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1.	2.	3.	Total

Physics 1CH Midterm #1

April 30, 2019

On all problems, you need to show your work to get full credit.

Below are a set of numerical constants. If you have any questions, please raise your hand to ask for help.

Acceleration of gravity (Earth)	g	10.0 m/s^2
Boltzmann constant	k	1.38 x 10^{-23} J/K
Electron charge	e	1.60 x 10^{-19} C
Electron mass	m_e	9.11 x 10^{-31} kg
		0.511 MeV/c^2
Electron-volt	eV	1.60 x 10^{-19} J
Permeability of free space	μ_o	4π x 10^{-7} N/A^2
Permittivity of free space	ε_o	8.85 x 10^{-12} $C^2/N\text{-}m^2$
Planck constant	h	6.63 x 10^{-34} J-s
Proton mass	m_p	1.67 x 10^{-27} kg
		938 MeV/c^2
Speed of light in vacuum	c	3.00 x 10^8 m/s
Speed of sound in air (20° C)	v_s	340 m/s
Temperature conversion		0° C = 273 K

Index of refraction: Air n ~ 1.0

Water n = 1.33

Problem 1: Short Answer (40 points total):

a) True or False? *Chromatic aberration can occur in simple lenses, but not in ordinary mirrors.* Explain your answer – i.e. if true, you need to explain why chromatic aberration occurs in one but not the other; if false, you need to explain why it is false.

True

an example of chromatic abberation is spherical which occurs when light goes through a dispersive medium (index of refraction depends on frequency & therefore wavelength). since light never goes through a mirror but can go through lenses, chromatic aberrations can only occur in lenses.

b) A plane electromagnetic wave with angular frequency $\omega = 3.9 \times 10^{15}$ rad/s is traveling through a transparent medium with index of refraction $n = 1.2$. What is the shortest distance between two points along the wave that are separated by a phase difference of 45°?

wavelength is distance between two points seperated by 2π (360°)

distance between two points separated by phase difference of 45° is

$$\frac{\lambda}{8}$$

$\omega = 3.9 \times 10^{15}$ rad/s $\quad n = 1.2$

$$\frac{\lambda}{8} = \boxed{5.035 \times 10^{-8}\ m}$$

$$V = \frac{c}{n} = 2.5 \times 10^{8}\ m/s$$

$$\omega = 2\pi \nu$$

$$\nu = \frac{\omega}{2\pi} = 6.207 \times 10^{14}\ Hz$$

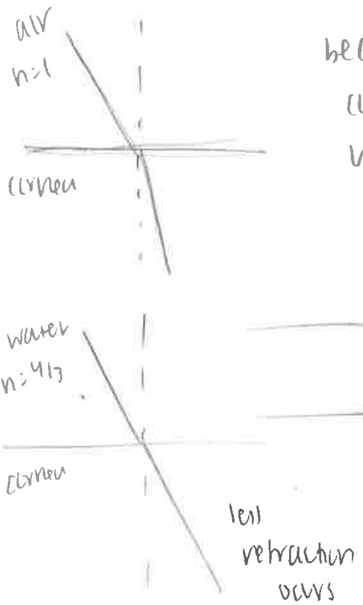
$$V = \nu\lambda$$

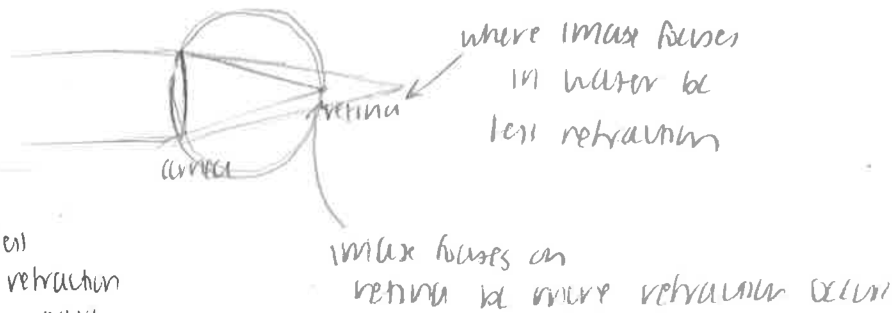
$$\lambda = \frac{V}{\nu} = \frac{2.5 \times 10^{8}\ m/s}{6.207 \times 10^{14}\ Hz} = 4.0277 \times 10^{-7}\ m$$

Problem 1 (continued):

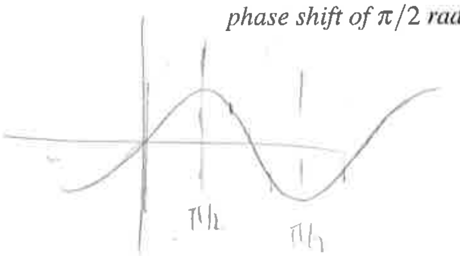
c) The index of refraction of the human cornea is about 1.38. If you can see clearly in air, why can't you see clearly underwater? Why do goggles help? Drawing a picture will help and you can consider an object at infinity to make things easier.

the water distorts light that comes through the cornea and forms images on the retina of the eye. This is because water has a different index of refraction compared to air and the refraction that occurs under water is different than in air (something the cornea is designed to handle)

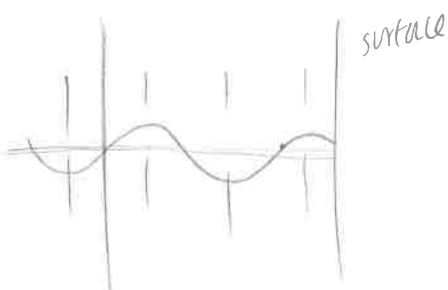




d) True or False? *When sunlight reflects off the top surface of a swimming pool, the reflected wave incurs a phase shift of* $\pi/2$ *radians.* Explain your answer.



True, when light reflects it undergoes a phase shift of $\frac{\pi}{2}$ radians. This is because the direction of the wave reverses



$\psi(x,t) = A\sin(kx - \omega t)$ travels to right

$\downarrow$

$\psi(x,t) = A\sin(kx + \omega t)$ travels to left.

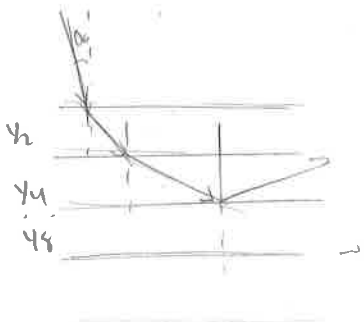
this wave undergoes a phase shift of $\pi/2$

-1.14

Problem 2: Mystery Lake (25 Points Total):

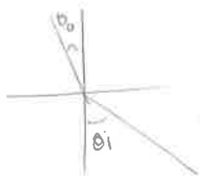
Deep in the Amazon rainforest is a lake containing a mysterious transparent liquid. The local villagers know that the top 1 m layer has an index of refraction n=1/2, the next 1 m layer has an index of refraction n=1/4, then n=1/8, then n=1/16, and so forth. The lake is very deep, and the index of refraction approaches zero near the bottom. Standing on a ledge above the lake, you take a laser pointer from your pocket and direct it into the lake. Assume that the laser beam hits the planar surface of the lake at an incident angle (measured with respect to the normal) of θ_0, where $0° < \theta_0 < 90°$.

a) What happens to the laser beam for $\theta_0 = 10°$? Support your answer with a calculation that determines the path of the beam in the lake. Illustrate the path of the beam with a figure.



$OPL = \int_A^B n(s)\, ds$

the laser beam eventually undergoes total internal reflection

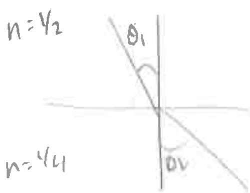


$\theta_0 = 10°$

$n_0 \sin\theta_0 = n_i \sin\theta_i$

$\sin 10° = \frac{1}{2} \sin\theta_i$

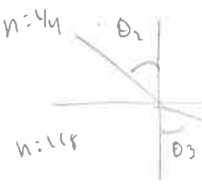
$\theta_i = 20.322°$



$\theta_1 = 20.322°$

$\frac{1}{2} \sin\theta_1 = \frac{1}{4} \sin\theta_2$

$\theta_2 = 43.99°$



$\frac{1}{4} \sin\theta_2 = \frac{1}{8} \sin\theta_3$

$\theta_3 > 90°$

undergoes total internal reflection

Problem 2 (continued)

b) Now discuss what happens to the laser beam for all values of θ_0. Support your answer with a calculation and discuss the path of the beam for various ranges of θ_0.

$0 < \theta_0 < 90$

1	θ_0
1/2	θ_1
1/4	θ_2
1/8	θ_3
1/16	θ_4

$\sin\theta_0 = \frac{1}{2}\sin\theta_1 = \frac{1}{4}\sin\theta_2 = \frac{1}{8}\sin\theta_3 = \frac{1}{16}\sin\theta_4$

the laser beam will undergo total internal reflection at a certain level in the liquid regardless of the value of θ_0 as long as it satisfies the requirement

$0° < \theta_0 < 90°$

$n = 1/2$: $\sin\theta_1 = 2\sin\theta_c = 1$, $\theta_0 = \theta_c$, $\theta_1 = 90°$, $\sin\theta_c = 1/2$, $\theta_c = 30°$

$90° > \theta_0 > 30°$
total internal reflection occurs when it reaches the boundary of the $n = 1/2$ layer

$n = 1/4$: $\sin\theta_0 = \frac{1}{4}\sin\theta_2$, $\sin\theta_c = \frac{1}{4}\sin\theta_2 = \frac{1}{4}$, $\theta_c = 14.4°$

$14.4° < \theta_0 < 30°$
total internal reflection occurs when it reaches the boundary of the $n = 1/4$ layer

$n = 1/8$: $\sin\theta_c = \frac{1}{8}\sin\theta_3 = \frac{1}{8}$, $\theta_c = 7.18°$

$7.18° < \theta_c < 14.4°$
total internal reflection occurs when it reaches boundary of $n = 1/8$ layer

$n = 1/16$: $\sin\theta_c = 1/16$, $\theta_c = 3.583°$

$3.583° < \theta_c < 7.18°$
TIR (total int. ref) occurs at $n = 1/16$ boundary

$n = 1/32$: $\sin\theta_c = 1/32$, $\theta_c = 1.79$

$1.79° < \theta_c < 3.583°$
TIR at $n = 1/32$ boundary

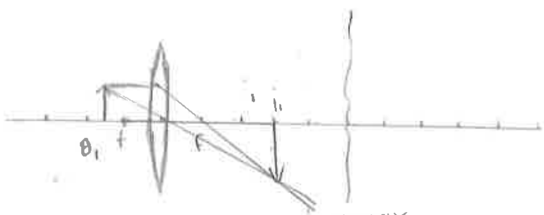
Problem 3: Objects and Images (35 points total)

convex

On an optical bench is a lens with a focal length, $f_l = +10\,\text{cm}$, and a mirror of unknown type (convex, concave, or planar) and unknown focal length. An upright object, O_1, is located 15 cm to the left of the lens. The mirror is located 50 cm to the right of the lens. The image formed by the mirror, I_2, is inverted and is the same size when compared to the original object O_1.

a) Determine the position of the image formed by the lens, I_1. Is it real/virtual, upright/inverted, and what is its magnification?

real
inverted
$M = -2$



lens:

$\frac{1}{o_1} + \frac{1}{i_1} = \frac{1}{f}$

$\frac{1}{15} + \frac{1}{i_1} = \frac{1}{10}$

$i_1 = 30\text{ cm}$

$M = -\frac{i}{o} = \frac{-30}{15} = -2$

b) Given what you know about the second image, determine the parameters of the mirror: is it convex, concave, or planar, and what is its radius of curvature, R? Determine the position of the image formed by the mirror, I_2. Is it real/virtual?

$M_T = -1$ $M_T = M_1 M_L$ $M_1 = -2$

$-1 = -2 M_1$

$M_2 = 1/2$

convex

$R = 40\text{cm}$

mirror:

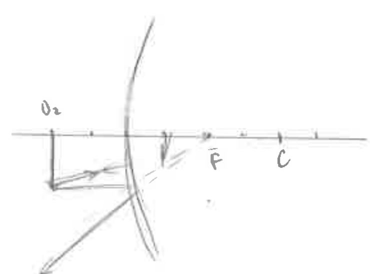
$o_2 = 20\text{ cm}$

$i_2 = ?$

$M_2 = \frac{1}{2} = \frac{-i_2}{20}$

$i_2 = -10\text{ cm}$

virtual image



$\frac{1}{o_2} + \frac{1}{i_2} = \frac{1}{f}$ $f = -\frac{R}{2}$

$\frac{1}{20} - \frac{1}{10} = \frac{1}{f}$ $f = -20$

convex bc $f < 0$

$f = -\frac{R}{2} = -20$ $R = 40$

Problem 3 (continued)

c) On the horizontal axis below, draw the lens and the mirror. Then, provide a to-scale ray-trace for at least two rays from the initial object to the first image and then at least two rays from the second object to the second image. Indicate the positions of O_1, I_1, O_2, I_2, F_1, F_2, and C, where F_1 identifies the focal points of the lens, F_2 identifies the focal point of the mirror, and C identifies the center of the mirror. (You do not have to ray-trace back through the lens to form a third image).

F1 = 10cm
f2 = -20 cm
O1 = 15 cm left of lens
R = 40
i1 = 30cm