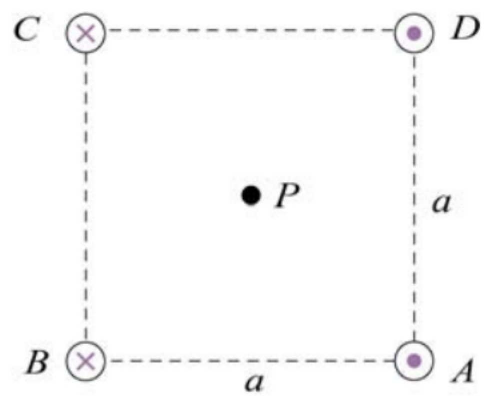
You must show your work to receive credit. An answer written down with no work will receive no credit.

Problem 1

30 points

Consider a configuration of infinitely long parallel current-carrying wires shown in the diagram below:



Wires B and C carry current I into the page, and wires A and D carry current I out of the page. What are the magnitude and direction of the magnetic field at point P? You may use the expression for the magnetic field of infinitely long wires derived in class.

The direction of the magnetic field at P from each point can be determined using the right-hand rule. Let $\hat{x}$ be the direction from B to A and let $\hat{y}$ be the direction from B to C. The distance from A, B, C, and D to P is $\frac{a}{\sqrt{2}}$. Thus the magnitude of B from each wire is $\frac{\mu_0 I}{\sqrt{2}\pi a}$. Then we can compute:

$$\vec{B}_A = \frac{\mu_0 I}{\sqrt{2}\pi a}\frac{1}{\sqrt{2}}(-\hat{x} - \hat{y})$$
$$\vec{B}_B = \frac{\mu_0 I}{\sqrt{2}\pi a}\frac{1}{\sqrt{2}}(\hat{x} - \hat{y})$$
$$\vec{B}_C = \frac{\mu_0 I}{\sqrt{2}\pi a}\frac{1}{\sqrt{2}}(-\hat{x} - \hat{y})$$
$$\vec{B}_D = \frac{\mu_0 I}{\sqrt{2}\pi a}\frac{1}{\sqrt{2}}(\hat{x} - \hat{y}).$$

Adding these up, we get the net magnetic field:

$$\vec{B}_P = -\frac{2\mu_0 I}{\pi a}\hat{y}$$

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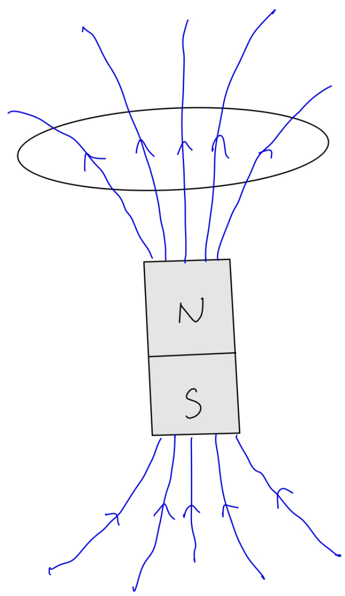
Problem 2

30 points

Consider a loop of wire adjacent to a permanent magnet in the configuration shown.



(a): 10 pts

Hold the magnet stationary and move the loop of wire towards the magnet as shown. Faraday's law,

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

tells us that an emf is induced in the wire due to the change in magnetic flux through the wire. There must be some force that drives the electrons in the wire to form a current. What is this force?

With the magnet stationary, there is no induced electric field. The force driving the current is the **magnetic** force $\vec{F} = q\,\vec{v} \times \vec{B}$ acting on the current-carriers in the conductor.

(a): 10 pts

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(b): 10 pts

Determine the direction of the current that flows by considering the force you found in part (a).

In this case, the magnetic field at the wire is pointing "up and out" at the location of the wire, and the velocity of the particles is downard towards the magnet. Using the right-hand rule, we can determine that $\vec{v} \times \vec{B}$ points in the **clockwise** (as seen from above) direction, which will be the direction of current flow. We can confirm this using Lenz's law: as the loop moves towards the wire, the magnetic flux is positive and increasing, and so the induced current should be such to produce a *negative* magnetic flux, which a clockwise current acheives.

(b): 10 pts

(c): 10 pts

Now consider an equivalent situation in which you hold the loop of wire still and move the magnet towards the loop. Faraday's law still tells us that a current flows due to the changing magnetic flux. What force acts on the electrons to drive the current in this case? Justify your answer.

Since the loop is stationary, we don't have the same magnetic force as in part (a). In this case, an electric field is induced, as evidenced by Faraday's law for a stationary loop,

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}.$$

Thus the force driving the current in this case is the **electric** force. This is remarkable! This tells us that depending on whether we are seeing the situation from the perspective of the magnet or the wire, there is either an induced electric field or no induced electric field. The value of the electric and magnetic fields depend on the speed at which the observer moves! This situation will be clarified when we discuss special relativity.

(c): 10 pts

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Problem 3

40 pts

A long cylindrical conductor of radius R carries a uniform current density $\vec{J} = J\hat{z}$ that runs parallel to the axis of the cylinder (the z-axis). A time-varying electric field is established everywhere in space and is given by $\vec{E} = E_0 \cos(\omega t)\hat{z}$. Using Ampere's law, compute the magnetic field in the following regions: [YOU MUST SHOW ALL WORK; YOU MAY NOT USE RESULTS FROM LECTURE OR PSETS]

(a): 20 pts

$r > R$

For this we use the complete Ampere's law,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (i_C + \epsilon_0 \frac{d\Phi_E}{dt}).$$

Take as our Amperean path a circle of radius $r > R$ centered at the axis of the cylinder. The current passing through this path is $\pi R^2 J$. The electric flux through this circle is $\Phi_E = (\pi r^2)E_0 \cos(\omega t)$. By symmetry, the magnitude of this magnetic field is constant, and so we have

$$\begin{aligned}(2\pi r)B &= \mu_0 \left((\pi R^2)J + (\pi r^2)E_0 \frac{d}{dt} \cos(\omega t) \right) \\ &= \pi \mu_0 \left(JR^2 - r^2 E_0 \omega \sin(\omega t) \right).\end{aligned}$$

Rearranging, we find

$$B = \frac{1}{2}\mu_0 \left(\frac{JR^2}{r} - rE_0\omega \sin(\omega t) \right).$$

The direction of $\vec{B}$ is either circulating clockwise or counterclockwise around the wire, but which direction may depend on time and also on the relative magnitudes of E_0 and J.

(a): 20 pts

(b): 20 pts

$r < R$

This is the same as above, except that the current enclosed by our Amperean path is $\pi r^2 J$, and so

$$B = \frac{1}{2}\mu_0 \left(J - E_0\omega \sin(\omega t) \right) r$$

(b): 20 pts