# PHYSICS 1C Mid-term 2

ALEXANDER SWERDLOW

TOTAL POINTS

**60 / 60**

QUESTION 1

## Problem 1 12 pts

### 1.1 1a 2 / 2

✓ **- 0 pts** Correct

**- 1 pts** partial credit

**- 2 pts** wrong

### 1.2 1b 4 / 4

✓ **- 0 pts** Correct

**- 1 pts** 3 right

**- 2 pts** 2 right

**- 3 pts** 1 right

**- 4 pts** wrong

### 1.3 1c 2 / 2

✓ **- 0 pts** Correct

**- 2 pts** wrong

### 1.4 1d 2 / 2

✓ **- 0 pts** Correct

**- 1 pts** 1 right

**- 2 pts** wrong

### 1.5 1e 2 / 2

✓ **- 0 pts** Correct

**- 1 pts** 1 right

**- 2 pts** wrong

QUESTION 2

## Problem 2 8 pts

### 2.1 2a 3 / 3

✓ **- 0 pts** Correct

**- 1 pts** Area

**- 1 pts** flux

**- 1 pts** current

**+ 0.5 pts** formula

### 2.2 2b 3 / 3

✓ **- 0 pts** Correct

**- 1 pts** identify parameters

**- 1 pts** integration

**- 1 pts** direction

### 2.3 2c 2 / 2

✓ **- 0 pts** Correct

**- 2 pts** formula

**- 1 pts** wrong final answer

QUESTION 3

## Problem 3 10 pts

### 3.1 3a 2 / 2

✓ **- 0 pts** Correct

**- 1.5 pts** partial credit

**- 2 pts** wrong

### 3.2 3b 3 / 3

✓ **- 0 pts** Correct

**- 1 pts** partial credit

**- 2.5 pts** some attempt

**- 3 pts** wrong

### 3.3 3c 2 / 2

✓ **- 0 pts** Correct/expression is right

**- 2 pts** wrong

### 3.4 3d 3 / 3

✓ **- 0 pts** Correct

**- 1.5 pts** calculation error/right expression/incomplete

- **2.5 pts** some attempt
- **3 pts** wrong

QUESTION 4

## Problem 4 10 pts

### 4.1 4a **3 / 3**

✓ **- 0 pts Correct**
- **1.5 pts** partial
- **2.5 pts** some attempt
- **3 pts** wrong

### 4.2 4b **3 / 3**

✓ **- 0 pts Correct**
- **1.5 pts** partial
- **2.5 pts** some attempt
- **3 pts** wrong

### 4.3 4c **2 / 2**

✓ **- 0 pts Correct**
- **1 pts** wrong direction/ wrong value/incomplete
- **2 pts** wrong

### 4.4 4d **2 / 2**

✓ **- 0 pts Correct**
- **1 pts** partial credit
- **2 pts** wrong

QUESTION 5

## Problem 5 10 pts

### 5.1 5a **3 / 3**

✓ **- 0 pts Correct**
- **3 pts** wrong equation

### 5.2 5b **4 / 4**

✓ **- 0 pts Correct**
- **2 pts** Electric field
- **1 pts** displacement current
- **1 pts** differentiation

### 5.3 5c **3 / 3**

✓ **- 0 pts Correct**
- **1 pts** Ic formula
- **1 pts** differentiate
- **1 pts** IC = ID

QUESTION 6

## Problem 6 10 pts

### 6.1 6a **3 / 3**

✓ **- 0 pts correct**
- **1.5 pts** frequnecy
- **0.5 pts** calculation
- **1 pts** z formula

### 6.2 6b **2 / 2**

✓ **- 0 pts Correct**
- **2 pts** tan
- **1 pts** pf
- **0.5 pts** calculation

### 6.3 6c **2 / 2**

✓ **- 0 pts Correct**
- **2 pts** wrong

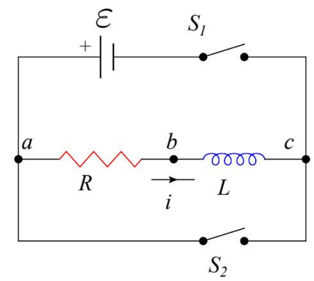
### 6.4 6d **3 / 3**

✓ **- 0 pts Correct**
- **1 pts** current
- **1 pts** voltage
- **0.5 pts** phase
- **0.5 pts** calculation

Two constants for your reference: $\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$, $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$

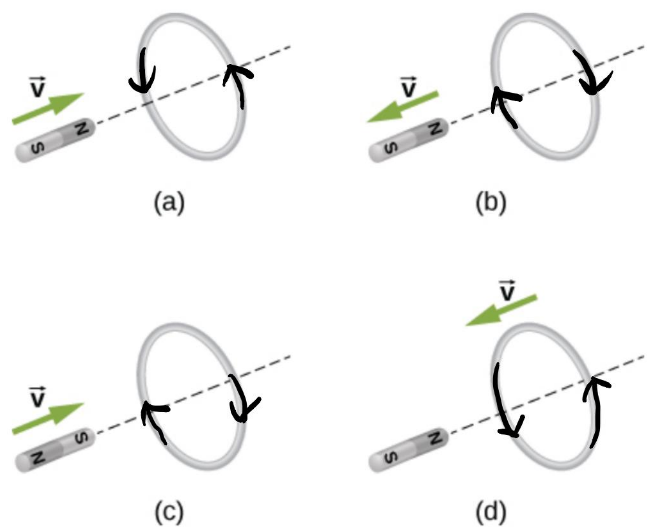
Problem 1 (12 pts): for the questions below, please give *__your final answers only__*.

(a) (2 pts) In the following figure, the current is flowing in the direction shown, what are the algebraic signs of the potential differences $v_{ab}$ and $v_{bc}$ when switch $S_1$ is closed and switch $S_2$ is open: ____a____? What about when $S_1$ is open and $S_2$ is closed: ____b____?




a. $v_{ab} > 0$, $v_{bc} > 0$ b. $v_{ab} > 0$, $v_{bc} < 0$ c. $v_{ab} < 0$, $v_{bc} > 0$ d. $v_{ab} < 0$, $v_{bc} < 0$

(b) (4 pts) For each of the following situations shown in the figure, please indicate the direction of the induced current with an arrow on the conducting loop, observing from the side of the magnet. Please make a clean drawing so that the arrow is unambiguous.




(c) (2 pts) An electromagnetic plane wave propagates in the vacuum. Its electric field $\vec{E}(x,t) = E_{\max}\cos(kx + \omega t)\,\hat{j}$, please determine the direction of the Poynting vector. Your choice: ____c____

a. $+y$ b. $-y$ c. $-x$ d. $+x$ e. $-z$ f. $+z$

g. not enough information, cannot be determined

### 1.1 1a 2 / 2

✓ - **0 pts** Correct

- **1 pts** partial credit

- **2 pts** wrong

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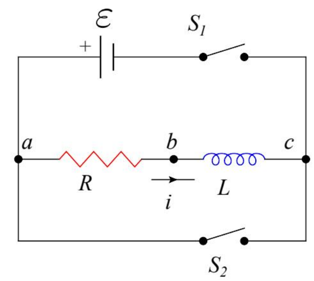


a. $v_{ab} > 0, v_{bc} > 0$ b. $v_{ab} > 0, v_{bc} < 0$ c. $v_{ab} < 0, v_{bc} > 0$ d. $v_{ab} < 0, v_{bc} < 0$

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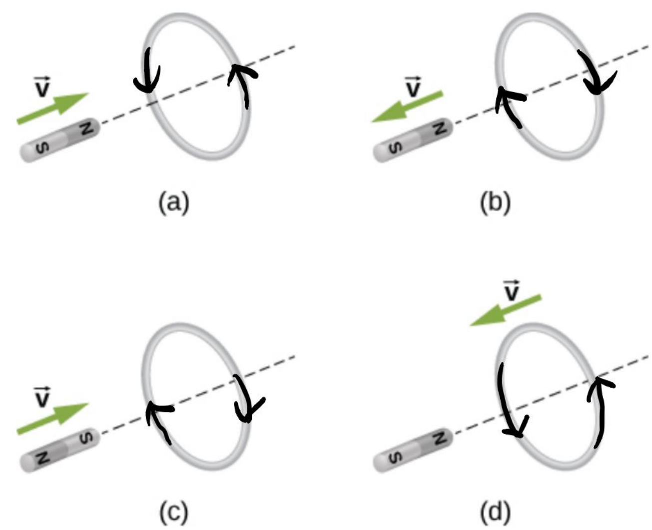


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a. $+y$ b. $-y$ c. $-x$ d. $+x$ e. $-z$ f. $+z$

g. not enough information, cannot be determined

### 1.2 1b 4 / 4

✓ **- 0 pts** Correct

**- 1 pts** 3 right

**- 2 pts** 2 right

**- 3 pts** 1 right

**- 4 pts** wrong

Two constants for your reference: $\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$, $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$

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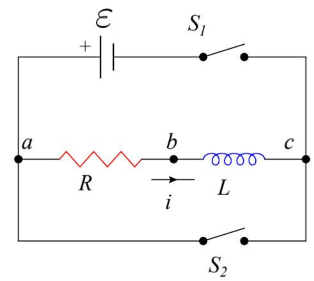


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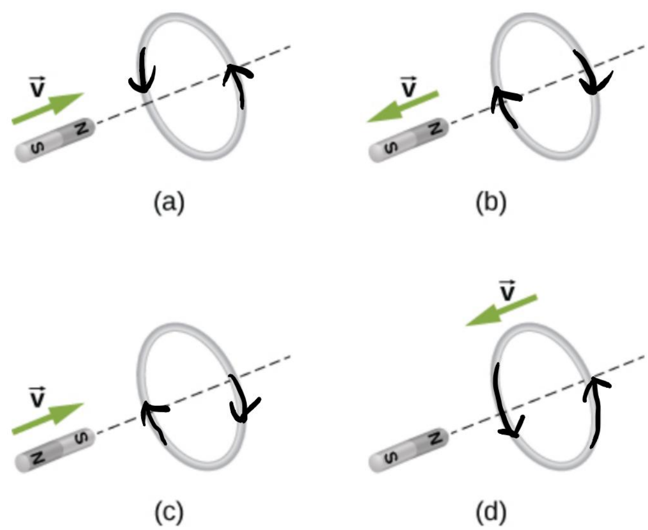


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a. $+y$  b. $-y$  c. $-x$  d. $+x$  e. $-z$  f. $+z$

g. not enough information, cannot be determined

### 1.3 1c 2 / 2

- ✓ **- 0 pts** Correct
- **- 2 pts** wrong

(d) (2 pts) In a *L-R-C* series circuit with AC current, the voltage amplitudes $V_R$, $V_L$, $V_C$ for resistor *R*, the inductor *L*, and the capacitor *C* depend on the angular frequency $\omega$. As $\omega$ increases, $V_R$ will be___C___; $V_L$ will be___A___

a. increasing    b. decreasing    c. unchanged    d. undetermined

(e) (2 pts) A transformer on a utility pole steps the rms voltage down from 12 kV to 240 V. What is the ratio of the number of secondary turns to the number of primary turns? $\frac{N_2}{N_1} =$ __1/50__

If the input current to the transformer is 2.0 A, what is the output current? $I_{out} =$ __100A__

$X_C = \omega L$

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2}$$

$$\frac{240V}{12 \cdot 10^3 V} = 0.02 = \frac{1}{50}$$

$$\frac{1}{50} = \frac{2A}{I_2}$$

$I_2 = 100A$

1.4 1d **2 / 2**

- ✓ **- 0 pts** Correct
- **- 1 pts** 1 right
- **- 2 pts** wrong

(d) (2 pts) In a *L-R-C* series circuit with AC current, the voltage amplitudes $V_R$, $V_L$, $V_C$ for resistor *R*, the inductor *L*, and the capacitor *C* depend on the angular frequency $\omega$. As $\omega$ increases, $V_R$ will be__C__; $V_L$ will be__A__

a. increasing b. decreasing c. unchanged d. undetermined

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If the input current to the transformer is 2.0 A, what is the output current? $I_{\text{out}} =$ __100A__

$X_C = \omega L$

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2}$$

$$\frac{240V}{12 \cdot 10^3 V} = 0.02 = \frac{1}{30}$$

$$\frac{1}{50} = \frac{2A}{I_2}$$

$I_2 = 100A$

1.5 1e **2 / 2**

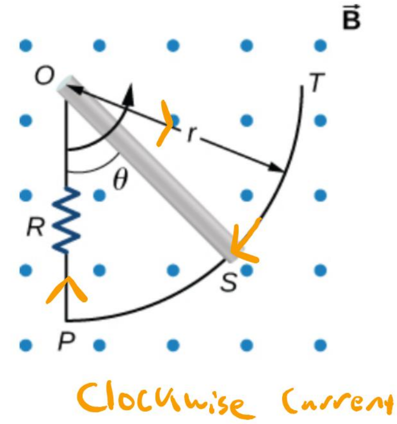
✓ **- 0 pts** Correct

**- 1 pts** 1 right

**- 2 pts** wrong

Problem 2 (8 pts)

As shown in the figure, a metal rod *OS* is rotating in a horizontal plane around the point *O*. The rod slides along a wire that forms a circular arc *PST* of radius *r*. The system is in a constant magnetic field $\vec{B}$ that is directed out of the page, and we rotate the rod at a constant angular velocity $\omega$. Please answer the following questions




a) (3 pts) Start specifically from Faraday's law $\varepsilon = -\frac{d\Phi_B}{dt}$ to compute the current *I* in the closed loop *OSPO*? Assume that the resistor *R* furnishes all of the resistance in the closed loop. Provide the magnitude only. (*hint: you can compute the magnetic flux* $\Phi_B$ *for the loop area covered by OSPO.*)

b) (3 pts) Start specifically from motional emf formula $\varepsilon = \int(\vec{v} \times \vec{B}) \cdot d\vec{l}$ to compute the current *I* in the closed loop *OSPO*? Indicate the direction of the current in the figure.

c) (2 pts) Calculate the power dissipated in the resistor.

a) $\mathcal{E} = \frac{-d\Phi_B}{dt}$ $\Phi_B = \int B \cdot dA = BA = B \cdot \frac{\theta}{2\pi} \cdot \pi r^2 = \frac{1}{2} B \theta r^2$

$\mathcal{E} = -\frac{d}{dt}[\Phi_B] = -\frac{d}{dt}[\frac{1}{2} B \theta r^2] = -\frac{1}{2} B \omega r^2$

$|\mathcal{E}| = IR = I = \frac{|\mathcal{E}|}{R} = \frac{B \omega r^2}{2R}$ $\boxed{I = \frac{B\omega r^2}{2R}}$

## 2.1 2a 3 / 3

✓ **- 0 pts** Correct

**- 1 pts** Area

**- 1 pts** flux

**- 1 pts** current

**+ 0.5 pts** formula

b) $\mathcal{E} = \int (\vec{v} \times \vec{B}) \cdot d\vec{l}$

$= B \int v \, dr = B \int \omega r \, dr = \omega B \int_0^r r' dr' = \boxed{\frac{1}{2} B \omega r^2}$ [Same as Before]

$\uparrow \; v = \omega r$

Current is clockwise as $\frac{d\Phi}{dt} > 0$ so induced B points into the page. $\left[\mathcal{E} = -\frac{d\Phi}{dt}\right]$

c) $P = \frac{\mathcal{E}^2}{R} = \frac{\left(\frac{1}{2} B \omega r^2\right)^2}{R} = \frac{\frac{1}{4} B^2 \omega^2 r^4}{R} = \boxed{\frac{B^2 \omega^2 r^4}{4R}}$

## 2.2 2b 3 / 3

- ✓ **- 0 pts** Correct
- **- 1 pts** identify parameters
- **- 1 pts** integration
- **- 1 pts** direction

b) $\mathcal{E} = \int (\vec{v} \times \vec{B}) \cdot d\vec{l}$

$$= B \int v \, dr = B \int \omega r \, dr = \omega B \int_0^r r' dr' = \boxed{\frac{1}{2} B \omega r^2}$$ [Same as Before]

$\uparrow \; v = \omega r$

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2.3 2c **2 / 2**

✓ **- 0 pts** Correct

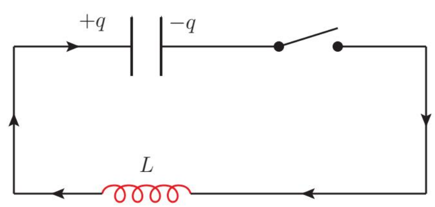
**- 2 pts** formula

**- 1 pts** wrong final answer

Problem 3 (10 pts)

In a *L-C* circuit as shown in the figure, one closes the switch at some initial time $t = 0$. We find that the maximum charge on the capacitor is $2.0 \times 10^{-6}$ C and the maximum current through the inductor is 8.0 mA. Please answer the following questions

a) (2 pts) *Briefly* describe *qualitatively* how the current behaves in such a *L-C* circuit. You are **not** supposed to compute anything.




b) (3 pts) What is the period of the circuit?

c) (2 pts) How much time elapses between an instant when the capacitor is fully charged and the next instant when it is completely uncharged?

d) (3 pts) If the capacitance is $C = 1.0 \mu F$, what is the self-inductance *L*?

a)

The current oscillates in SHM, starting at zero and increasing until the energy in the capacitor is stored in the inductor in the form of an induced magnetic field. The current then decreases, charging the capacitor which now has its polarity reversed. The decreasing current induces an emf in the same dir as the current, slowing the decrease of current. The current goes to zero just as the capacitor is fully charged and the cycle repeats, returning the circuit to its initial state.

### 3.1 3a 2 / 2

- ✓ **- 0 pts** Correct
- **- 1.5 pts** partial credit
- **- 2 pts** wrong

b) $-L\frac{di}{dt} - \frac{q}{c} = 0$

$$\frac{d^2q}{dt^2} + \frac{1}{LC} q = 0$$

$$\frac{d^2q}{dt^2} = \frac{-1}{LC} q$$

$$q(t) = Q\cos(\omega t + \phi)$$

$$i(t) = \frac{dq(t)}{dt} = -\omega Q \sin(\omega t + \phi)$$

$$= I_0 \sin(\omega t + \phi)$$

$\therefore \quad -\omega Q = I_0$

$$\omega = \left|\frac{-I_0}{Q}\right| = \left|-\frac{8\cdot 10^{-3} A}{2\cdot 10^{-6} C}\right|$$

$$= 4000 \text{ rad/s}$$

$$T = \frac{2\pi}{\omega} = \boxed{1.5707\cdot 10^{-3} \text{ sec}}$$

c) $q(t) = Q\cos(\omega t + \phi)$

$$\frac{d}{dt}(q(t)) = -\omega Q \sin(\omega t + \phi) = 0$$

Sin(x) has a max at $t = \frac{T}{4}$

$T = 1.5707\cdot 10^{-3}$ sec

$$\boxed{t = 3.927\cdot 10^{-4} \text{ sec}}$$

d) $C = 1.0 mF$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$4000 = \frac{1}{\sqrt{L\cdot 1mF}}$$

$$\boxed{L = 0.0625 H}$$

### 3.2 3b 3 / 3

- ✓ **- 0 pts** Correct
- **- 1 pts** partial credit
- **- 2.5 pts** some attempt
- **- 3 pts** wrong

b) $-L\frac{di}{dt} - \frac{q}{c} = 0$

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$$

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$$\omega = \frac{1}{\sqrt{LC}}$$

$$4000 = \frac{1}{\sqrt{L\cdot 1\mu F}}$$

$$\boxed{L = 0.0625 H}$$

### 3.3 3c 2 / 2

✓ **- 0 pts** Correct/expression is right

**- 2 pts** wrong

b) $-L\frac{di}{dt} - \frac{q}{c} = 0$

$$\frac{d^2q}{dt^2} + \frac{1}{LC} q = 0$$

$$\frac{d^2q}{dt^2} = \frac{-1}{LC} q$$

$$q(t) = Q\cos(\omega t + \phi)$$

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$$= I_0 \sin(\omega t + \phi)$$

$\therefore$ $-\omega Q = I_0$

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d) $C = 1.0 \text{ mF}$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$4000 = \frac{1}{\sqrt{L \cdot 1\text{mF}}}$$

$\boxed{L = 0.0625 \text{ H}}$

3.4 3d 3 / 3

✓ **- 0 pts** Correct

**- 1.5 pts** calculation error/right expression/incomplete

**- 2.5 pts** some attempt

**- 3 pts** wrong

Problem 4 (10 pts)

An electromagnetic wave traveling in $-z$ direction in vacuum has a wavelength $\lambda = 6 \times 10^{-7}$ m. The magnetic field associated with such an electromagnetic wave is along $+x$ direction and has an amplitude $B_{max} = 6 \times 10^{-8}$ T. Note for the vectors below, your expression should reflect the direction.

(a) (3 pts) Please derive the expression for the wave corresponding to the magnetic field $\vec{B}$.

(b) (3 pts) Please derive the expression for the wave corresponding to the electric field $\vec{E}$.

(c) (2 pts) Please derive the expression for the instantaneous Poynting vector $\vec{S}$.

(d) (2 pts) Find the instantaneous values of the total energy density $u$.

$-z$ dir so $+$ ↙

a) $B(z,t) = B_{max}\cos(kz+\omega t) = 6\cdot10^{-8}\,T\,\cos\left((1.0\cdot10^{7}\,m^{-1})z + (3.0\cdot10^{15}\,s^{-1})t\right)\hat{i}$

$B_{max} = 6\cdot10^{-8}\,T$

$\omega = kc$

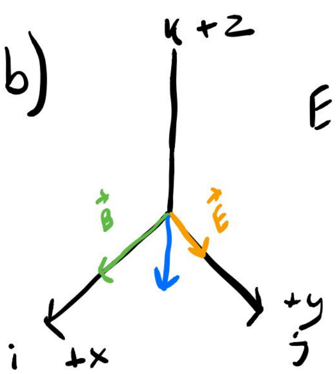
$k = \frac{2\pi}{\lambda} = \frac{2\pi}{6\cdot10^{-7}\,m} = 1.05\cdot10^{7}\,m^{-1}$

$\frac{\omega}{k} = c$

$\omega = 1.05\cdot10^{7}\,m^{-1}\cdot 3\cdot10^{8}\,\frac{m}{s}$

$= 3.14\cdot10^{15}$ rad/s

b)




$E(z,t) = E_{max}\cos(kz+\omega t) = cB_{max}\cos(kz+\omega t)$

$E(z,t) = 18\,\frac{V}{m}\cos\left((1.0\cdot10^{7}\,m^{-1})z + (3.0\cdot10^{15}\,s^{-1})t\right)\hat{j}$

$cB_{max} = 3\cdot10^{8}\,m/s\cdot 6\cdot10^{-8}\,T$

4.1 4a **3 / 3**

✓ **- 0 pts** Correct

**- 1.5 pts** partial

**- 2.5 pts** some attempt

**- 3 pts** wrong

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-z dir so +

a) $B(z,t) = B_{max}\cos(kz + \omega t) = 6\cdot10^{-8}\,T\ \cos\left((1.0\cdot10^{7}\,m^{-1})z + (3.0\cdot10^{15}\,s^{-1})t\right)\hat{i}$

$B_{max} = 6\cdot10^{-8}\,T$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{6\cdot10^{-7}\,m} = 1.05\cdot10^{7}\,m^{-1}$

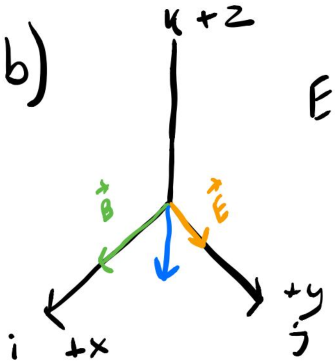
$\frac{\omega}{k} = c$

$\omega = kc$

$\omega = 1.05\cdot10^{7}\,m^{-1}\cdot3\cdot10^{8}\,\frac{m}{s}$

$= 3.14\cdot10^{15}$ rad/s

b)




$E(z,t) = E_{max}\cos(kz + \omega t) = cB_{max}\cos(kz + \omega t)$

$E(z,t) = 18\,\frac{V}{m}\cos\left((1.0\cdot10^{7}\,m^{-1})z + (3.0\cdot10^{15}\,s^{-1})t\right)\hat{j}$

$cB_{max} = 3\cdot10^{8}\,m/s\cdot6\cdot10^{-8}\,T$

4.2 4b **3 / 3**

- ✓ **- 0 pts** Correct
- **- 1.5 pts** partial
- **- 2.5 pts** some attempt
- **- 3 pts** wrong

c) $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$

[points in -z direction]

$$= \frac{-1}{\mu_0}\left(18\ \frac{V}{m} \cdot 6 \cdot 10^{-8}\ T\right)\cos^2\left((1.0 \cdot 10^7\ m^{-1})z + (3.0 \cdot 10^{15}\ s^{-1})t\right)\hat{k}$$

$[\mu_0 = 4\pi \cdot 10^{-7}\ T \cdot m/A]$

$$= -0.86 \cos^2\left((1.0 \cdot 10^7\ m^{-1})z + (3.0 \cdot 10^{15}\ s^{-1})t\right)\hat{k} \quad \frac{W}{m^2}$$

d) $U = \varepsilon_0 E^2 = \varepsilon_0 \left(18\ \frac{V}{m} \cos\left((1.0 \cdot 10^7\ m^{-1})z + (3.0 \cdot 10^{15}\ \frac{rad}{s})t\right)\right)^2$

$$= 8.854 \cdot 10^{-12}\ \frac{F}{m} \cdot \left(18\ \frac{V}{m}\right)^2 \cos^2\left((1.0 \cdot 10^7\ m^{-1})z + (3.0 \cdot 10^{15}\ s^{-1})t\right)$$

$$= 2.87 \cdot 10^{-9} \cos^2\left((1.0 \cdot 10^7\ m^{-1})z + (3.0 \cdot 10^{15}\ s^{-1})t\right) \left[\frac{J}{m^3}\right]$$

4.3 4c **2 / 2**

- ✓ **- 0 pts** Correct
- **- 1 pts** wrong direction/ wrong value/incomplete
- **- 2 pts** wrong

c) $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$

[points in -z direction]

$$= \frac{-1}{\mu_0}\left(18\,\frac{V}{m} \cdot 6 \cdot 10^{-8}\,T\right)\cos^2\left((1.0 \cdot 10^7\,m^{-1})z + (3.0 \cdot 10^{15}\,s^{-1})t\right)\hat{k}$$

[$\mu_0 = 4\pi \cdot 10^{-7}\,T \cdot m/A$]

$$= -0.86\cos^2\left((1.0 \cdot 10^7\,m^{-1})z + (3.0 \cdot 10^{15}\,s^{-1})t\right)\hat{k} \quad \frac{W}{m^2}$$

d) $U = \varepsilon_0 E^2 = \varepsilon_0 \left(18\,\frac{V}{m}\cos\left((1.0 \cdot 10^7\,m^{-1})z + (3.0 \cdot 10^{15}\,\frac{rad}{s})t\right)\right)^2$

$$= 8.854 \cdot 10^{-12}\,\frac{F}{m} \cdot \left(18\,\frac{V}{m}\right)^2 \cos^2\left((1.0 \cdot 10^7\,m^{-1})z + (3.0 \cdot 10^{15}\,s^{-1})t\right)$$

$$= 2.87 \cdot 10^{-9}\cos^2\left((1.0 \cdot 10^7\,m^{-1})z + (3.0 \cdot 10^{15}\,s^{-1})t\right)\left[\frac{J}{m^3}\right]$$

4.4 4d **2 / 2**

- ✓ **- 0 pts** Correct
- **- 1 pts** partial credit
- **- 2 pts** wrong

Problem 5 (10 pts)

A capacitor with capacitance $C$ whose plates have area $A$ and separation distance $d$ is connected to a resistor $R$ and a battery of voltage $V$. The current starts to flow at time $t = 0$. Please answer the following questions

a) (3 pts) What is the voltage $V_C(t)$ between the plates at time $t$? (*hint: recall what you have learned on RC circuit. You might refer to eTextbook section 26.4.*)

b) (4 pts) Compute the displacement current $I_d$ between the capacitor plates at time $t$.

c) (3 pts) From the properties of the capacitor, find the conducting current $I_c$ in the $RC$ circuit. Compare your answer to the displacement current, what is their relation?

let $\mathcal{E} = V$

$$\mathcal{E} - iR - \frac{q}{C} = 0$$

$$\mathcal{E} - \frac{dq}{dt}R - \frac{q}{C} = 0$$

$$\mathcal{E} - \frac{q}{C} = \frac{dq}{dt}R$$

$$\frac{\mathcal{E}}{R} - \frac{q}{RC} = \frac{dq}{dt}$$

$$\frac{1}{q - C\mathcal{E}}\,dq = \frac{-1}{RC}\,dt$$

$$\int_0^q \frac{1}{q' - C\mathcal{E}}\,dq' = -\int_0^t \frac{1}{RC}\,dt'$$

$$\ln\left(\frac{q - C\mathcal{E}}{-C\mathcal{E}}\right) = \frac{-t}{RC}$$

$$\frac{q - C\mathcal{E}}{-C\mathcal{E}} = e^{-t/RC}$$

cont. from bot.

$$q(t) = C\mathcal{E}(1 - e^{-t/RC})$$

$$V_c(t) = \frac{q(t)}{C} = V(1 - e^{-t/RC})$$

$$\boxed{V_c(t) = V(1 - e^{-t/RC})}$$

5.1 5a **3 / 3**

✓ **- 0 pts** Correct

**- 3 pts** wrong equation

b) $V = ED$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_C + \mu_0 \left( \varepsilon_0 \frac{d\Phi_E}{dt} \right)$$

$\uparrow$ $I_d$

$$E = \frac{V}{D} = \frac{V}{d}\left(1 - e^{-t/RC}\right)$$

$$C = \varepsilon_0 \frac{A}{d}$$

$$I_d(t) = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d}{dt}\left(E(t) \cdot A\right) = \varepsilon_0 A \cdot \frac{V}{D} \cdot \frac{1}{RC} e^{-t/RC} = \varepsilon_0 \frac{V}{D} \cdot \frac{1}{\left(\varepsilon_0 \frac{A}{d}\right) C} e^{-t/RC}$$

$$\boxed{I_d(t) = \frac{V}{R} e^{-t/RC}}$$

c) From pt A:

$$q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$

$$I_c(t) = \frac{dq(t)}{dt} = \frac{V}{R} e^{-t/RC} = I_d$$

$$\boxed{I_c(t) = I_d(t)}$$

5.2 5b **4 / 4**

- ✓ **- 0 pts** Correct
- **- 2 pts** Electric field
- **- 1 pts** displacement current
- **- 1 pts** differentiation

b) $V = ED$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_C + \mu_0 \left( \varepsilon_0 \frac{d\Phi_E}{dt} \right)$$

($\uparrow$ $I_d$)

$$E = \frac{V}{D} = \frac{V}{d}\left(1 - e^{-t/RC}\right)$$

$$C = \varepsilon_0 \frac{A}{d}$$

$$I_d(t) = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d}{dt}\left(E(t) \cdot A\right) = \varepsilon_0 A \cdot \frac{V}{D} \cdot \frac{1}{RC} e^{-t/RC} = \varepsilon_0 \frac{V}{D} \cdot \frac{1}{\left(\varepsilon_0 \frac{A}{d}\right) c} e^{-t/RC}$$

$$\boxed{I_d(t) = \frac{V}{R} e^{-t/RC}}$$

c) From pt A:

$$q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$

$$I_c(t) = \frac{dq(t)}{dt} = \frac{V}{R} e^{-t/RC} = I_d$$

$$\boxed{I_c(t) = I_d(t)}$$

5.3 5c **3 / 3**

- ✓ **- 0 pts** Correct
- **- 1 pts** Ic formula
- **- 1 pts** differentiate
- **- 1 pts** IC = ID

Problem 6 (10 pts)

A 200 Ω resistor, a 1.00 H inductor, and a 4.00 $\mu$F capacitor are connected in series across a voltage source that has voltage amplitude 40.0 V.

(a) (3 pts) At what angular frequency will the impedance be smallest? What is the impedance $Z$ at this angular frequency?

(b) (2 pts) Assume the angular frequency of 200 rad/s for the rest of questions. What is the power factor for this circuit?

(c) (2 pts) What is the average power delivered to the capacitor, and to the inductor?

(d) (3 pts) At time $t$ = 10.0 ms, please compute the instantaneous current $i$ and instantaneous voltage $v$.

a) $Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$

$Z_{min}$ when $\omega L = \frac{1}{\omega C}$

$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2H \cdot 4mF}} = \boxed{500 \text{ rad/s}}$

$Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} = \sqrt{R^2} = R = \boxed{200\,\Omega}$

## 6.1 6a **3 / 3**

- ✓ **- 0 pts** **correct**
- **- 1.5 pts** frequnecy
- **- 0.5 pts** calculation
- **- 1 pts** z formula

b)

$$\omega = 200 \text{ rad/s}$$

$$\phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

$$= \tan^{-1}(-5.25) = -1.38 \text{ rad}$$

$$\text{power factor} = \cos(\phi) = \cos(-1.38 \text{ rad}) = \boxed{0.187}$$

c)

$$P_{avg,\,capacitor} = 0 \text{ Watts}$$

$$P_{avg,\,inductor} = 0 \text{ Watts}$$

Avg power to capacitor/inductor is zero. only resistors have non-zero avg power.

d)

$$t = 10\,ms = 10 \cdot 10^{-3} \text{ sec}$$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = 1068.88\,\Omega$$

$$I_{max} = \frac{V}{Z} = \frac{40V}{1068.88\Omega} = 0.0374 A$$

$$i(t) = I\cos(\omega t) = 0.0374A \cdot \cos\left(200\tfrac{rad}{s} \cdot 0.01 \text{ sec}\right) = \boxed{-0.0156A}$$

$$v(t) = V\cos(\omega t + \phi) = 40V \cdot \cos\left(200\tfrac{rad}{s} \cdot 0.01 \text{ sec} - 1.38 \text{ rad}\right) = \boxed{32.61\,V}$$

↑

From pt b, $\phi = -1.38$ rad

6.2 6b **2 / 2**

✓ **- 0 pts** Correct

**- 2 pts** tan

**- 1 pts** pf

**- 0.5 pts** calculation

b)

$\omega = 200 \text{ rad/s}$

$\phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$

$= \tan^{-1}(-5.25) = -1.38 \text{ rad}$

$\text{power factor} = \cos(\phi) = \cos(-1.38 \text{ rad}) = \boxed{0.187}$

c)

$P_{avg,\ capacitor} = 0 \text{ Watts}$

$P_{avg,\ inductor} = 0 \text{ Watts}$

Avg power to capacitor/inductor is zero. only resistors have non-zero avg power.

d)

$t = 10 \text{ ms} = 10 \cdot 10^{-3} \text{ sec}$

$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = 1068.88\ \Omega$

$I_{max} = \frac{V}{Z} = \frac{40V}{1068.88\Omega} = 0.0374 A$

$i(t) = I\cos(\omega t) = 0.0374A \cdot \cos(200 \frac{\text{rad}}{s} \cdot 0.01 \text{ sec}) = \boxed{-0.0156 A}$

$v(t) = V\cos(\omega t + \phi) = 40V \cdot \cos(200 \frac{\text{rad}}{s} \cdot 0.01 \text{ sec} - 1.38 \text{ rad}) = \boxed{32.61 V}$

↑

From pt b, $\phi = -1.38 \text{ rad}$

6.3 6c 2 / 2

✓ - 0 pts Correct

- 2 pts wrong

b)

$\omega = 200\ rad/s$

$$\phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

$$= \tan^{-1}(-5.25) = -1.38\ rad$$

$$\text{power factor} = \cos(\phi) = \cos(-1.38\ rad) = \boxed{0.187}$$

c)

$$\boxed{P_{avg\ capacitor} = 0\ \text{Watts} \qquad P_{avg\ inductor} = 0\ \text{Watts}}$$

Avg power to capacitor/inductor is zero. only resistors have non-zero avg power.

d)

$t = 10\,ms = 10 \cdot 10^{-3}\ sec$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = 1068.88\ \Omega$$

$$I_{max} = \frac{V}{Z} = \frac{40V}{1068.88\Omega} = 0.0374\,A$$

$$i(t) = I\cos(\omega t) = 0.0374A \cdot \cos\left(200\tfrac{rad}{s} \cdot 0.01\,sec\right) = \boxed{-0.0156A}$$

$$v(t) = V\cos(\omega t + \phi) = 40V \cdot \cos\left(200\tfrac{rad}{s} \cdot 0.01\,sec - 1.38\,rad\right) = \boxed{32.61\,V}$$

↑

From pt b, $\phi = -1.38\,rad$

6.4 6d **3 / 3**

- ✓ **- 0 pts** **Correct**
- **- 1 pts** current
- **- 1 pts** voltage
- **- 0.5 pts** phase
- **- 0.5 pts** calculation