Cindy Chang — Name

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1.	2.	3.	4.	5.	6.	Total

Physics 1CH Final Exam

June 10, 2019

On all problems, you need to show your work to get full credit.

Please put a box around all final answers or expressions.

Acceleration of gravity (Earth)	g	10.0 m/s^2
Boltzmann constant	k	1.38×10^{-23} J/K
Electron charge	e	1.60×10^{-19} C
Electron mass	m_e	9.11×10^{-31} kg
		0.511 MeV/c^2
Electron-volt	eV	1.60×10^{-19} J
Permeability of free space	μ_o	$4\pi \times 10^{-7}$ N/A^2
Permittivity of free space	ε_o	8.85×10^{-12} $C^2/N\text{-}m^2$
Planck constant	h	6.63×10^{-34} J-s
		4.14×10^{-15} eV-s
Proton mass	m_p	1.67×10^{-27} kg
		938 MeV/c^2
Speed of light in vacuum	c	3.00×10^{8} m/s
Speed of sound in air (20° C)	v_s	340 m/s
Temperature conversion		0° C = 273 K

Small angle approximation: $\tan(\theta) = \sin(\theta) = \theta$ (for θ in radians)

$\cos(\theta) = \sin(90^\circ - \theta)$ $\sin^2\theta + \cos^2\theta = 1$

Cindy Chang

Name

Physics 1CH Final Exam

June 10, 2019

Problem 1: Short Answer (40 points total):

a) True or False? A converging lens cannot form a real image from a virtual object. Explain your answer (an equation or a figure might be useful).

converging lens:

$f > \emptyset$

virtual object:

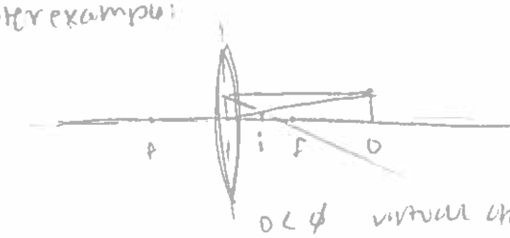
$o < \emptyset$

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{o}$$ since o is negative and f is positive

$$\frac{1}{i} = \frac{1}{|f|} + \frac{1}{|o|}$$ i will always be positive. so it is a real image.

counterexample:



$o = -10$

$f = 5$

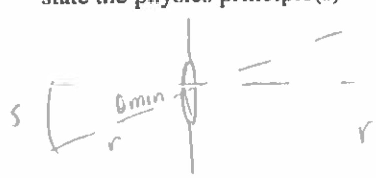
$\frac{1}{i} = \frac{1}{5} - \frac{1}{-10} =$

$o < \emptyset$ virtual obj.

$i > \emptyset$ real image

$\boxed{\text{False}}$

b) Astronaut Rita is in a low-Earth orbit (altitude = 300 km) and passes over her home town. Can she use her unaided eyes to identify her (single-family) house? Use an estimate for the size of her house, the wavelength of visible light, and a pupil diameter of 4 mm. If she cannot resolve her house, explain why not. (Make sure to state the physics principle(s) involved and to show your work).



$\theta_{min} = \frac{1.22\lambda}{D}$

$r = 300000 \text{ m}$

$\theta = \frac{s}{r}$

$D = .004 \text{ m}$

s = size of house = 60 ft.

λ visible light $= \sim 600$ nm $= 600 \times 10^{-9}$ m.

$$\theta_{min} = \frac{1.22\lambda}{D} = \frac{1.22 \times 600 \times 10^{-9} \text{m}}{.004 \text{ m}} = 1.83 \times 10^{-4}$$

$\theta_{min} = \frac{s_{min}}{r}$ $\quad s_{min} = 54.9$ ft.

She cannot resolve her house because if we use the estimates for λ visible light and the size of her house from above, the minimum size the house can be for it to still be resolvable from a distance of 300 km is 54.9 ft. My estimate for her house was 50 ft which is less than the minimum

Cindy Chang
Name

Physics 1CH Final Exam

June 10, 2019

Problem 1 (continued):

c) Physics student Chuck moonlights as a security guard at a retirement community. While he is driving slowly through the community, he turns on the car siren for fun. The siren operates at a characteristic frequency of 600 Hz. As he approaches a reflective wall directly in front of him, Chuck hears a sound pattern having 18 beats per second. How fast is he going in km/hr? (Make sure to state the physics principle(s) involved and to show your work).

$f_1 = 600 \text{ Hz}$

beat frequency = 18 Hz

Chuck is the surce and detector of the sund.

$f_2 - f_1 = 18 \text{ Hz}$

$f_2 - 600 \text{ Hz} = 18 \text{ Hz}$

$f_2 = 618 \text{ Hz}$

surce and detector are going towards each other and are equal.

$$f_2 = f_1 \left(\frac{v \pm u_D}{v \pm u_s} \right)$$

$u_D = u_s = V_c$

$$= f_1 \left(\frac{v + u_D}{v - u_s} \right)$$

$V = 340 \text{ m/s}$

$$f_2 = f_1 \left(\frac{v + V_c}{v - V_c} \right)$$

$$f_2 (v - V_c) = f_1 (v + V_c)$$

$$f_2 v - f_2 V_c = f_1 v + f_1 V_c$$

$$f_2 V - f_1 v = f_1 V_c + f_2 V_c$$

$$V_c = \frac{f_2 v - f_1 v}{f_1 + f_2} = \frac{v (18 \text{ Hz})}{(600 + 618) \text{ Hz}} = 5.025 \text{ m/s}$$

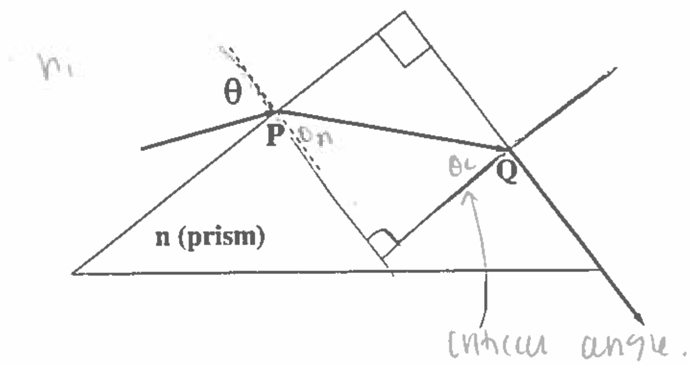
$$\frac{5.025 \text{ m}}{\text{s}} \left| \frac{1 \text{ km}}{1000 \text{ m}} \right| \frac{3600 \text{ s}}{1 \text{ hr}} = \boxed{18.09 \text{ km/hr}}$$

Cindy Cheung
Name

Physics 1CH Final Exam — June 10, 2019

Problem 2: (25 points total)

As shown in the figure below, a ray of light, initially in air, strikes a 90° prism at point P. It refracts there and travels through the prism to refract again at point Q, whereupon it travels along the right-side prism surface.



a) Determine an expression for the index of refraction of the prism, n_{prism}, in terms of the angle of incidence θ. Your expression should not depend on angles other than the angle of incidence. For an angle of incidence of 60°, what must the index of refraction be for the light ray to take this path?

$n_1 \sin\theta = n_2 \sin\theta_n$ $\qquad$ $\theta_n + \theta_c = 90$

$\sin\theta = n_{prism} \sin\theta_n$ $\qquad$ $n_2 = n_{prism}$

$n_{air} = n_1 \approx 1$

critical angle $\qquad$ $\theta_n = 90 - \theta_c$

$n_p \sin\theta_c = n_1 \sin\theta_1 = n_1 = 1$

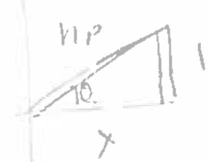
$\theta_c = \sin^{-1}\left(\frac{1}{n_2}\right)$ $\qquad$ $n_p = \frac{1}{\sin\theta_c}$ $\qquad$ $n_{prism} = \frac{\sin\theta_n}{\sin\theta}$

$\theta_n = 90 - \theta_c = 90 - \sin^{-1}\left(\frac{1}{n_p}\right)$

$\sin\theta_n = \sin(90 - \theta_c) = \cos\theta_c$ $\qquad$ $\sin(90° - \theta) = \cos(\theta)$

$\sin\theta = n_{prism} \cos\theta_c$

$\sin\theta = \frac{1}{\tan\theta_c}$ $\qquad$ $\theta_c = \tan^{-1}\left(\frac{1}{\sin\theta}\right)$



$\frac{x}{n_p}$

$\sin\theta_c = \frac{1}{n_p}$

$n_p = \frac{1}{\sin\theta_c} = \boxed{\frac{1}{\sin\left[\tan^{-1}\left(\frac{1}{\sin\theta}\right)\right]}}$

$\theta = 60°$

$\theta_c = 49.1°$

$\sin\theta_c = \frac{1}{n_p}$

$n_p = \frac{1}{\sin\theta_c}$

$= 1.323$

4

Cindy Cheung
Name

Problem 2 (continued)

b) What is the upper bound on the value of the index of refraction, for light to have such a path through the prism?

$$\sin\theta = \frac{1}{\tan\theta_c} = n_{prism}\cos\theta_c$$

$$n_p = \frac{1}{\sin\left[\tan^{-1}\left(\frac{1}{\sin\theta}\right)\right]}$$

$$0 < \theta < 90$$

within these bands for θ n_p maxes at

$$\boxed{n_p = 1.414}$$

Cindy Chung
Name

Physics 1CH   Final Exam
June 10, 2019

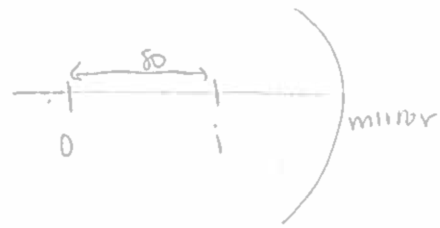
Problem 3: Geometrical and Physics Optics (35 points total)

This problem consists of two separate, and unrelated, problems in optics.

a) A woman uses a concave mirror of radius 1.5 m to examine the makeup on her face. How far from the mirror should her face be for the image to be 80 cm from her face? Please show your work.

$r = -1.5$

$f = \frac{+1.5}{2} = .75$



$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$

$\frac{1}{.80+i} + \frac{1}{i} = \frac{1}{f}$

$o - i = .80\ m$

$o = 80 + i$

$\frac{i}{(.80+i)i} + \frac{.80+i}{i(.80+i)} = \frac{1}{.75}$

$\frac{.80+2i}{i(.80+i)} = \frac{1}{.75}$

$\frac{.80+2i}{.80i+i^2} = \frac{1}{.75} = \frac{4}{3}$

$.8+2i = \frac{4}{3}(.80i+i^2)$

$0 = 1.0667i + \frac{4}{3}i^2 - .8 - 2i$

$= -.933i - .8 + \frac{4}{3}i^2$

$\frac{-b \pm \sqrt{b^2-4ac}}{2a} =$

negative #s don't work

$i = 1.20, \cancel{-.388}$

1.19

6

$o = 80 + i = 80 + 1.2 = \boxed{2\ m}$

Cindy Chang
Name

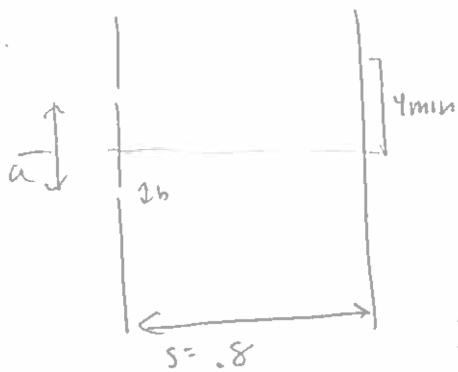
Physics 1CH Final Exam

June 10, 2019

Problem 3 (continued)

b) Two narrow slits, with a spacing of 0.080 mm, are illuminated by light having a wavelength of 550 nm and the resulting pattern is viewed on a screen 80 cm from the slits. The width of the central diffraction maximum on the screen is 4.4 cm. What is the ratio of the irradiance of the 5th interference maximum (not counting the central interference maximum) to the irradiance of the central interference maximum?

$\lambda = 550$ nm $\qquad a = .00008\ m$



$y_{min} = .022\ m \qquad m=1$

$= \dfrac{m \lambda s}{b} \qquad b = 2 \times 10^{-5}\ m$

$I = 4 I_0 \cos^2\left(\dfrac{y a \pi}{s \lambda}\right) \dfrac{\sin^2\beta}{\beta}$

$\dfrac{2\pi}{\lambda}$

$\dfrac{a \pi \sin\theta}{\lambda} = \dfrac{y a \pi}{s \lambda}$

$y_{5th\ int.} = \dfrac{N \lambda s}{a} = \dfrac{5(550 \times 10^{-9} m)(.8 m)}{.00008\ m}$

$= .0275\ m$

$\dfrac{I}{I_0} = 4 \cos^2\left(\dfrac{.0275 m (.00008 m)\pi}{.8 m\ (550 \times 10^{-9} m)}\right) \dfrac{\sin^2\beta}{\beta}$

$= 4 \cdot \dfrac{\sin^2\beta}{\beta^2}$

$\beta = k \dfrac{b}{2} \sin\theta = k \dfrac{b}{2} \dfrac{y_{5th}}{s}$

$= 4 \dfrac{\sin^2(3.927)}{(3.927)^2}$

$= \dfrac{2\pi}{\lambda} \dfrac{b}{2} \dfrac{y_{5th}}{s}$

$= \dfrac{2 \times 10^{-5} m\ \pi}{550 \times 10^{-9} m} \left(\dfrac{.0275 m}{.8}\right)$

$= \boxed{.1297}$

$= 3.927$

Cindy Chang
Name

Physics 1CH Final Exam June 10, 2019

Problem 4: Short Answer (40 points total):

a) You are outside viewing a rainbow that forms a large semi-circular arc in the sky. Does the top of the arc appear red or violet to you? Or, does it depend on your orientation to the rainbow? Explain your answer (with a figure, if you like).

$\boxed{\text{red}}$ because the water acts like a dispersive medium where the index of refraction depends on the wavelength and since violet light has a shorter wavelength it gets refracted more so it will always be at the bottom of the arc.

b) True or False? For circularly polarized light indicident on two linear polarizers whose polarization axes have a relative angle of 60°, the overall transmitted irradiance is one-quarter of the incident irradiance. Explain your answer (numerically).

circularly polarized light acts like natural light since the polarization orientation is constantly changing.

the irradiance after the first polarizer is $I_1 = I_0 \langle \cos^2\theta \rangle$
$= \frac{1}{2} I_0$

second polarizer: $I_2 = I_1 \cos^2\theta = \frac{1}{2} I_0 \cos^2 60$
$= \boxed{\frac{1}{8} I_0}$

Cindy Chung
Name

Physics 1CH Final Exam

June 10, 2019

Problem 4 (continued):

c) True or False? The Michelson-Morley experiment proved that light did not obey Galilean relativity. Explain your answer.

True

It assumed that light traveled through a medium called the aether and that the speed of light would change in response to the relative motion between two objects which is described by Galilean relativistic transformations. However, the resulting interference pattern showed that the speed of light is a universal constant and does not change based on relative motion

d) True or False? In Compton scattering, the kinetic energy gained by the electron depends on the scattering angle of the photon, θ, but not on the wavelength of the incident light.

$$E_\gamma + E_{e^-} = E_\gamma' + E_{e^-}'$$

$$\frac{hc}{\lambda} + \cancel{m_e c^2} = \frac{hc}{\lambda'} + \cancel{m_e c^2} + ke'$$

$$ke' = \frac{hc}{\lambda} - \frac{hc}{\lambda'} = \frac{hc\lambda'}{\lambda\lambda'} - \frac{hc\lambda}{\lambda'\lambda} = \frac{hc(\lambda'-\lambda)}{\lambda\lambda'}$$

$$\lambda'-\lambda = \frac{h}{m_e c}(1-\cos\theta)$$

$$\lambda' = \lambda + \frac{h}{m_e c}(1-\cos\theta)$$

$$ke' = \frac{hc\left[\frac{h}{m_e c}(1-\cos\theta)\right]}{\lambda\left[\lambda + \frac{h}{m_e c}(1-\cos\theta)\right]}$$

$\boxed{\text{False:}}$

It depends on both the scattering angle & the wavelength of the incident light

Cindy Chung
Name

Physics 1CH Final Exam June 10, 2019

Problem 5: Fourier's string (30 points total)

In this problem, consider a string that vibrates in the $x-y$ plane. The string has length L and is held down at $x=0$ and $x=L$. At time $t=0$, the string has the following shape:

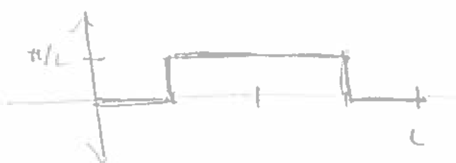
$$y(x)=0,\quad 0\le x\le \frac{L}{4}$$

$$y(x)=\frac{\pi}{2},\quad \frac{L}{4}<x<\frac{3L}{4}$$

$$y(x)=0,\quad \frac{3L}{4}\le x\le L$$

Consider the spatial profile of the string as being described by a Fourier sum of standing wave modes, i.e. a Fourier sine series in wave number $k_n=n\pi/L, n=1,2,3...$

a) Determine the Fourier sine series that describes the spatial profile at time $t=0$ and provide the values of the first three non-zero terms.



$$f(x)=\sum_{n=1}^{\infty} b_n \sin(k_n x) \qquad k_n=\frac{n\pi}{L}$$

$$b_n=\frac{2}{L}\int_0^L f(x)\sin(k_n x)\,dx$$

$$b_n=\frac{\cancel{2}}{L}\int_{L/4}^{3L/4}\frac{\pi}{\cancel{2}}\sin\left(\frac{n\pi}{L}x\right)dx=\frac{\pi}{L}\int_{L/4}^{3L/4}\sin\left(\frac{n\pi}{L}x\right)dx$$

$$=\frac{\cancel{\pi}}{\cancel{L}}\left(\frac{\cancel{L}}{n\cancel{\pi}}\right)\left[-\cos\left(\frac{n\pi}{L}x\right)\right]_{L/4}^{3L/4}$$

$$=\frac{1}{n}\left[-\cos\left(\frac{n\pi}{\cancel{L}}\frac{3\cancel{L}}{4}\right)+\cos\left(\frac{n\pi}{\cancel{L}}\frac{\cancel{L}}{4}\right)\right]=\frac{1}{n}\left[\cos\left(\frac{n\pi}{4}\right)-\cos\left(\frac{3}{4}n\pi\right)\right]$$

$$\boxed{f(x)=\sum_{n=1}^{\infty}\frac{1}{n}\left[\cos\left(\frac{n\pi}{4}\right)-\cos\left(\frac{3}{4}n\pi\right)\right]\sin\left(\frac{n\pi}{L}\right)}$$

$n=1$

$$b_1=\cos\left(\frac{\pi}{4}\right)-\cos\left(\frac{3\pi}{4}\right)$$

$$=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}=\sqrt{2}$$

$n=2$

$$b_2=\frac{1}{2}\left[\cos\frac{\pi}{2}-\cos\left(\frac{3\pi}{2}\right)\right]$$

$$=0$$

$n=3$

$$b_3=\frac{1}{3}\left[\cos\frac{3\pi}{4}-\cos\frac{9}{4}\pi\right]$$

$$=\frac{1}{3}\left[-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}\right]$$

$$=\frac{1}{3}\left[-\frac{2\sqrt{2}}{2}\right]=-\frac{\sqrt{2}}{3}$$

$n=4$

$$b_4=\frac{1}{4}\left(\cos\pi-\cos 3\pi\right)$$

$$=0$$

$$b_5=\frac{1}{5}\left[\cos\frac{5\pi}{4}-\cos\frac{15\pi}{4}\right]$$

$$=\frac{1}{5}\left(-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}\right)=\frac{-2\sqrt{2}}{2(5)}=\frac{-\sqrt{2}}{5}$$

$$b_6=0$$

10

$$\boxed{f(x)\approx\sqrt{2}\sin\left(\frac{\pi}{L}x\right)-\frac{\sqrt{2}}{3}\sin\left(\frac{3\pi}{L}x\right)-\frac{\sqrt{2}}{5}\sin\left(\frac{5\pi}{L}x\right)}$$

Cindy Chang
Name

Problem 5 (continued)

b) Now consider the string evolving in time. Assume that $L = 1$ m and that the speed of propagation of waves on the string is 100 m/s. What is $y(x,t)$? Write out the first three non-zero terms of the series for $y(x,t)$ and substitute for all known quantities.

(Hint: the form of the time dependence of the string will depend on the initial conditions).

since the function isnt 0 at $t=0$

$$\psi(x,t) = \sum_{n=1}^{\infty} b_n \sin(k_n x)\cos(\omega_n t)$$
$$= \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{L}x\right)\cos(\omega_n t)$$

$$\psi(x,t) \approx \sqrt{2}\sin\left(\frac{\pi}{L}x\right)\cos(\omega_1 t) - \frac{\sqrt{2}}{3}\sin\left(\frac{3\pi}{L}x\right)\cos(\omega_3 t) - \frac{\sqrt{2}}{5}\sin\left(\frac{5\pi}{L}x\right)\cos(\omega_5 t)$$

$L=1$

$$\psi(x,t) = \sqrt{2}\sin(\pi x)\cos(\omega_1 t) - \frac{\sqrt{2}}{3}\sin(3\pi x)\cos(\omega_3 t) - \frac{\sqrt{2}}{5}\sin(5\pi x)\cos(\omega_5 t)$$

take time derivative of each term and set equal to speed.

$$\sqrt{2}\,\omega_1 = -\frac{\sqrt{2}}{3}\omega_3 = -\frac{\sqrt{2}}{5}\omega_5 = 100 \text{ m/s}$$

$\omega_1 = 70.71$ $\omega_3 = -212.132$ $\omega_5 = -353.553$

$$\boxed{\psi(x,t) = \sqrt{2}\sin(\pi x)\cos(70.71t) - \frac{\sqrt{2}}{3}\sin(3\pi x)\cos(-212.132t) - \frac{\sqrt{2}}{5}\sin(5\pi x)\cos(-353.553t)}$$

Cindy Chung
Name

Physics 1CH Final Exam

June 10, 2019

Problem 6: Sticky Collision (30 points total)

A particle with a rest mass of 1 MeV/c^2 and a kinetic energy of 2 MeV collides with a stationary particle of rest mass 2 MeV/c^2. After the collision, the particles stick together.

a) Determine the speed of the moving particle before the collision and the initial total momentum of the system. Express your answers in appropriate units (i.e. speed in units of the speed of light and momentum in units of MeV/c).

A B

A B

composite particle.

$E_i = E_f$

$$\frac{1}{\sqrt{1-\beta^2}} = \gamma$$

$$E_{A\,before} = M_A c^2 + 2\,MeV = \gamma m_a c^2$$

$$1\,MeV + 2\,MeV = \gamma (1\,MeV)$$

$$\frac{1}{\gamma} = \sqrt{1-\beta^2}$$

$$\frac{1}{\gamma^2} = 1-\beta^2$$

$$\gamma = 3 \qquad \beta = .943 = \frac{v}{c}$$

$$\beta^2 = 1 - \frac{1}{\gamma^2}$$

$$\boxed{v = 2.83 \times 10^8\ m/s}$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}}$$

$$P_{initial} = \gamma m_A v_A = \gamma m_A \beta c$$

$$= 3\,(1\,MeV/c^2)(.943)(c)$$

$$= \boxed{2.829\ MeV/c}$$

Cindy Cheung
Name

Problem 6 (continued)

b) Determine the speed of the particles and the rest mass of the system after the collision. Again, express your answers in appropriate units.

$M_A c^2 + K_{eA} + M_B c^2 = M_{TOT} c^2 + K_e$

$1 \text{ MeV} + 2 \text{ MeV} + 2 \text{ MeV} = M_{TOT} c^2 + K_e$

$5 \text{ MeV} = M_{TOT} c^2 + K_e$

$K_e + M_{TOT} c^2 = \gamma_{TOT} M_{TOT} c^2 = 5 \text{ MeV}$

$K_e = M_{TOT} c^2 (\gamma_{TOT} - 1)$

$P_{initial} = P_{final}$

$\gamma_A M_A V_A = \gamma_{TOT} M_{TOT} V_{TOT}$

$2.829 = \gamma_{TOT} M_{TOT} V_{TOT}$

$\beta = \frac{v}{c}$

$= \gamma_{TOT} M_{TOT} \beta c$

$= 5 \text{ MeV}/c \ \beta$

$\beta = .5658 = \frac{V_{final}}{c}$ $\boxed{V_{final} = 1.697 \times 10^8 \text{ m/s}}$

$\gamma_{TOT} = \frac{1}{\sqrt{1-\beta^2}} = 1.213$

$\gamma_{TOT} M_{TOT} c^2 = 5 \text{ MeV}$

$\gamma_{TOT} M_{TOT} = 5 \text{ MeV}/c^2$

$\boxed{M_{TOT} = 4.12 \text{ MeV}/c^2}$

Cindy Chang

Name

Physics 1CH Final Exam June 10, 2019

Cindy Chang
Name

Physics 1CH   Final Exam

June 10, 2019

Space for extra work or poetry