Name (clearly please!) Groucho Marx
UID:

MIDTERM #1
Version C
Physics 1C, Prof. David Saltzberg
November 7, 2019

Time: 90 minutes. Closed Notes. Closed Book. One 3"x5" sheet. Calculators (even graphing) are allowed. Show your work. *Please write your name on every page just in case.*

If a problem is confusing or ambiguous, notify the professor. Clarifications will be written on the blackboard. Check the board.

Problem	**Points**
1	15 /15
2	20 /20
3	20 /20
4	15 /15
5	15 /15
6	15 /15
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TOTAL	100 /100

1) Short answers:

a) (5 pts) In class we performed a demonstration where a student pedaled a bicycle connected to a generator to power light bulbs in parallel. The reason he had to pedal harder as more bulbs were added was:

1) due to increasing back-EMF (or "counter-EMF") of the generator
2) due to increasing back-torque (or "counter-torque") of the generator
3) due to the presence of the Maxwell term $+\mu_0\varepsilon_0(d\Phi_E/dt)$ in the Ampère-Maxwell Law
4) due to the presence of the conduction current term $\mu_0 I_C$ in the Ampère-Maxwell Law
5) because aluminum is diamagnetic and like-poles (e.g., N and N) repel.

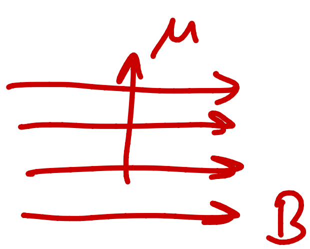
You apply torque, so generator provides counter-torque

b) (5 pts) A block of material is placed near a bar magnet and is weakly attracted to it. When the bar magnet is removed, the block of material is found not to be magnetized. This is an example of:

1) diamagnetism
2) ferromagnetism
3) paramagnetism
4) animal magnetism
5) a magnetic monopole

c) (5 pts) A magnetic dipole **μ** is placed in a uniform magnetic field, **B**. Draw a configuration where the magnitude of the torque on the dipole is maximal.

$\vec{\tau} = \vec{\mu} \times \vec{B}$ so $\vec{\mu}$ perpendicular to $\vec{B}$ is maximal



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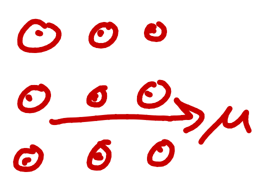


or

etc.

2) A sinusoidal electromagnetic wave with wavelength 2.0 m travels in vacuum in the +x direction with its electric field having an amplitude of 300 V/m that points along the y-axis. At *x=0, t=0* the electric field is zero and decreasing in value (i.e., becoming more negative).

a) (10 pts) Write down the equation for the magnetic field in terms of its amplitude, wavenumber *k*, and angular frequency ω. [Hint: your answer should be a vector. It should have numerical values with units for *k*, ω, and the field amplitude.]

$\hat{i} = -\hat{j} \times (?)$ must be $-\hat{k}$

$t=0, x=0 \Rightarrow$ sine

minus sign means propagating in positive x direction

$$\vec{B} = -\hat{k} B_0 \sin(kx - \omega t)$$

assures $|\vec{B}|$ decreasing at $t=0$, $x=0$

$$B_0 = E_0/v = \frac{300\,V/m}{3\times10^8\,m/s} = 100\times10^{-8} = 10^{-6}\,T$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{2m} = \pi = 3.14\,m^{-1}$$

$$c = \lambda\nu = \frac{\omega}{k} \Rightarrow \omega = ck = (3\times10^8)(3.14) = 9.42\times10^8\,rad/s$$

$$\vec{B} = -\hat{k}\left(10^{-6}\,T\right)\sin\left(\left(3.14\,m^{-1}\right)x - \left(9.42\times10^8\,s^{-1}\right)t\right)$$

$1\mu T$

b) (10 pts) Suppose this wave is perfectly absorbed uniformly over an entire square-shaped sail with dimensions 2.0m × 2.0m. What is the force on the sail in Newtons? [Hint: this one may need your calculator.]

$$\text{Pressure} = \frac{I}{c} \quad \text{(absorbtion)}$$

$$I = \langle S_{AV} \rangle = \frac{1}{2\mu_0}|E||B| = \frac{1}{\mu_0 c}|E|^2$$

$$\text{Pressure} = \frac{|E|^2}{2\mu_0 c^2}$$

$$\text{Force} = \text{Pressure} \times \text{Area}$$

$$= \frac{|E|^2 A}{2\mu_0 c^2} \quad \left(\text{or } \epsilon_0 |E|^2 A \text{ is equivalent}\right)$$

$$= \frac{(\cancel{3}00)^2(\cancel{2}\times\cancel{2})}{\cancel{2}(\cancel{4}\pi\times10^{-7})(\cancel{3}\times10^8)^2}$$

$$= \frac{10^4}{(2\pi\times10^{-7})(10^{16})}$$

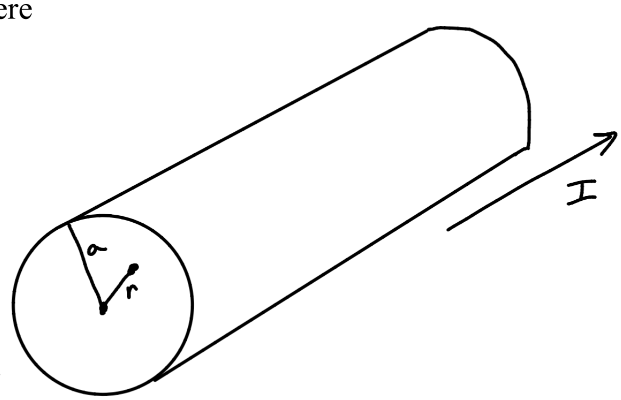
$$= \frac{1}{2\pi}\times10^{-5}$$

$$= 0.16\times10^{-6}\ N$$

$$= \boxed{1.6\times10^{-6}\ N}$$

3) A long cylindrical wire of radius, a, carries a current density $J(r)$ along its axis that is proportional to the cube of the radius, r, that is:

$$J(r) = J_0({}^{r}/_{a})^3\,, r < a,$$
$$J(r) = 0, \text{elsewhere}$$



a) (10 pts) Suppose a=5.0 m and the total current carried in the wire is I_0=3.14 A. What is J_0? [Answer is a numerical value with units]

$$I_0 = \int_S J(r)\, \underbrace{dA}_{r\,dr\,d\theta}$$

$$= \int_0^{2\pi} d\theta \int_0^a J_0 \left(\frac{r}{a}\right)^3 r\,dr$$

$$= 2\pi J_0 \int \frac{r^4}{a^3}\,dr$$

$$= 2\pi J_0 \frac{1}{5} \frac{r^5}{a^3}\Bigg]_0^a$$

$$\cancel{3.14} = \frac{2\cancel{\pi} J_0}{5} a^2$$

$$J_0 = \frac{5}{2a^2} = \frac{5}{2(5^2)} = \frac{1}{(2)(5)} = \boxed{0.1\ A/m^2}$$

b) (10 pts) Suppose the current is changed so that $J_0=10\ A/m^2$. [Hint: this part does not depend on your answer for part a]. What is the value B at r=2.0m? [This is a numerical value with units.]

$$\oint B\cdot dl = \mu_0 \int \vec{J}\cdot d\vec{A}$$

$$(B)(2\pi r) = \mu_0 \int_0^{2\pi} d\theta \int_0^r J_0 \left(\frac{r'}{a}\right)^3 r' dr'$$

$$(B)(\cancel{2\pi} r) = \mu_0 \cancel{2\pi} \frac{J_0}{5} \frac{r'^5}{a^3}\Big|_0^r$$

$$B = \mu_0 \frac{J_0}{5} \frac{r^4}{5^3}$$

$$= (4\pi \times 10^{-7})\left(\frac{10}{5}\right)\left(\frac{2^4}{125}\right)$$

$$= \frac{(4\pi)(32)}{125} \times 10^{-7}$$

$$= \frac{128\pi}{125} \times 10^{-7}$$

$$\simeq \boxed{3.2 \times 10^{-7}\ T}$$

4) An AC wall socket in Europe provides a voltage that is sinusoidal in time with a frequency of 50 Hz and root-mean-square (R.M.S.) voltage of 240 V between its two conductors.

a) (5 pts) What is the amplitude of the voltage?

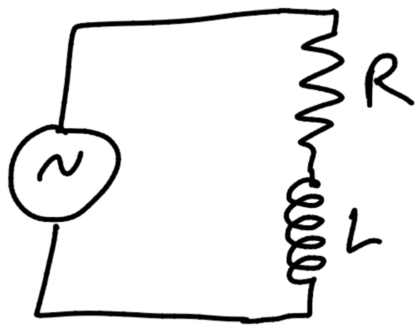
V_0 = amplitude

$V_{Rms} = V_0/\sqrt{2} \Rightarrow V_0 = \sqrt{2}\, V_{rms} = 1.4 \times 240\ V$

$= 336\ V$

(exact = 339.4)

b) (10 pts) Now you connect an inductor and resistor in series to this European AC wall socket, as shown below:



Where R=62.8 Ω and L= 100 mH. The current is at its maximum positive value at *t=0*. What is the RMS voltage across the inductor? [Hint: you may need your calculator a little.]

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$V_{source}^{Rms} = I_{Rms} Z$

$V_L^{Rms} = I_{Rms} X_L$

$V_L^{Rms} = V_{source}^{Rms} \dfrac{X_L}{Z}$

$= (240) \dfrac{X_L}{Z}$

(next page)

[extra space]

$$\omega = 2\pi f = (6.28)(50\ Hz) = 100\pi$$
$$= 314\ rad/s$$

$$X_L = \omega L = (314)(0.1) = 31.4\ \Omega$$
$$R = 62.8\ \Omega$$
$$Z = \sqrt{62.8^2 + 31.4^2} = 31.4\sqrt{2^2+1}$$
$$= 31.4\sqrt{5}$$

$$V_L^{Rms} = (240)\left(\frac{\cancel{31.4}}{\cancel{31.4}\sqrt{5}}\right)$$

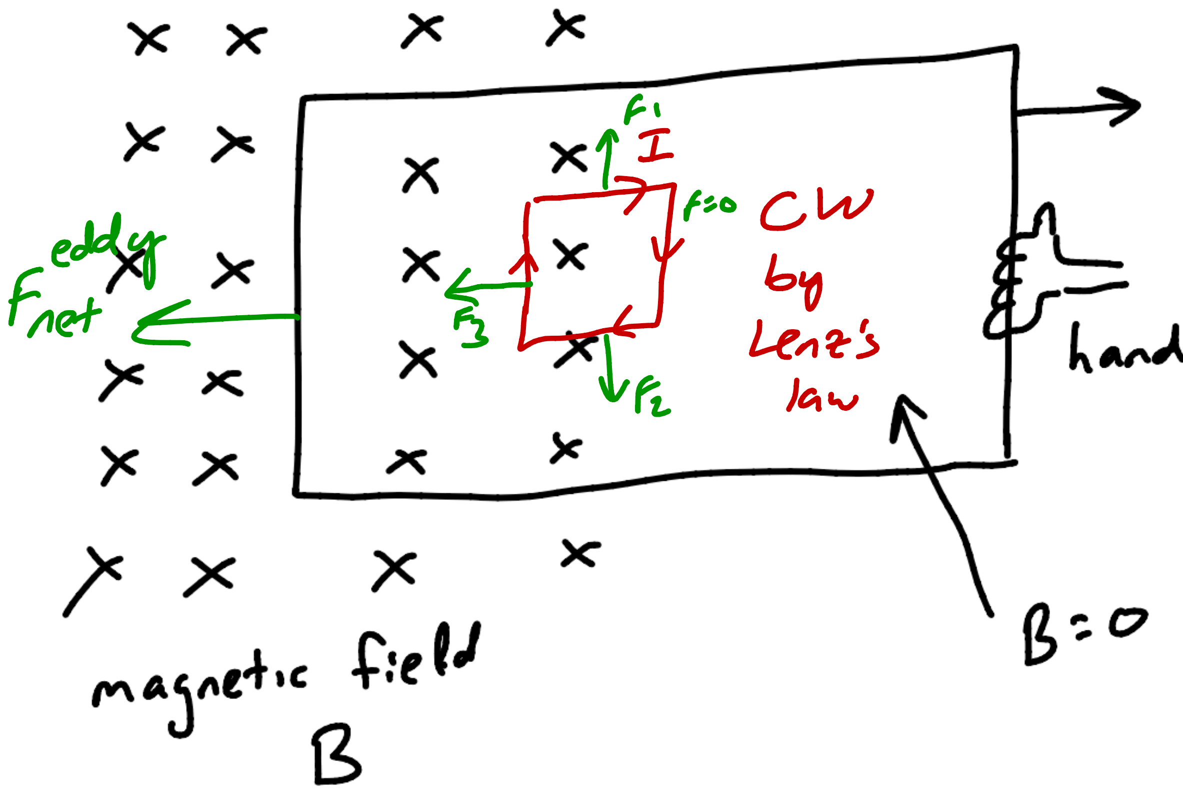
$$V_L^{Rms} = \frac{240}{\sqrt{5}} = \frac{240}{2.24}$$

$$= \boxed{107\ V}$$

5) You are pulling a rectangular plate of copper out of a region with uniform magnetic field B with a constant speed v.

a) (10 pts) On the figure below draw one of the eddy current loops showing the direction of current flow.

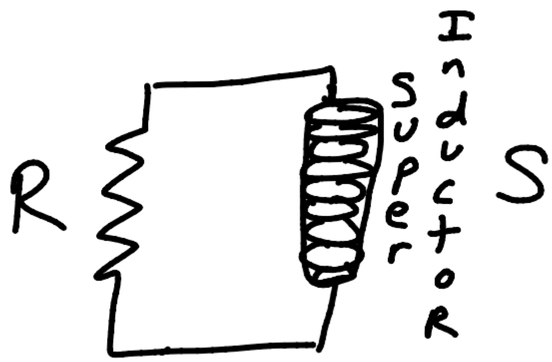
b) (5 pts) <u>Also</u> indicate the direction of the net force on the copper due to Eddy currents



$\vec{F}_1$ cancels $\vec{F}_2$

$\vec{F}_{net}$ must be in direction of $\vec{F}_3$ (opposes $\vec{v}$)

6) (15 pts) Scientists at a top-secret government laboratory recently invented a *super-inductor*. The voltage drop across a super-inductor is given by $V(t) = S\frac{d^3}{dt^3}I(t)$, where S is a constant known as the *super-inductance*, and $I(t)$ and $V(t)$ are the current and voltage. In the circuit below, a super-inductor with super-inductance $S=800{,}000$ (in the appropriate S.I. units) is in series with a resistor with resistance $R=100\ \Omega$ and the initial current at time $t=0$ is I_0.



At what time does the current become $(1/e)I_0$?

$$-IR - S\frac{d^3I}{dt^3} = 0$$ loop rule

$$I(t) = -\frac{S}{R}\frac{d^3I}{dt^3}$$

Try $I(t) = I_0 e^{-t/\tau}$ ← given on board

$$\cancel{I_0}\cancel{e^{-t/\tau}} = \left(-\frac{1}{\tau}\right)^3 \frac{S}{R}\cancel{I_0}\cancel{e^{-t/\tau}}$$

$$\tau^3 = S/R \qquad \tau = \sqrt[3]{S/R}$$

Drops by 1/e factor in time $\tau = \sqrt[3]{S/R}$

$$\sqrt[3]{\frac{S}{R}} = \sqrt[3]{\frac{8000\cancel{00}}{\cancel{100}}} = \sqrt[3]{8}\sqrt[3]{1000} = \boxed{20s}$$