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Name UID

1.	2.	3.	Total
34	25	32	91

Excellent

Physics 1CH Midterm #1 April 30, 2019

On all problems, you need to show your work to get full credit.

Below are a set of numerical constants. If you have any questions, please raise your hand to ask for help.

Acceleration of gravity (Earth)	g	10.0 m/s^2
Boltzmann constant	k	1.38 x 10^{-23} J/K
Electron charge	e	1.60 x 10^{-19} C
Electron mass	m_e	9.11 x 10^{-31} kg
		0.511 MeV/c^2
Electron-volt	eV	1.60 x 10^{-19} J
Permeability of free space	μ_o	4π x 10^{-7} N/A^2
Permittivity of free space	ε_o	8.85 x 10^{-12} C^2/N-m^2
Planck constant	h	6.63 x 10^{-34} J-s
Proton mass	m_p	1.67 x 10^{-27} kg
		938 MeV/c^2
Speed of light in vacuum	c	3.00 x 10^8 m/s
Speed of sound in air (20^o C)	v_s	340 m/s
Temperature conversion		0^o C = 273 K

Index of refraction: Air n ~ 1.0

Water n = 1.33

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Problem 1: Short Answer (40 points total):

a) True or False? *Chromatic aberration can occur in simple lenses, but not in ordinary mirrors.* Explain your answer – i.e. if true, you need to explain why chromatic aberration occurs in one but not the other; if false, you need to explain why it is false.

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False. Chromatic aberration occurs because refracting indexes often depend on frequency of light. However, there is no change in index in the case of reflection, so there should be no aberration because there is no reflection. Furthermore, because of the law of reflection, the reflected angle must equal the incident angle, so there is only one path the light can take for any given incidence, regardless of light frequency.

Contradicts

b) A plane electromagnetic wave with angular frequency $\omega = 3.9 \times 10^{15}$ rad/s is traveling through a transparent medium with index of refraction $n = 1.2$. What is the shortest distance between two points along the wave that are separated by a phase difference of 45°?

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$n = \frac{c}{v} \Rightarrow v = \frac{c}{n} = \frac{3\times10^8}{1.2} = 2.5\times10^8$

$v = \frac{\omega}{k} \Rightarrow k = \frac{\omega}{v} = \frac{3.9\times10^{15}}{2.5\times10^8} = 1.56\times10^7$

$k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{k} = 4.03\times10^{-7}$ m

A phase difference of 45° corresponds to $\frac{45}{360} = 0.125$ wavelengths $\Rightarrow$ The distance between them is $0.125 \times 4.03\times10^{-7}$ m $= 5.03\times10^{-8}$ m

Problem 1 (continued):

c) The index of refraction of the human cornea is about 1.38. If you can see clearly in air, why can't you see clearly underwater? Why do goggles help? Drawing a picture will help and you can consider an object at infinity to make things easier.

Underwater, the index of refraction is approximately 1.33, which is much closer to the index of refraction for the human cornea than air is. Thus light from infinity does not converge as well from water to the eye, and the image forms farther behind the retina. Goggles help because they have a flat boundary, so they don't refract any light from infinity in the water, but still allows for light in the air of the goggles to refract in the human eye as if they were seeing in air. They also keep water out of your eyes, which is uncomfortable.

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d) True or False? *When sunlight reflects off the top surface of a swimming pool, the reflected wave incurs a phase shift of $\pi/2$ radians.* Explain your answer.

False, reflected waves always have a phase shift of π radians.

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NOT SO

Only externally.

Internally $\Delta\phi = \emptyset$

Problem 2: Mystery Lake (25 Points Total):

Deep in the Amazon rainforest is a lake containing a mysterious transparent liquid. The local villagers know that the top 1 m layer has an index of refraction n=1/2, the next 1 m layer has an index of refraction n=1/4, then n=1/8, then n=1/16, and so forth. The lake is very deep, and the index of refraction approaches zero near the bottom. Standing on a ledge above the lake, you take a laser pointer from your pocket and direct it into the lake. Assume that the laser beam hits the planar surface of the lake at an incident angle (measured with respect to the normal) of θ_0, where $0° < \theta_0 < 90°$.

a) What happens to the laser beam for $\theta_0 = 10°$? Support your answer with a calculation that determines the path of the beam in the lake. Illustrate the path of the beam with a figure.

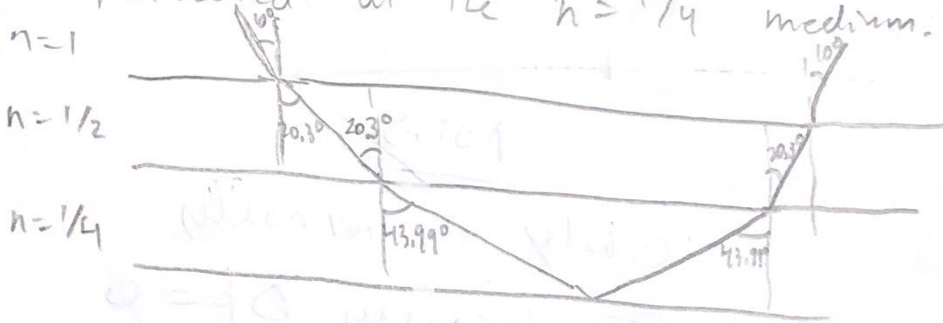
$n_1 \sin\theta_1 = n_2 \sin\theta_2 \Rightarrow \sin\theta_2 = \frac{n_1 \sin\theta_1}{n_2} = \frac{\sin(10)}{0.5} \Rightarrow$

$\theta_2 = 20.3° \Rightarrow$

$n_2 \sin\theta_2 = n_3 \sin\theta_3 \Rightarrow \sin\theta_3 = \frac{n_2 \sin\theta_2}{n_3} = \frac{0.5\sin(20.3)}{0.25} \Rightarrow$

$\theta_3 = 43.99° \Rightarrow$

For a medium that has half the index of refraction of the previous medium, the critical angle equals $\sin^{-1}(0.5) = 30° \Rightarrow$ Any incident angle above that gets totally internally reflected. So the beam gets reflected at the n = 1/4 medium.



Problem 2 (continued)

b) Now discuss what happens to the laser beam for all values of θ_0. Support your answer with a calculation and discuss the path of the beam for various ranges of θ_0.

For $\theta_0 > 30°$, the light will be reflected because the critical angle as previously calculated ($\theta_c = \sin^{-1}(0.5)$) will always be 30° for each medium.

For $\theta_0 = 30°$, the incident light will appear to refract at 90° and move horizontally across the water.

$\sin\theta_0 = 0.5\sin(30°) \Rightarrow \theta_0 = \sin^{-1}(0.5\sin(30°)) = 14.5°$

For $14.5° < \theta_0 < 30°$, the light will be internally reflected at the $n=\frac{1}{2}$ medium since the refracted light will be equal to the critical angle.

$\sin\theta_0 = 0.5\sin(14.5°) \Rightarrow \theta_0 = 7.18° \Rightarrow$

For $7.18° < \theta_0 < 14.5°$, light is totally internally reflected at the $n=\frac{1}{4}$ medium.

This shows for index of refraction n, the incident angle to cause total internal reflection at that medium is about $30 \times n < \theta_0 < 30 \times 2n$.

Problem 3: Objects and Images (35 points total)

On an optical bench is a lens with a focal length, $f_1 = +10\,\text{cm}$, and a mirror of unknown type (convex, concave, or planar) and unknown focal length. An upright object, O_1, is located 15 cm to the left of the lens. The mirror is located 50 cm to the right of the lens. The image formed by the mirror, I_2, is inverted and is the same size when compared to the original object O_1.

a) Determine the position of the image formed by the lens, I_1. Is it real/virtual, upright/inverted, and what is its magnification?

$$\frac{1}{o} + \frac{1}{i_1} = \frac{1}{f} \Rightarrow \frac{1}{15} + \frac{1}{i_1} = \frac{1}{10} \Rightarrow \frac{1}{i_1} = \frac{1}{10} - \frac{1}{15} = \frac{1}{30} \Rightarrow$$

$i_1 = 30$ cm

Because $i > 0$, the image is **real**

$M = \frac{-i}{o} = \frac{-30}{15} = \boxed{-2}$

The image is inverted since $M < 0$

b) Given what you know about the second image, determine the parameters of the mirror: is it convex, concave, or planar, and what is its radius of curvature, R? Determine the position of the image formed by the mirror, I_2. Is it real/virtual?

$M_{tot} = M_1 M_2 \Rightarrow M_{tot} = -1 \Rightarrow M_1 = -2 \Rightarrow M_2 = -\frac{1}{-2} = 0.5$

$M_2 = \frac{-i_2}{o_2} \Rightarrow o_2 = d - i_1 = 50 - 30 = 20\text{ cm} \Rightarrow$

$0.5 = -\frac{i_2}{20} \Rightarrow \boxed{i_2 = -10\text{ cm}}$

$\frac{1}{20} - \frac{1}{10} = \frac{1}{f} \Rightarrow f = -20$ cm

The mirror is convex with $R = -2(f) = 40$ cm.

Because $i_2 < 0$, it is a virtual image.

Problem 3 (continued)

c) On the horizontal axis below, draw the lens and the mirror. Then, provide a to-scale ray-trace for at least two rays from the initial object to the first image and then at least two rays from the second object to the second image. Indicate the positions of O_1, I_1, O_2, I_2, F_1, F_2, and C, where F_1 identifies the focal points of the lens, F_2 identifies the focal point of the mirror, and C identifies the center of the mirror. (You do not have to ray-trace back through the lens to form a third image).