SOLUTIONS

Name UID

1.	2.	3.	Total
40	30	30	100

Physics 1CH Midterm #2

May 24, 2018

On all problems, you need to show your work to get full credit.

Below are a set of numerical constants. If you have any questions, please raise your hand to ask for help.

Acceleration of gravity (Earth)	g	10.0 m/s^2
Boltzmann constant	k	1.38×10^{-23} J/K
Electron charge	e	1.60×10^{-19} C
Electron mass	m_e	9.11×10^{-31} kg
		0.511 MeV/c^2
Electron-volt	eV	1.60×10^{-19} J
Permeability of free space	μ_o	$4\pi \times 10^{-7}$ N/A^2
Permittivity of free space	ε_o	8.85×10^{-12} $C^2/N\text{-}m^2$
Planck constant	h	6.63×10^{-34} J-s
Proton mass	m_p	1.67×10^{-27} kg
		938 MeV/c^2
Speed of light in vacuum	c	3.00×10^8 m/s
Speed of sound in air (20° C)	v_s	340 m/s

Small angle approximation (θ in radians):

$$\sin(\theta) \approx \tan(\theta) \approx \theta$$

Problem 1: Short Answer (40 points total):

a) True or False. Since a standing wave does not travel, it is not truly a wave and does not satisfy the wave equation. Explain your answer.

$\boxed{\text{FALSE}}$

A STANDING WAVE IS A WAVE AND IT DOES SATISFY THE WAVE EQN. ψ_r ψ_L

STANDING WAVES ARE SUPERPOSITION OF RT + LEFT MOVING TRAVEL WAVES. IF ψ_r, ψ_L BOTH SATISFY W.E. THEN $\psi_{TOT} = \psi_r + \psi_L$ SATISFIES WAVE EQN.

b) You are in a store examining sunglasses displayed in a glass case. The salesperson claims that the sunglasses have Polaroid filters. You suspect that the sunglasses are just tinted plastic. You ask to see a couple of the sunglasses. Name two ways you could find out for sure (in the store). Explain your answer.

1) TRANSMISSION

TAKE TWO SUNGLASSES AND LOOK AT LIGHT GOING THROUGH BOTH AS YOU ROTATE ONE - CHECK IF TRANSMISSION $\to 0$ (MALUS' LAW)

2 REFLECTION

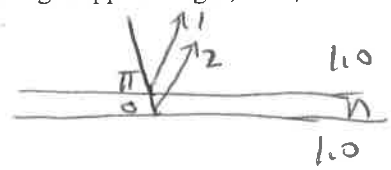
USE ONE PAIR OF SUNGLASSES TO OBSERVE LIGHT REFLECTED OFF GLASS. FOR $n_1 = 1.0$ $n_2 \simeq 1.5$ $\theta_B \simeq 56°$ BREWSTER'S Angle.

LIGHT INCIDENT ON GLASS AT $\theta \simeq 56°$ will BE Reflected WITH LINEAR POLARIZATION. CHECK THIS WITH SUNGLASSES.

Problem 1 (continued):

c) Imagine a soap bubble formed in air. As the bubble is just about to pop (i.e. as its thickness goes to zero), will the reflected light appear bright, dark, or neither of these? Explain your answer.

$\boxed{\text{DARK}}$



FOR SOAP BUBBLE IN AIR, THERE IS A RELATIVE PHASE SHIFT OF π BETWEEN REFLECTING RAYS 1 and 2.

THUS FOR VERY THIN FILMS ($d \to 0$), THERE IS NO PHYSICAL PATH DIFFERENCE BETWEEN 1 AND 2, AND 1 and 2 HAVE SIMPLY A PHASE DIFFERENCE OF π

⟹ DESTRUCTIVE INTERFERENCE FOR ALL λ → BLACK OR DARK

d) A student is working with a double-slit interference experiment. Instead of using light of a single wavelength, a light source having wavelengths of 400 nm, 500 nm and 600 nm is used. Describe the interference pattern you would see on the screen, if any. You can assume that the light hitting the two slits is coherent and has the same linear polarization.

BLUE GREEN RED

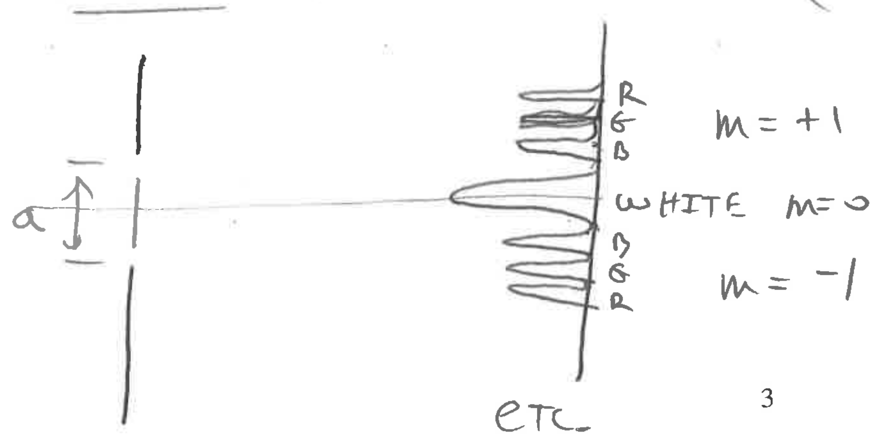
$\boxed{\text{YOU WOULD SEE AN INTERFERENCE PATTERN}}$

CENTER - WOULD BE MAX. FOR ALL λ ⟹ R+G+B FRINGES = WHITE

AWAY FROM CENTER YOU WOULD SEE SEPARATE BRIGHT FRINGES, FIRST BLUE, THEN GREEN THEN RED

MAXIMA $d \sin\theta = m\lambda$

so larger λ ⟹ larger θ for same m



CENTER WOULD ALSO BE BRIGHTER THAN m=+1,−1 FRINGES

Problem 2: (30 points total)

A road tunnel leading straight through a mountain greatly amplifies tones at frequencies of 135 Hz and 138 Hz.

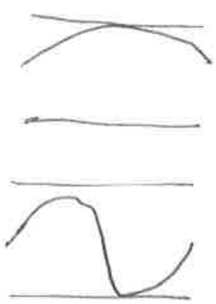
a) Find the shortest length the tunnel can be. Explain why you know that this is indeeed the shortest length. Note that the speed of sound is given on page 1.

$V_s = 340$ m/s Speed of sound

TUNNEL = OPEN-OPEN PIPE ⇒ STANDING WAVE

n=1 $\lambda = 2L$ $\lambda_n = \frac{2L}{n}$ $n = 1, 2, 3 \ldots$

n=2 $\lambda = L$



Let $f_a = 135$ Hz, $f_b = 138$ Hz — 2 modes of system

for a, b INTEGERS

a) Assume $b = a+1$ modes differ by 1

$$V_s = f_a \lambda_a = f_b \lambda_b \Rightarrow f_a \frac{2L}{a} = f_b \frac{2L}{a+1}$$

Algebra ⇒ $a = \left(\frac{f_b}{f_a} - 1\right)^{-1} = 45$

$b = 46$

$$\lambda_a = \frac{V_s}{f_a} = 2.52\,m = \frac{2L}{a} \Rightarrow \boxed{L = 56.7\,m}$$

Now try $b = a+2$, you get $a = 90$, $b = 92$ ⇒ $L = 113\,m$

Any larger spacing of modes will give you higher mode #'s and larger values of L

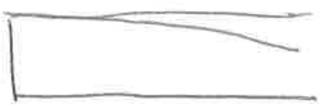
Hence solution we found for $b = a+1$ is for shortest length

Problem 2 (continued)

b) Suppose one end of the tunnel is completely closed off due to a rock slide. What would be the lowest frequency tone the tunnel would greatly amplify now? Would either of the tones of 135 Hz or 138 Hz be amplified in this case?

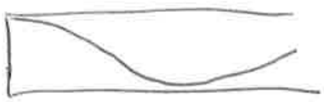
NOW TUNNEL IS OPEN-CLOSED PIPE

$n = 1$



$$\lambda_n = \frac{4L}{n} \quad n = 1, 3, 5 \ldots$$

$n = 3$



LOWEST FREQ CORRESPONDS TO LONGEST λ

SO

$$\lambda_1 = 4L = 227\,m$$

$$\boxed{f_1 = \frac{V_s}{\lambda_1} = 1,5\ Hz}$$

LOWEST TONE AMPLIFIED

CHECK 135 Hz, 138 Hz

f_a f_b

THESE YIELD $\left(\begin{array}{l} a = 90 \\ b = 92 \end{array} \right)$ NOT MODES OF OPEN-CLOSED PIPE

$\boxed{\text{SO TONES OF 135 Hz, 138 Hz WOULD NOT BE AMPLIFIED IN THIS CASE}}$

Problem 3: Fourier Techniques (30 points total)

Consider a function defined by:

$$\psi(x) = h, \quad 0 < x < L$$
$$\psi(x) = 0, \quad \text{elsewhere},$$

for h a constant.

a) Determine the Fourier *sine series* expansion of $\psi(x)$ in the interval (0,L). (Note: there is not a trivial solution to this problem).

FOURIER SINE SERIES OVER $(0,L)$

$$\psi(x) = \sum_{n=1}^{\infty} b_n \sin k_n x \qquad k_n = \frac{n\pi}{2}, \quad n = 1,2,3..$$

$$b_n = \frac{2}{L}\int_0^L \psi(x)\sin(k_n x)\,dx = \frac{2}{L}\int_0^L h \sin(k_n x)\,dx$$

$$b_n = \frac{2h}{Lk_n}\left(-\cos k_n x\right)\Big|_0^L = \frac{2h}{n\pi}\left(1 - \cos n\pi\right)$$

for n odd, $\cos(n\pi) = -1$, $b_n = \frac{4h}{n\pi}$

for n even, $\cos(n\pi) = 0$, $b_n = 0$

So

$$\boxed{\psi(x) = \sum_{n=1,3,5..}^{\infty} \frac{4h}{n\pi}\sin\left(\frac{n\pi x}{L}\right)}$$

Problem 3 (continued)

b) For $h = \frac{\pi}{4}$ m, evaluate $\psi(\frac{L}{2})$ using what you found in part a), and thus determine a series expansion for π. Estimate π using the first five terms of the expansion.

TAKE $h = \frac{\pi}{4}$ m, $\psi(x) = \sum_{n=1,3,5\ldots}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi x}{L}\right)$

@ $x = \frac{L}{2}$ $\psi(\frac{L}{2}) = \sum_{n=1,3,5\ldots}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi}{2}\right)$

So $\frac{\pi}{4} = \sum_{n=1,3,5\ldots}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi}{2}\right)$

$$\boxed{\pi = 4 \sum_{n=1,3,5\ldots}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi}{2}\right)}$$

First few terms:

$$\pi = 4\left[\sin\left(\frac{\pi}{2}\right) + \frac{1}{3}\sin\left(\frac{3\pi}{2}\right) + \frac{1}{5}\sin\left(\frac{5\pi}{2}\right) + \ldots\right]$$

$$= 4\left(\underbrace{1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9}}_{0.835} - \ldots\right)$$

$\boxed{\pi \simeq 3.34}$ 1ST five Terms