MT 1 Physics 1C Fall 2013

Name________________________________

Signature____________________________

9-digit SID #_________________________

Class Statistics
High: 100 (100%)
Low: 3 (3%)
Median: 72 (72%)
Mean: 70.6 (71%)

1	18	/20
2	29	/30
3	18	/20
4	26	/30
Total	91	/100

Turn off and put away all electronics, phones, cameras, calculators.

You may have a single sided, ½ page of notes, pencils and eraser on your desk and this exam.

Put all work and calculations on the pages of the exam. It's good advice not to erase anything, just cross out. You may get partial credit on some problems.

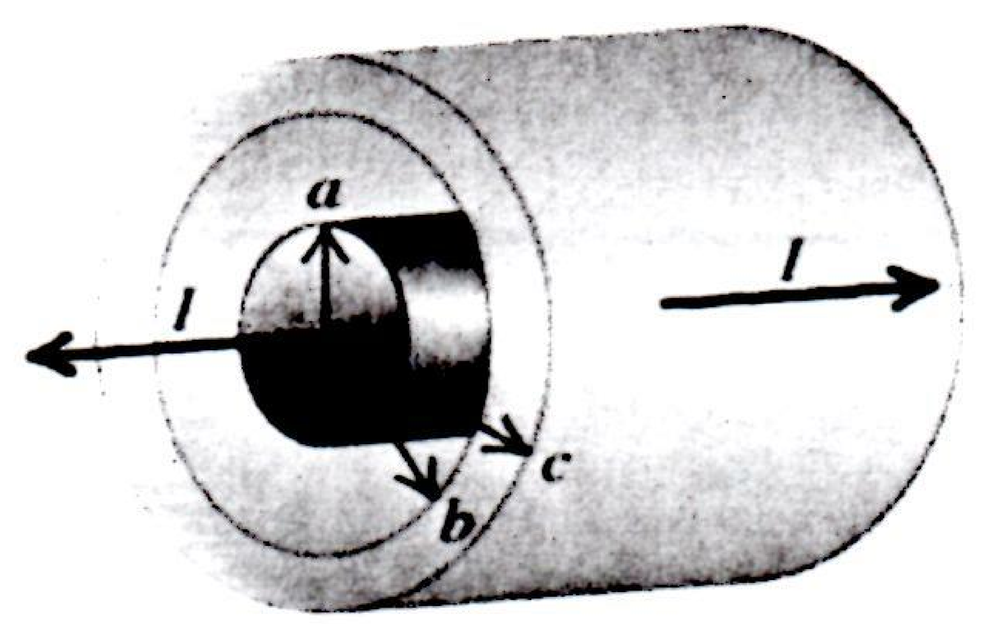
The following information may or may not be helpful.

$$\sin 30 = \frac{1}{2} \qquad \sin 60 = \frac{\sqrt{3}}{2} \qquad \cos 30 = \frac{\sqrt{3}}{2} \qquad \cos 60 = \frac{1}{2}$$

$$\cos 45 = \sin 45 = \frac{\sqrt{2}}{2} \approx 0.707 \approx \frac{1}{1.4}$$

$\mu_0 = 4\pi\times10^{-7}$ Tm/A (Tm/A = N/A^2) $\qquad$ $\epsilon_0 = 8.85\times10^{-12}$ C^2/Nm^2

1 eV= 1.602×10^{-19} J $\qquad$ 1 MeV= 1.602×10^{-13} J $\qquad$ 1 nm=10^{-9} m

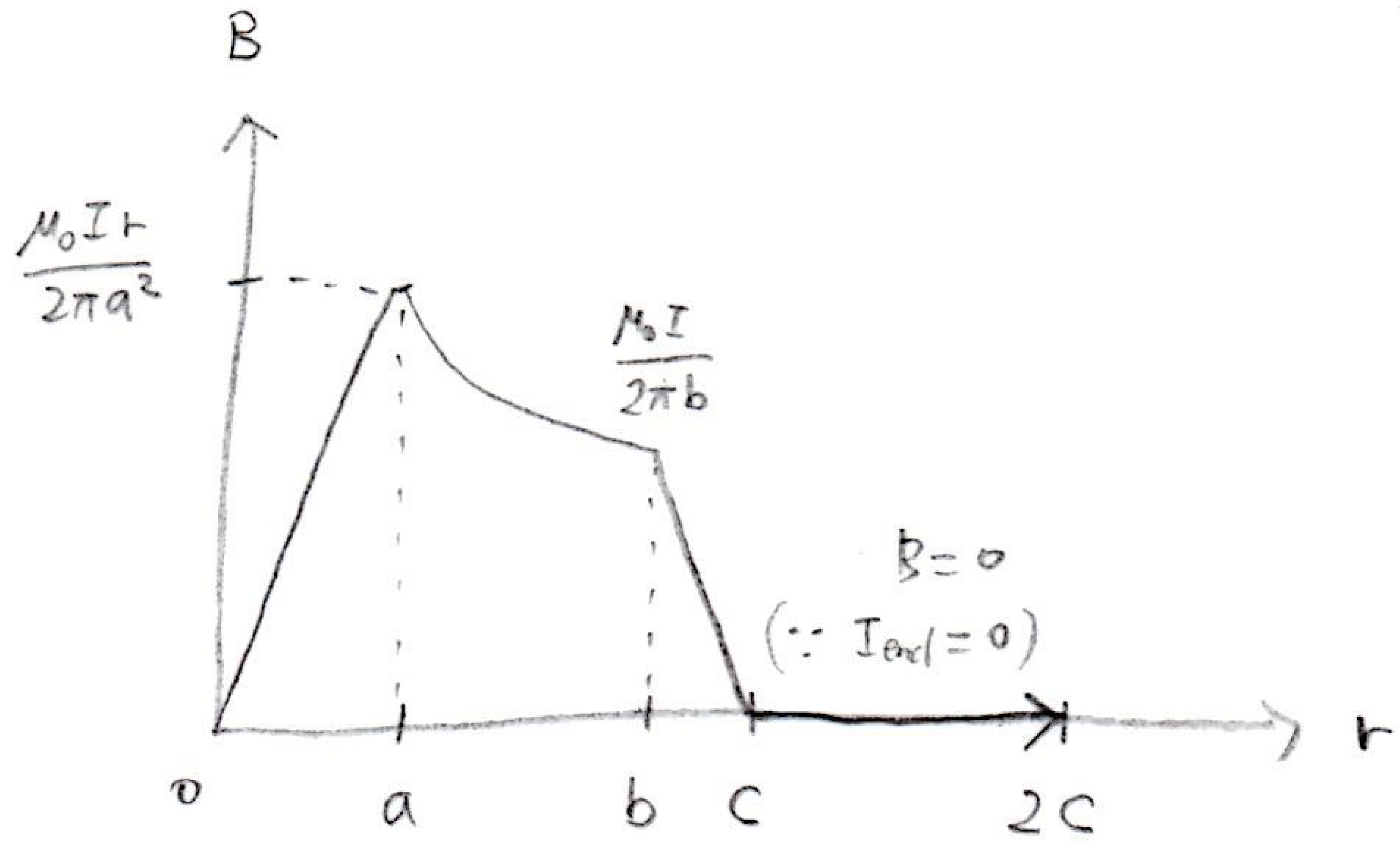


1a. (10 pts) Above is a coax cable carrying a uniform current I in the center conductor and a uniform return current I in the outer conductor. Draw a graph showing the Magnetic Field B(r) from r = 0 to 2c. Evaluate and label the max magnetic field and the field at a, b, and c.

10

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$B = \frac{\mu_0 I_{encl}}{2\pi r}$$



$0 < r < a$,

$$B = \frac{\mu_0 I}{2\pi r} \left(\frac{r^2 \pi}{a^2 \pi}\right) = \frac{\mu_0 I r}{2\pi a^2}$$

$a < r < b$,

$$B = \frac{\mu_0 I}{2\pi r}$$

$b < r < c$

$$B = \frac{\mu_0 I - I\left(\frac{r^2}{c^2}\right)}{2\pi r}$$

1b. (10 pts) Use the Biot-Savart law to solve for the magnetic field at point P.

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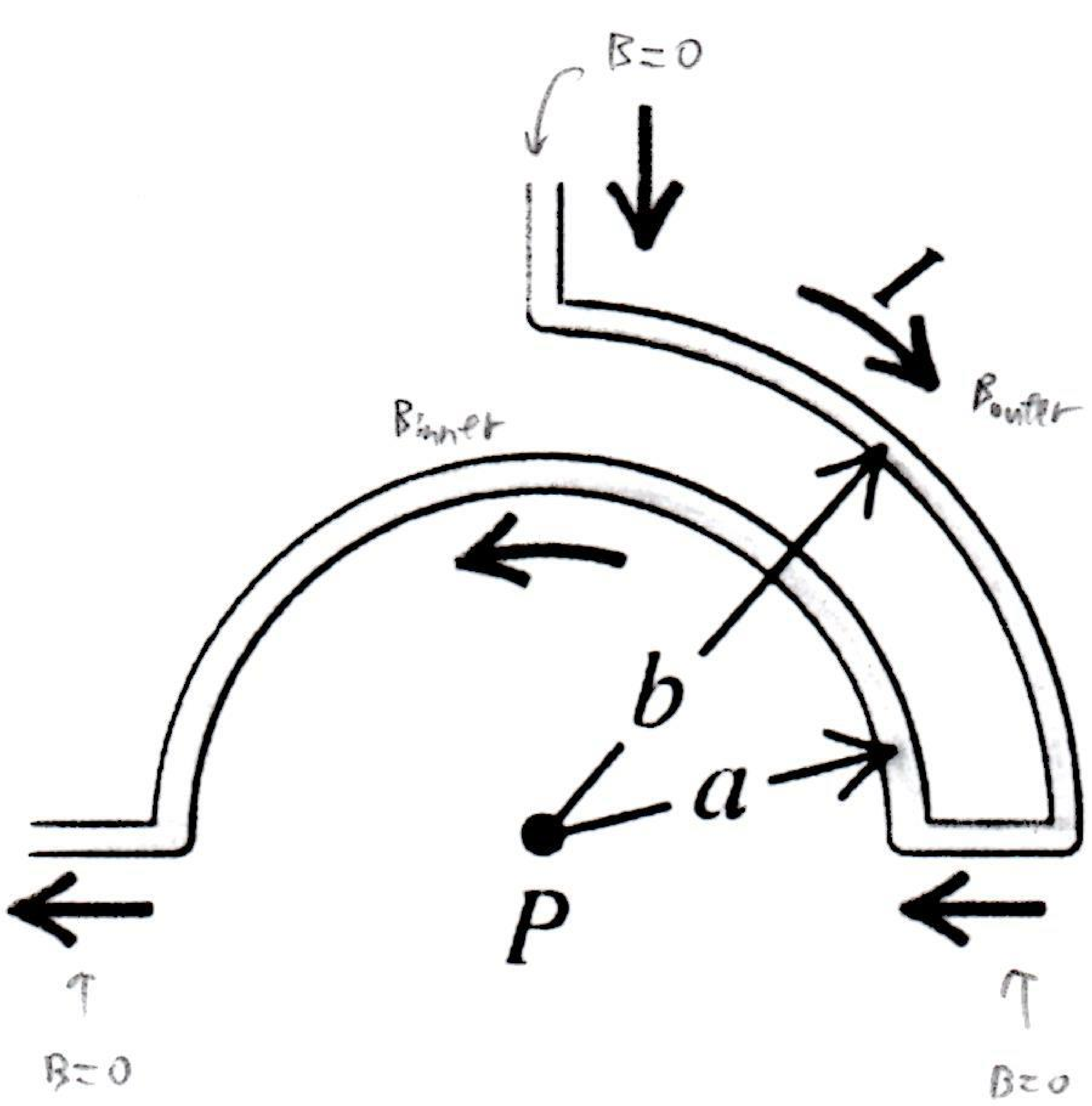


$$B = \frac{\mu_0}{4\pi} \int \frac{I \, d\vec{l} \times \hat{r}}{r^2}$$

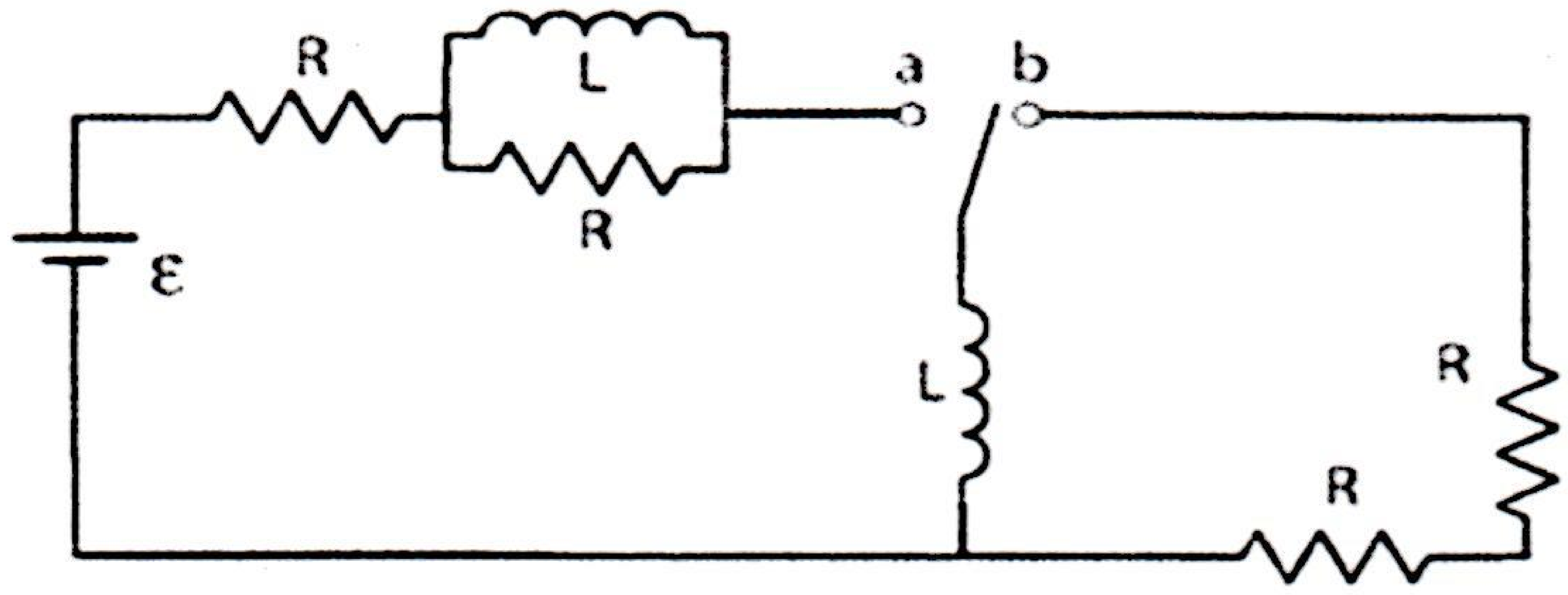
$$B_{outer} = \frac{\mu_0 I}{4\pi} \int_0^{\pi/2} \frac{r \, d\theta}{r^2} = \frac{\mu_0 I}{4\pi r} \int_0^{\pi/2} d\theta = \frac{\mu_0 I}{4\pi r} \cdot \frac{\pi}{2} = \frac{\mu_0 I}{8 r}$$

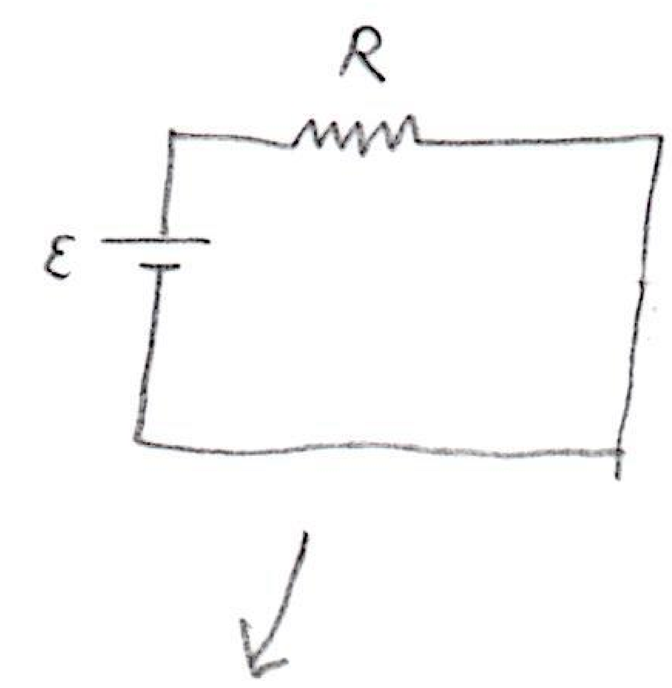
into the page

$$B_{inner} = \frac{\mu_0 I}{4\pi} \int_0^{\pi} \frac{r \, d\theta}{r^2} = \frac{\mu_0 I}{4\pi r} \cdot \pi = \frac{\mu_0 I}{4 r}$$ into the page

$$\boxed{\vec{B} = \left(\frac{\mu_0 I}{4a} - \frac{\mu_0 I}{8b}\right) \text{into } \cancel{\text{the}} \text{ page}}$$

Problem 2





Consider the circuit above.

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2a. (5 pts) I throw the switch to position "a" and wait a long time. What is the current flowing from the battery?

L acts as short circuit.

$\mathcal{E} = IR \qquad \boxed{I = \frac{\mathcal{E}}{R}}$

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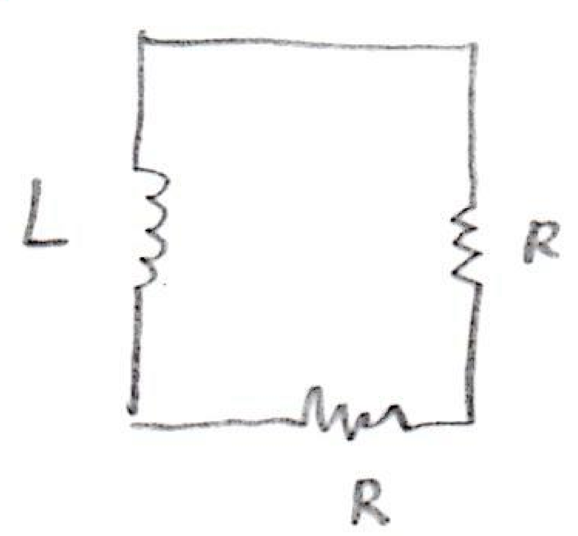
2b. (5 pts) I then throw the switch to position "b" and reset time t = 0. Write the differential equation that describes the current I.



$$-L\frac{dI}{dt} - IR - IR = 0$$

$$L\frac{dI}{dt} + 2IR = 0$$

$$\boxed{\frac{dI}{dt} = \frac{-2R}{L} I}$$

5

2c. (5 pts) Solve the equation to give I(t) =

$$\frac{dI}{dt} = \frac{-2R}{L} I$$

$$\int_{I_0}^{I} \frac{dI}{I} = \int_0^t \frac{-2R}{L} dt$$

$$\ln\left(\frac{I}{I_0}\right) = \frac{-2R}{L} t$$

$$\frac{I}{I_0} = e^{\frac{-2R}{L} t}$$

$$\boxed{I(t) = I_0 e^{\frac{-2R}{L} t}}$$

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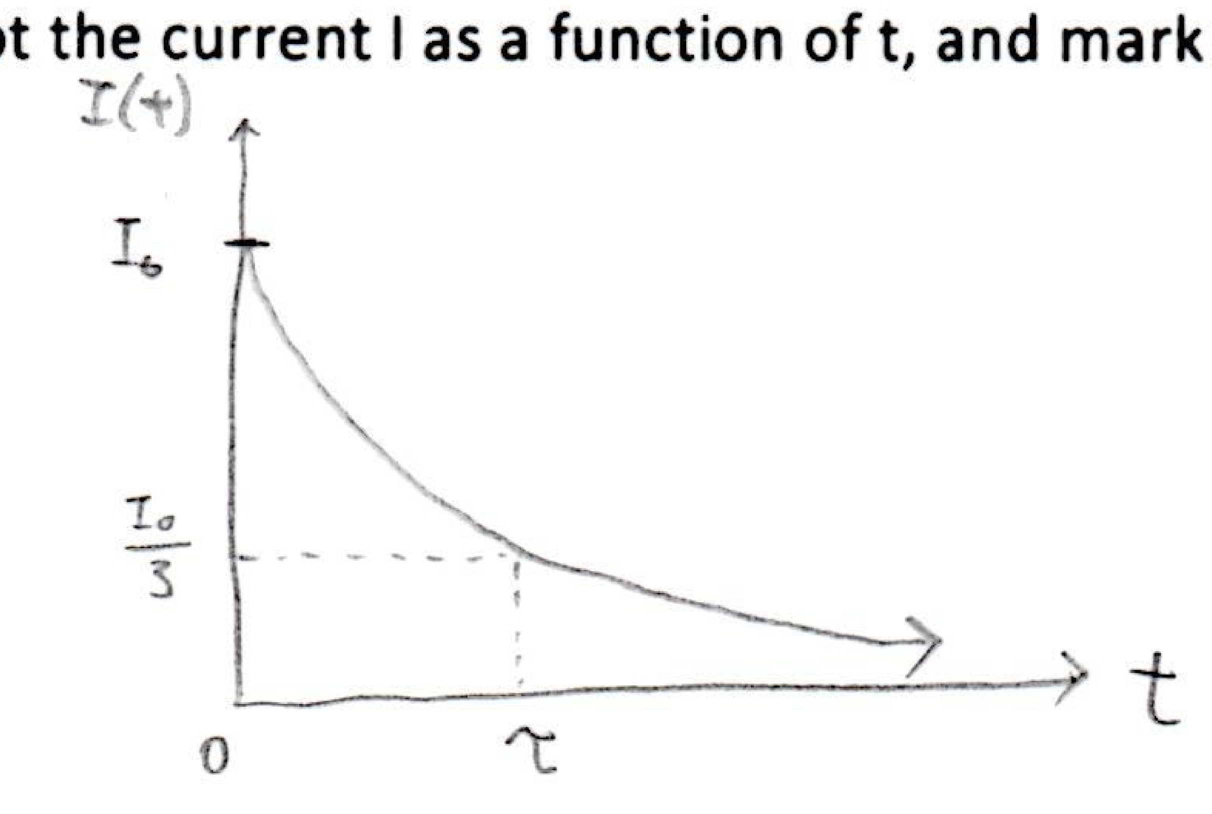
2d. (5 pts) Plot the current I as a function of t, and mark any time constants on the graph.



$$\tau\left(\frac{2R}{L}\right) = 1$$

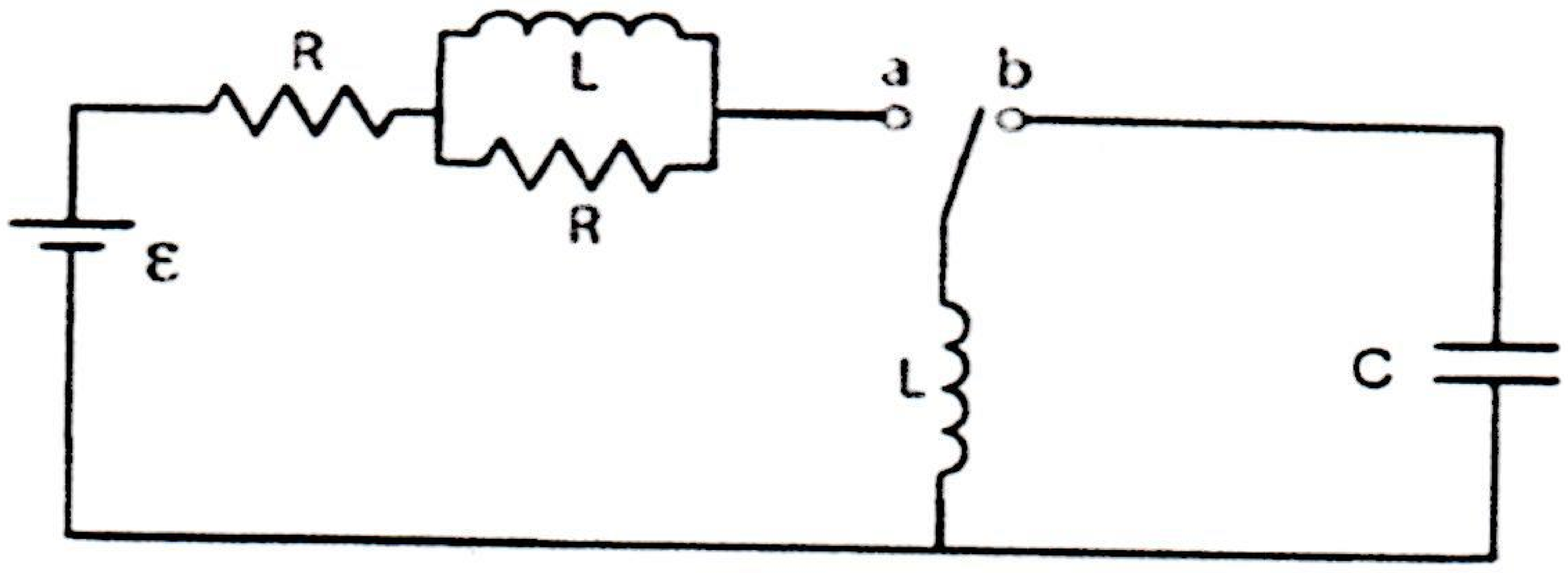
$$\tau = \frac{L}{2R}$$

At τ,

$$I = I_0 e^{-1}$$

$$I \approx \frac{I_0}{3}$$

Now, before we throw the switch to "b", we replace the resistors R with capacitor C as shown in the diagram below. Again, after a long time with the switch in position "a", we connect to "b" at t=0 so only the inductor and capacitor are in the circuit.



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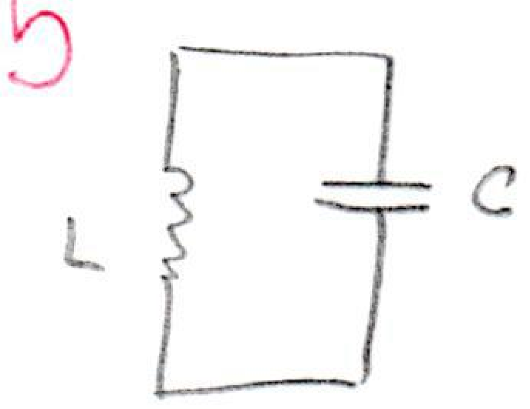
2e. (5 points) Write down the differential equation for the charge Q on the capacitor.



$$0 = -L\frac{dI}{dt} - \frac{q}{C}$$

$$0 = L\frac{d^2q}{dt^2} + \frac{q}{C}$$

$$\boxed{0 = \frac{d^2q}{dt^2} + \frac{q}{LC}}$$

4

2f. (5 points) Solve for Q(t)

Let's guess $q = Q_0 \cos(\omega t)$ $\qquad Q_0 = CV$

$$\frac{dq}{dt} = -Q_0\omega \sin(\omega t)$$

$$\frac{d^2q}{dt^2} = -Q_0\omega^2\cos(\omega t) = -\omega^2 q$$

$$0 = -\omega^2 q + \frac{q}{LC}$$

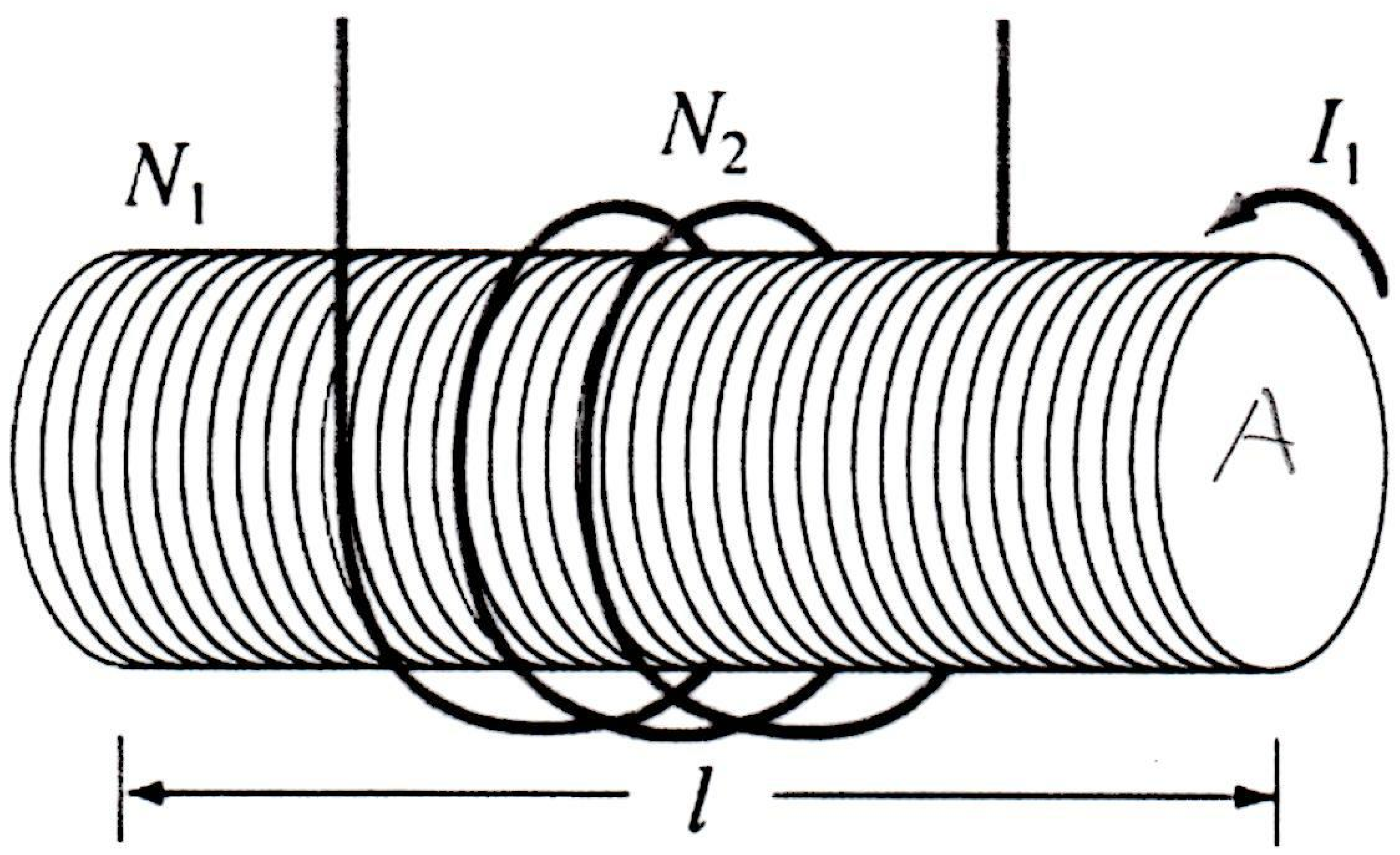
$$\omega^2 = \frac{1}{LC} \checkmark$$

$$Q(t) = Q_0 \cos(\omega t)$$

$$Q(t) = Q_0 \cos\left(\frac{t}{\sqrt{LC}}\right)$$

$$\boxed{Q(t) = C\varepsilon \overset{\sin}{\cancel{\cos}}\left(\frac{t}{\sqrt{LC}}\right)}$$

3a. (10 pts) Coil 1 is a long straight solenoid with N_1 windings and cross sectional area A.



$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$

$B_1 \, l = \mu_0 I_1 N_1$

$B_1 = \frac{\mu_0 I_1 N_1}{l}$

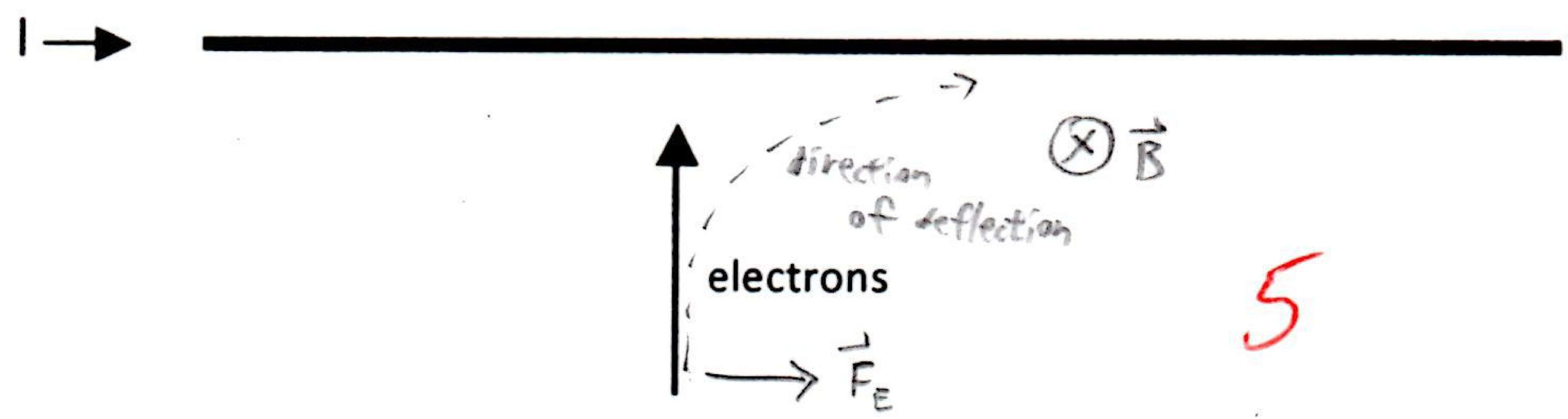
N_2 windings are wrapped along a short length of the solenoid. What is the mutual inductance M due to the current I_1 in the solenoid.

$M = \frac{N_2 \Phi_{B on 2}}{I_1}$ $\Phi_{B on 2} = B_1 A = \frac{\mu_0 I N_1}{l} A$

$M = \frac{\mu_0 I N_1 N_2 A}{I_1} = \boxed{\mu_0 N_1 N_2 A}$

8

3b. (5pt)



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An electron beam approaches a long wire carrying current I to the right as shown. Which way is the electron beam deflected? Draw the direction on the figure.

2.4
×1.6
240
144
3.84

3c. (5pt) An electron moves with velocity **v** = (4.0 **i** -6.0 **j**) x 10^4 m/s in a magnetic field **B** = (-0.80 **i** + 0.60 **j**) T.

What is the magnitude and direction of the force?

$\vec{F} = q \vec{v} \times \vec{B}$

$\vec{v} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -6 & 0 \\ -0.8 & 0.6 & 0 \end{vmatrix} \times 10^4 \, T \cdot m/s = (2.4 - 4.8) \times 10^4 \hat{k} \; T \cdot m/s$

$\vec{F} = (-1.6 \times 10^{-19} c)(-2.4 \times 10^4 \, T \cdot m/s)\hat{k} = \boxed{(3.84 \times 10^{-15} N)\hat{k}}$

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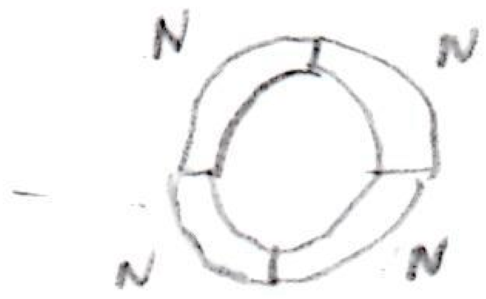
Analogy :



$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$

$B(2\pi r) = \mu_0 I (4N)$

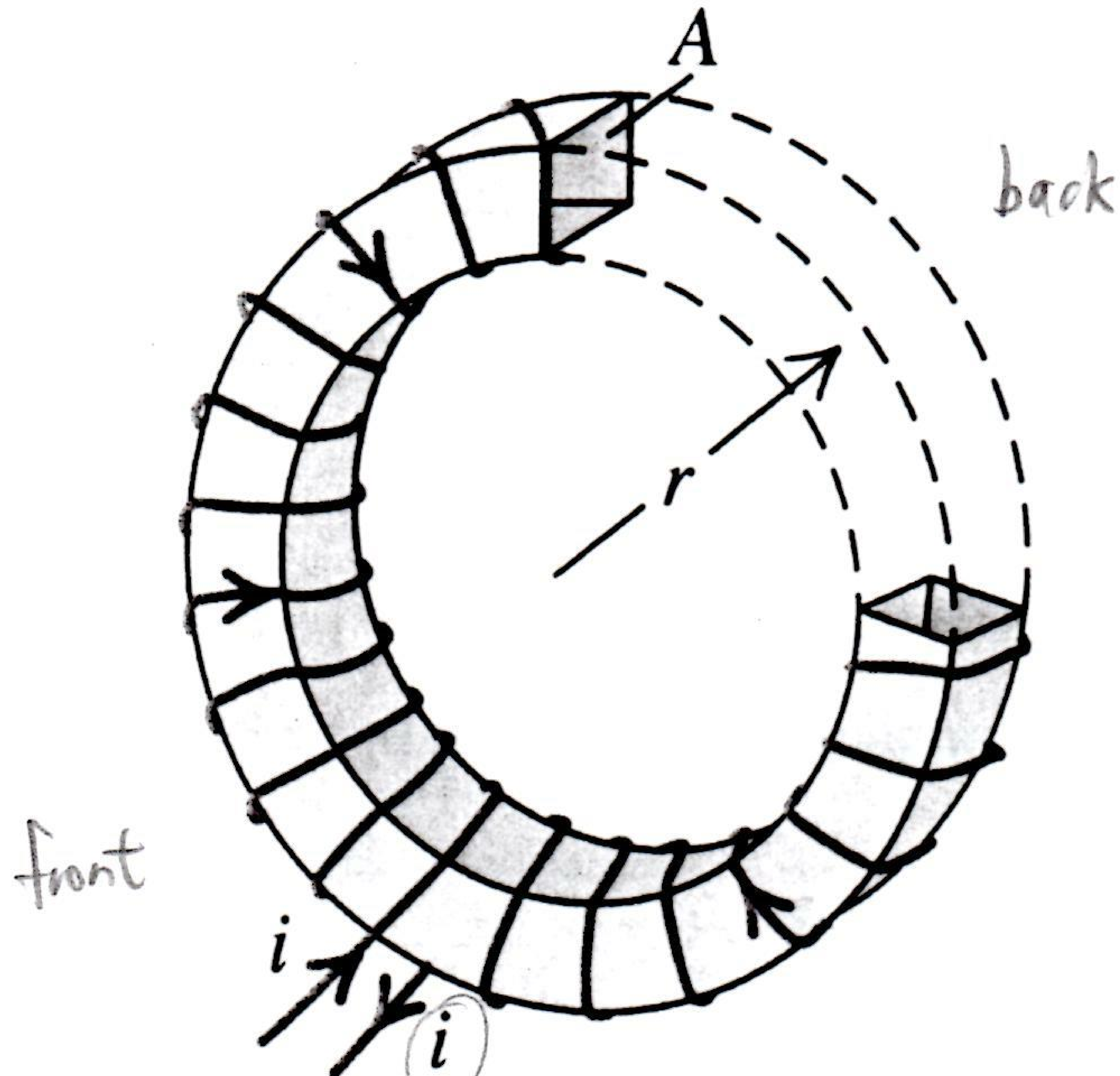
$B = 4\left(\frac{\mu_0 I N}{2\pi r}\right)$

Each: $B = \frac{\mu_0 I N}{2\pi r}$

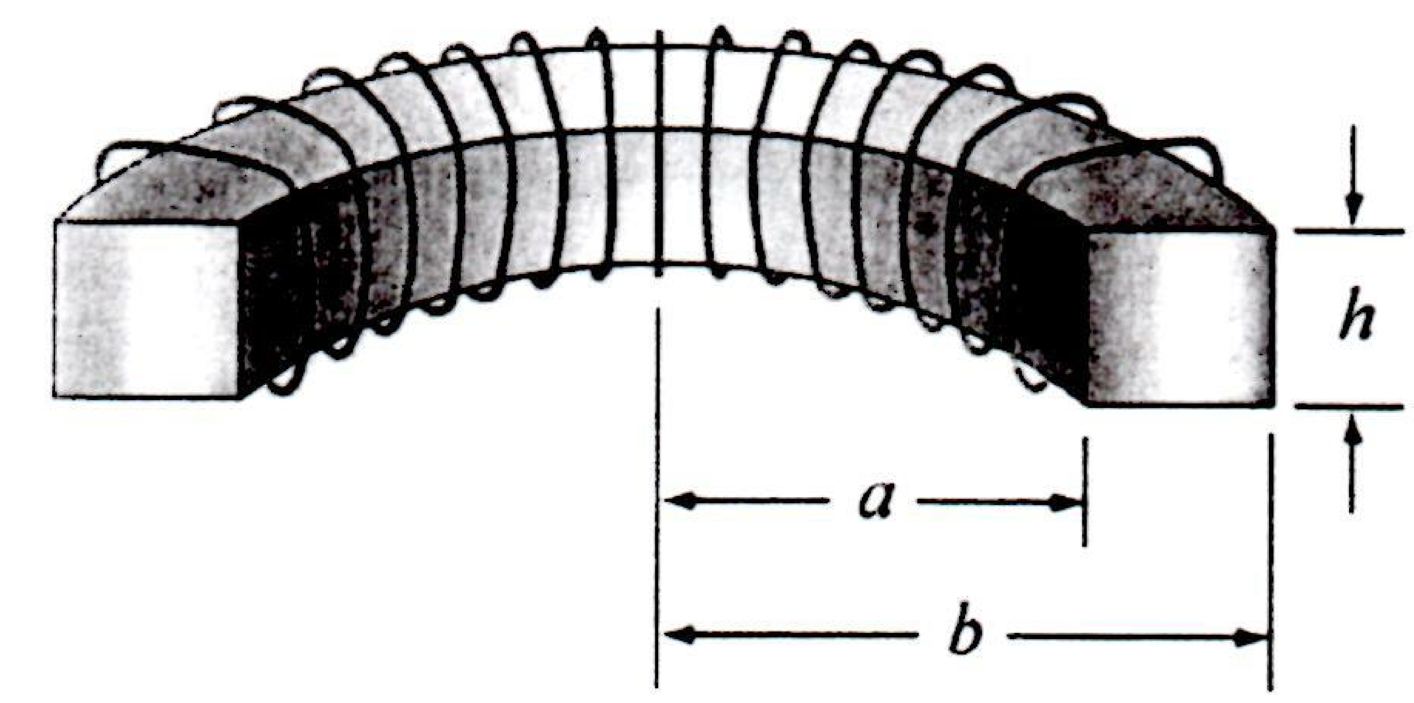
Here, $\frac{3}{4}$ toroidal solenoid $= N$,

$B = \frac{\mu_0 I N}{2\pi r}$

4. Consider the N toroidal windings shown below.



→ i is given



4a. (6 points) What is the magnetic field B(r) for a ≤ r ≤ b?

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$

$B(2\pi r) = \mu_0 i N$

$B = \frac{\mu_0 i N}{2\pi r}$

For $\frac{3}{4}$ of a toroidal solenoid, $\boxed{B = \text{same} = \frac{\mu_0 i N}{2\pi r} \ (T)}$

5

~~Not true~~

Can't use Amperes, but we want full solenoid

4b. (6 points) What is the magnetic flux through each winding of the toroid?

$d\Phi_B = B\, dA$

2

$= \int_a^b \frac{\mu_0 i N}{2\pi r}\, dr \ \underbrace{\left(\frac{1}{N}\right)}_{\text{each winding}} (A)$

$\Phi = \int_0^h dz \int_a^b dr\, (B)$

1

no A, just h

$$\boxed{\Phi_B = h \frac{\mu_0 i}{2\pi} \ln\left(\frac{b}{a}\right)}$$

4c. (6points) What is the self inductance L of the toroid?

$$L = \frac{N\Phi_B}{i} = \frac{N}{i}\left[\frac{\mu_0 i A}{2\pi}\ln\left(\frac{b}{a}\right)\right]$$

(3)

$$\boxed{L = \frac{\mu_0 N A}{2\pi}\ln\left(\frac{b}{a}\right)}$$

(3)

↳ wrong & wrong units
but right for (6)

4d. (6 points) What is the magnetic energy density u_B inside the toroid?

$$u_B = \frac{B^2}{2\mu_0} = \left[\frac{\mu_0 i N}{2\pi r}\right]^2 \frac{1}{2\mu_0}$$

(3)

$$= \frac{\mu_0^2 i^2 N^2}{8\pi r^2 \cdot 2\mu_0}$$

$$\boxed{u_B = \frac{\mu_0 i^2 N^2}{8\pi r^2}\ (J/m^3)}$$

(3)

4e. (6 points) Integrate the magnetic energy density over the volume to get the magnetic energy U_B stored in the toroid.

$$U_B = \int_0^h \int_0^{\frac{3\pi}{2}} \int_a^b u_B \, r\,dr\,d\phi\,dh$$

(2)

→ we wanted full but O.K.

(2)

Just know your B is incorrect for a 3/4

$$= h\left(\frac{3\pi}{2}\right)\int_a^b \frac{\mu_0 i^2 N^2}{8\pi r}\,dr$$

$$= h\left(\frac{3\pi}{2}\right)\frac{\mu_0 i^2 N^2}{8\pi}\ln\left(\frac{b}{a}\right)$$

$$= \boxed{\left[\frac{3\mu_0 i^2 N^2 h}{16}\ln\left(\frac{b}{a}\right)\right] J}$$

(2)