e^t}{t}\right) - \frac{3}{2}\left[x^2\frac{e^{tx}}{t}\Big|_0^1 - \int_0^1 \underbrace{\frac{2e^{tx}}{t}}_{dv}\underbrace{x}_{u}dx\right]$$
$$= \left(\frac{3}{2}\right)\left(\frac{e^t}{t}\right) - \frac{3}{2}\left[\frac{e^t}{t} - 2\left[\int_0^1 \frac{e^{tx}}{t^2}dx\right]\right]$$
$$= \left(\frac{3}{2}\right)\left(\frac{e^t}{t}\right) - \frac{3}{2}\left[\frac{e^t}{t} - 2\frac{e^{tx}}{t^3}\right] = 2\frac{e^t}{t^3}$$