20S-PHYSICS1CH-1 Midterm Exam

[redacted]

TOTAL POINTS

136 / 150

QUESTION 1

1 Problem 1 32 / 35

- **- 1.5 pts** mistake in phase velocity (a)
- **- 1.5 pts** mistake in spatial dependence (a)
- **- 1.5 pts** mistake in group velocity (b)
- **- 3 pts** mistake in linear combination (c)
- ✓ **- 1.5 pts mistake in propagation velocity (c)**
- ✓ **- 1.5 pts mistake in comparison (c)**
- **- 5 pts** no phase velocity (a)
- **- 5 pts** no spatial dependence (a)
- **- 5 pts** no group velocity (b)
- **- 0 pts** no mistakes
- **- 10 pts** no linear combination (c)
- **- 5 pts** no propagation velocity (c)
- **- 5 pts** no comparison (c)

QUESTION 2

2 Problem 2 40 / 45

- **- 0 pts** Correct
- **- 3 pts** mistake in (a)
- **- 6 pts** 2 independent mistakes in (a)
- **- 9 pts** 3 independent mistakes in (a)
- **- 12 pts** 4 independent mistakes in (A)
- **- 20 pts** no (a)
- ✓ **- 3 pts 1 mistake in (b)**
- **- 6 pts** 2 mistakes in (b)
- **- 9 pts** 3 mistakes in (b)
- **- 15 pts** no (b)
- ✓ **- 2 pts 1 mistake in (c)**
- **- 4 pts** 2 mistakes in (c)
- **- 6 pts** 3 mistakes in (C)
- **- 10 pts** no (c)

QUESTION 3

3 Problem 3 39 / 45

- **- 0 pts** Correct
- ✓ **- 1.5 pts mistake in (a)**
- **- 5 pts** no (a)
- **- 1.5 pts** mistake in (b)
- **- 3 pts** 2 mistakes in (b)
- **- 10 pts** no (b)
- ✓ **- 1.5 pts mistake in (c)**
- **- 3 pts** 2 mistakes in (c)
- **- 10 pts** no (c)
- ✓ **- 3 pts mistake in (d)**
- **- 6 pts** 2 mistakes in (d)
- **- 9 pts** 3 mistakes in (d)
- **- 15 pts** no (d)
- **- 3 pts** 2 mistakes in (a)
- **- 6 pts** 3 mistakes in (c)

QUESTION 4

4 Problem 4 20 / 20

- ✓ **- 0 pts Correct**
- **- 2 pts** wrong polarization (a)
- **- 5 pts** no (a)
- **- 2 pts** wrong angle
- **- 5 pts** no (b)
- **- 4 pts** wrong shape
- **- 10 pts** no (c)

QUESTION 5

5 Notes submission 5 / 5

- ✓ **- 0 pts Correct**

1. (a) $v_p = \frac{w}{k}$, $w = \sqrt{k^2c^2 + w_c^2}$ $v_p = \frac{\sqrt{|\vec{k}|^2 c^2 + w_p^2}}{k} = \boxed{\sqrt{c^2 + \left(\frac{w_p}{|\vec{k}|}\right)^2}}$

If $w^2 < w_c^2$ the phase of the electric field $\boxed{\text{exponentially decays}}$.

(b) $v_g = \frac{\partial w}{\partial k} = \frac{\partial}{\partial k}\sqrt{k^2c^2 + w_c^2} = \frac{|\vec{k}|c^2}{\sqrt{k^2c^2 + w_c^2}} = \boxed{\frac{c}{\sqrt{1 + \left(\frac{w_c}{kc}\right)^2}}}$

(c) Wave 1: $E_0 \cos(\vec{k}\cdot\vec{x} - 4w_c t)$

Wave 2: $E_0 \cos(\vec{k}\cdot\vec{x} - 5w_c t)$

Superposition: $E_0\left(\cos(\vec{k}\cdot\vec{x} - 4w_c t) + \cos(\vec{k}\cdot\vec{x} - 5w_c t)\right)$

Using $\cos(A) + \cos(B) = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$:

$$\Rightarrow E_0\left[2\cos\left(\frac{\vec{k}\cdot\vec{x} - 4w_c t + \vec{k}\cdot\vec{x} - 5w_c t}{2}\right)\cos\left(\frac{\vec{k}\cdot\vec{x} - 4w_c t - \vec{k}\cdot\vec{x} + 5w_c t}{2}\right)\right]$$

$$= 2E_0\left[\cos\left(\frac{2\vec{k}\cdot\vec{x} - 9w_c t}{2}\right)\cos\left(\frac{w_c t}{2}\right)\right]$$

$$= \boxed{2E_0\left[\cos\left(\vec{k}\cdot\vec{x} - 4.5w_c t\right)\cos(0.5w_c t)\right]}$$

Velocity of propagation of the envelope : $\cos(0.5w_c t)$ is as follows

velocity of propagation At $0.5w_c = w$,

At the average of the two frequencies at $w = 4.5w_c$, $w^2 = 20.25w_c^2 = k^2c^2 + w_c^2$

$19.25w_c^2 = k^2c^2$ $w_c = 0.23kc$ group velocity is $\frac{\partial w_c}{\partial k} = \boxed{0.23c}$

2. (a) $|\vec{E}| = v|\vec{B}|$, where $v^2 = \frac{1}{\epsilon\mu}$ thus $\boxed{\frac{|\vec{E}|}{|\vec{B}|} = v \text{ where } v = \frac{1}{\sqrt{\epsilon_1\mu_0}}}$

$$\boxed{\text{Poynting Vector: } \vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}}$$

Energy Density of a wave is $\frac{1}{2}\left(\epsilon_1|\vec{E}|^2 + \frac{1}{\mu_0}|\vec{B}|^2\right)$

Relationship between v, energy Density and Poynting Vector

$$\text{Energy Density} = \frac{S}{v}$$

(b) Since the wave impinges normally to the surface thus $\theta_i = \theta_t = 0^\circ$

we have $r_\perp \equiv \left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{n_i - n_t}{n_i + n_t}$, $t_\perp \equiv \left(\frac{E_{0t}}{E_{0i}}\right)_\perp = \frac{2n_i}{n_i + n_t}$

$n = \frac{c}{v}$ and $v_1 = \frac{1}{\sqrt{\epsilon_1\mu_0}}$, $v_2 = \frac{1}{\sqrt{\epsilon_2\mu_0}}$ $n_i = \frac{c}{\sqrt{\epsilon_1\mu_0}}$ $n_t = \frac{c}{\sqrt{\epsilon_2\mu_0}}$

$$\boxed{r_\perp = \frac{\frac{c}{\sqrt{\epsilon_1\mu_0}} - \frac{c}{\sqrt{\epsilon_2\mu_0}}}{\frac{c}{\sqrt{\epsilon_1\mu_0}} + \frac{c}{\sqrt{\epsilon_2\mu_0}}} \qquad t_\perp = \frac{\frac{2c}{\sqrt{\epsilon_1\mu_0}}}{\frac{c}{\sqrt{\epsilon_1\mu_0}} + \frac{c}{\sqrt{\epsilon_2\mu_0}}}}$$

(c) Reflected Poynting : $\vec{S}_{ref} = \frac{1}{\mu_0}\vec{E}_R\times\vec{B}_R$

Transmitted Poynting : $\vec{S}_t = \frac{1}{\mu_0}\vec{E}_t\times\vec{B}_t$

where they relations is given by: $\vec{E}_r = \vec{E}_{inc}\cdot r_\perp$ from part (b)

and $\vec{E}_t = \vec{E}_{inc}\cdot t_\perp$ from part (b)

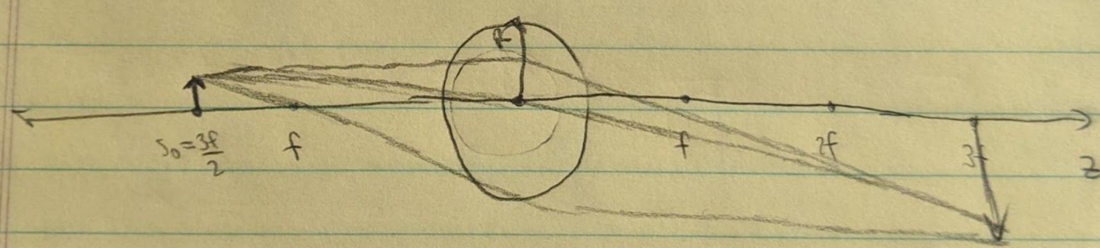
3. a. For radius r focal length for thin lens of paraxial rays is simply r/2. This in this instance $\boxed{f = R/2}$

b. The image is at s_1 where $\frac{1}{f} = \frac{1}{s_0} + \frac{1}{s_1}$

with $s_0 = \frac{3f}{2}$ we have $\frac{1}{f} = \frac{1}{3f/2} + \frac{1}{s_1}$

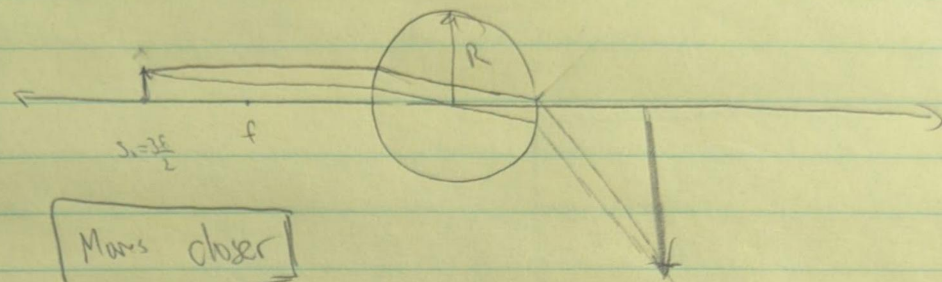
$$\frac{(3f/2)s_1}{f} = s_1 + \frac{3f}{2}, \quad \frac{3fs_1}{2f} = s_1 + \frac{3f}{2} \Rightarrow \frac{3}{2}s_1 = s_1 + \frac{3f}{2}$$

$\Rightarrow 3s_1 = 2s_1 + 3f \Rightarrow s_1 = 3f$. The image is at a distance 3f or $\frac{3R}{2}$ to the right of the dielectric sphere.



The image is $\boxed{\text{real}}$ since it converges.

c.



$\boxed{\text{Moves closer}}$

d. Order of what happens 1. Ray gets translated to sphere by distance s_0 2. Refraction of outer surface of sphere. 3. Translation within sphere. 4. Refraction of inner surface of sphere. 5. Translation of ray by distance s_1

$$\hat{T}_1 = \begin{pmatrix} 1 & 3f/2 \\ 0 & 1 \end{pmatrix}, \quad L_1 = \begin{pmatrix} 1 & 0 \\ -2/R & 1 \end{pmatrix} \quad \hat{T}_2 = \begin{pmatrix} 1 & 2R \\ 0 & 1 \end{pmatrix} \quad \hat{L}_2 = \begin{pmatrix} 1 & 0 \\ 2/R & 1 \end{pmatrix} \quad \hat{T}_3 = \begin{pmatrix} 1 & 3f \\ 0 & 1 \end{pmatrix}$$

Thus we have $\begin{pmatrix} 1 & 3f \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2/R & 1 \end{pmatrix}\begin{pmatrix} 1 & 2R \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -2/R & 1 \end{pmatrix}\begin{pmatrix} 1 & 3f/2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} y_0 \\ 0 \end{pmatrix}$

Simplifying: $\begin{pmatrix} 1+6f/R & 3f \\ 2/R & 1 \end{pmatrix}\begin{pmatrix} 1 & 2R \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -2/R & 1 \end{pmatrix}\begin{pmatrix} 1 & 3f/2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} y_0 \\ 0 \end{pmatrix}$

3d: Cont

$$\begin{pmatrix} 1+6f/R & 2R+15f \\ 2/R & 5 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -2/R & 1 \end{pmatrix}\begin{pmatrix} 1 & 3f/2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} y_0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1+\frac{6f}{R}-4-\frac{30f}{R} & 2R+15f \\ -\frac{8}{R} & 5 \end{pmatrix}\begin{pmatrix} 1 & 3f/2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} y_0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} -3-\frac{24f}{R} & 2R+15f \\ -8/R & 5 \end{pmatrix}\begin{pmatrix} 1 & 3f/2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} y_0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} -3-\frac{24f}{R} & -\frac{9f}{2}-\frac{36f^2}{R}+2R+15f \\ -\frac{8}{R} & 5-\frac{12f}{R} \end{pmatrix}\begin{pmatrix} y_0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} -3-\frac{24f}{R} & \frac{21f}{2}+2R-\frac{36f^2}{R} \\ -\frac{8}{R} & 5-\frac{12f}{R} \end{pmatrix}\begin{pmatrix} y_0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} \left(-3-\frac{24f}{R}\right)y_0 \\ -\frac{8y_0}{R} \end{pmatrix} \qquad f = R/2$$

Position: $\begin{pmatrix} (-3-12)\,y_0 \\ \frac{-8y_0}{R} \end{pmatrix} = \boxed{\begin{pmatrix} -15y_0 \\ \frac{-8y_0}{R} \end{pmatrix}}$

4. a. We should choose $\boxed{\text{p-polarization}}$

b. Angle $= \arctan\left(\frac{n_2}{n_1}\right) = \arctan\left(\frac{1.5}{1}\right) = \arctan(1.5) \approx \boxed{56.3^\circ}$

c. The shape of the laser in the glass will be $\boxed{\text{elliptical}}$.

Generic Maxwell's Equations:

Gauss Law: $\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$

Solenoidal Law: $\nabla \cdot \vec{B} = 0$

Faraday's Law: $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

Ampere-Maxwell: $\nabla \times \vec{B} = \mu_0 \vec{J} + \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$

In Matter replace ϵ_0 and μ_0 with ϵ and μ, where

$\epsilon = \epsilon_0(1+\chi_e)$ which relates $\vec{D} = \epsilon \vec{E}$

$\mu = \mu_0(1+\chi_m)$ which relates $\vec{H} = \frac{1}{\mu}\vec{B}$

Also replace $\vec{J}$ with $\vec{J}_f$

Speed of Light in matter $v^2 = \frac{1}{\epsilon \mu}$

Index of Refraction: $n = \frac{c}{v_p}$

$$n = \sqrt{\frac{\epsilon \mu}{\epsilon_0 \mu_0}} = \sqrt{(1+\chi_e)(1+\chi_m)}$$

Fourier Space Maxwell Eqns:

$\vec{E} \to \vec{\tilde{E}}, \vec{B} \to \vec{\tilde{B}}, \nabla \to i\vec{k}, \frac{\partial}{\partial t} = -i\omega$

Maxwell Eqns in Fourier:

$\vec{k} \cdot \vec{\tilde{E}} = 0$

$\vec{k} \cdot \vec{\tilde{B}} = 0$

$\vec{k} \times \vec{\tilde{E}} = \omega \vec{\tilde{B}}$

$\vec{k} \times \vec{\tilde{B}} = -\epsilon \mu \omega \vec{\tilde{E}}$

Dispersion Relation: $\omega^2(\vec{k}) = \frac{|\vec{k}|^2}{\epsilon \mu}$

and get $v_g = \frac{\partial \omega}{\partial k} = \pm \frac{\partial}{\partial \vec{k}} v|\vec{k}| = \pm v \frac{\vec{k}}{|\vec{k}|}$

also find $|\vec{\tilde{E}}| = v|\vec{B}|$ where v = speed of light

Maxwell in Ohmic Matter:

$\nabla \cdot \vec{E} = 0$ since $\vec{J} = \sigma \vec{E}$

$\nabla \cdot \vec{B} = 0$

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\nabla \times \vec{B} = \mu \sigma \vec{E} + \epsilon \mu \frac{\partial \vec{E}}{\partial t}$

Fourier Transform

$\vec{k} \cdot \vec{\tilde{E}} = 0$

$\vec{k} \cdot \vec{B} = 0$

$\vec{k} \times \vec{\tilde{E}} = \omega \vec{\tilde{B}}$

$\vec{k} \times \vec{\tilde{B}} = -i\mu\sigma \vec{\tilde{E}} - \epsilon \mu \omega \vec{\tilde{E}}$

Dispersion Relation: $\omega = \pm \sqrt{\frac{|\vec{k}|^2}{\epsilon \mu} - \frac{\sigma^2}{4\epsilon^2}} - \frac{i\sigma}{2\epsilon}$

Real Part

$\omega_R = \pm \sqrt{\frac{|\vec{k}|^2}{\epsilon \mu} - \frac{\sigma^2}{4\epsilon^2}}$ $\quad \frac{|\vec{k}|^2}{\epsilon \mu} - \frac{\sigma^2}{4\epsilon^2} > 0$

$\to |\vec{k}| > \sqrt{\frac{\mu}{\epsilon}} \frac{\sigma}{2}$ $\quad |\vec{k}| = \frac{2\pi}{\lambda}$ $\quad \frac{2\pi}{\lambda} > \sqrt{\frac{\mu}{\epsilon}} \frac{\sigma}{2}$

$\to \lambda < \sqrt{\frac{\epsilon}{\mu}} \frac{4\pi}{\sigma}$

In plasma:

$\nabla \cdot \vec{E} = 0$

$\nabla \cdot \vec{B} = 0$

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\nabla \times \vec{B} = \mu_0 \vec{J} + \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$

Plasma Constitutive Relation:

$$\frac{\partial \vec{J}}{\partial t} = \frac{q^2 n_0}{m} \vec{E}$$

Plasma Dispersion Relation:

$\omega = \sqrt{c^2 k^2 + \omega_p^2}$ $\quad \omega_p^2 = \frac{q^2 n_0}{\epsilon_0 m}$

or $\omega^2 = c^2 |\vec{k}|^2 + \omega_p^2$

$v_g = \frac{\partial \omega}{\partial \vec{k}} = \frac{c^2 \vec{k}}{\omega} = \frac{c}{\sqrt{1 + \left(\frac{\omega_p}{ck}\right)^2}}$

$v_p = \frac{\omega}{k} = \frac{\sqrt{c^2|\vec{k}|^2 + \omega_p^2}}{k}$

Wave Eqn:

$$\frac{1}{v^2} \frac{\partial^2 \phi}{\partial t^2} - \nabla^2 \phi = 0$$

In one-d: $\phi(t,x) = f(x+vt) + g(x-vt)$

BAC-CAB:

$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$

Fresnel Coefficients:

$$r_{\parallel} = \frac{n_t \cos(\theta_i) - n_i \cos(\theta_t)}{n_i \cos(\theta_t) + n_t \cos(\theta_i)} \qquad r_{\perp} \equiv \left(\frac{E_{0r}}{E_{0i}}\right)_{\perp} = \frac{n_i \cos(\theta_i) - n_t \cos(\theta_t)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)}$$

$$t_{\parallel} = \frac{2 n_i \cos(\theta_i)}{n_i \cos(\theta_t) + n_t \cos(\theta_i)} \qquad t_{\perp} \equiv \left(\frac{E_{0t}}{E_{0i}}\right)_{\perp} = \frac{2 n_i \cos(\theta_i)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)}$$

$\vec{E}_i = \vec{\tilde{E}}_{inc} e^{i(\vec{k}_i \cdot \vec{x} - \omega t)} + \vec{\tilde{E}}_r e^{i(\vec{k}_r \cdot \vec{x} - \omega t)}$

$\vec{E}_t = \vec{\tilde{E}}_t e^{i(\vec{k}_t \cdot \vec{x} - \omega t)}$

Power

$P_r = R_{\parallel} P_{\parallel} + R_{\perp} P_{\perp}$

$P_t = T_{\parallel} P_{\parallel} + T_{\perp} P_{\perp}$

$R_{\parallel} + T_{\parallel} = 1$

$R_{\perp} + T_{\perp} = 1$

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Snell: $n_i \sin\theta_i = n_t \theta_t$

Paraxial: $\sin\theta = \tan\theta = \theta$ $\quad \vec{r} = \begin{pmatrix} y_0 \\ y_0' \end{pmatrix}$

$\vec{r}' = \begin{pmatrix} y_0 \\ y_0' \frac{n_i}{n_t} \end{pmatrix}$ Action of crossing interface:

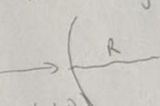
is $\begin{pmatrix} 1 & 0 \\ 0 & \frac{n_i}{n_t} \end{pmatrix} \begin{pmatrix} y(0) \\ y'(0) \end{pmatrix}$

$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f_1} + \frac{1}{f_2}$ $\quad f_1 = \frac{R_1}{n-1}$ $\quad f_2 = \frac{-R_2}{n-1}$

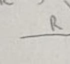
Translation $= \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$

Lens $= \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$ $\quad$ Mirror $= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Mirror with radius of curvature R is actually a lens with focal length $f = R/2$



$M = \begin{pmatrix} 1 & 0 \\ -\frac{2}{R} & -1 \end{pmatrix}$

if flipped  $M = \begin{pmatrix} 1 & 0 \\ \frac{2}{R} & -1 \end{pmatrix}$

Reflecting mirror with curvature is Reflecting the Lensing

$\hat{Ref}\,\hat{L} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2/R & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2/R & -1 \end{pmatrix}$

$v_p = \frac{\omega}{k} = \frac{c}{n} \Rightarrow \omega = \frac{kc}{n}$

$\omega_{inc} = \omega_{refracted} = \omega_{reflected}$

$\Rightarrow \frac{k_{inc}\, c}{n_i} = \frac{k_{refrac}\, c}{n_2} = \frac{k_{reflected}\, c}{n_1}$

$R_{\parallel} = r_{\parallel}^2, \; R_{\perp} = r_{\perp}^2$

$T_{\parallel} = \frac{n_t \cos(\theta_t)}{n_i \cos(\theta_i)} |t_{\parallel}|^2$

$T_{\perp} = \frac{n_t \cos(\theta_t)}{n_i \cos(\theta_i)} |t_{\perp}|^2$

$\vec{\tilde{E}}_{inc} = \vec{\tilde{E}}_{0\parallel} + \vec{\tilde{E}}_{0\perp}$

$\vec{\tilde{E}}_r = r_{\parallel} \vec{\tilde{E}}_{0\parallel} + r_{\perp} \vec{\tilde{E}}_{0\perp}$

$\vec{\tilde{E}}_t = t_{\parallel} \vec{\tilde{E}}_{0\parallel} + t_{\perp} \vec{\tilde{E}}_{0\perp}$