Physics 1C: Midterm 2

There are 170 points on the exam, and you have 90 minutes. The exam is closed book and closed notes. The use of any form of electronics is prohibited, except for a basic scientific calculator. To receive full credit, show all your work and reasoning. No credit will be given for answers that simply "appear." If you need extra space, use the backside of the page with a note to help the grader see that the work is continued elsewhere.

Name: Solutions Signature:__________ ID:________

Problem	*Your Score*	*Max Score*
1	_________	*25*
2	_________	*55*
3	_________	*40*
4	_________	*25*
5	_________	*25*
Total	_________	*170*

Fundamental Constants

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T\cdot m/A} \quad \& \quad \varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/(N\cdot m^2)}$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3.00 \times 10^8\ \mathrm{m/s}$$

Electric and Magnetic Force

$$\vec{F}_E = q\vec{E}$$

$$\vec{F}_B = q\vec{v} \times \vec{B} \quad \text{or} \quad \vec{F}_B = \int_c I d\vec{\ell} \times \vec{B}$$

Magnetic Torque

$$\vec{\tau} = \vec{\mu} \times \vec{B} \quad \text{with} \quad \vec{\mu} = \int \hat{n} A dI$$

$$U_B = -\vec{\mu} \cdot \vec{B}$$

Gauss' Law

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\varepsilon_0} \quad \text{or} \quad \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\oint_S \vec{B} \cdot d\vec{A} = 0 \quad \text{or} \quad \vec{\nabla} \cdot \vec{B} = 0$$

Biot-Savart Law

$$\vec{B} = \frac{\mu_0}{4\pi} I \int_c \frac{d\vec{\ell} \times \hat{r}}{r^2}$$

Ampere-Maxwell Law

$$\oint_c \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{encl}} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \quad \text{or} \quad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Faraday's Law and Motional EMF

$$\mathcal{E} = \oint_c \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt} \quad \text{or} \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\mathcal{E} = \int_c \left(\vec{v} \times \vec{B}\right) \cdot d\vec{\ell}$$

Mutual- and Self-Inductance

$$\Phi_{B1} = M_{12} I_2 \quad \& \quad \Phi_{B2} = M_{21} I_1$$

$$\Phi_B = LI$$

$$\mathcal{E}_L = -L\frac{dI}{dt} \quad \& \quad V_L = L\frac{dI}{dt}$$

Energy in an Inductor and Magnetic Energy Density

$$U_B = \frac{1}{2}LI^2$$
$$u_B = \frac{B^2}{2\mu_0}$$

Time Constant and Damping Coefficient in RL Circuit and Oscillation Frequency in LC Circuit

$$\tau = \frac{L}{R} = \frac{1}{2\beta}$$
$$\omega_0^2 = \frac{1}{LC}$$

Complex Numbers

$$\tilde{z} = x + iy = re^{i\phi} \quad \& \quad \tilde{z}^* = x - iy = re^{-i\phi}$$
$$\tilde{z}\tilde{z}^* = |\tilde{z}|^2 = r^2 = x^2 + y^2$$
$$\tan\phi = \frac{\Im\{\tilde{z}\}}{\Re\{\tilde{z}\}} = \frac{y}{x}$$

Complex Ohm's Law, Average Power, Complex Impedances, and Reactances

$$\tilde{V} = \tilde{I}\tilde{Z}$$
$$P = \frac{1}{2}\Re\left\{\tilde{V}\tilde{I}^*\right\} = \frac{1}{2}V_0 I_0 \cos\phi$$
$$\tilde{Z}_R = R$$
$$\tilde{Z}_C = \frac{1}{i\omega C} \quad \& \quad X_C = \frac{1}{\omega C}$$
$$\tilde{Z}_L = i\omega L \quad \& \quad X_L = \omega L$$

Poynting's Theorem and Vector, Intensity, and Radiation Pressure

$$P = \int \frac{d^2W}{dt} = -\frac{d}{dt}\int_{\mathcal{V}} \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{B^2}{\mu_0}\right) dV - \oint_{\mathcal{S}} \vec{S}\cdot\hat{n}\,dA \quad \text{with} \quad \vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$
$$\mathcal{I} = \left\langle \left|\vec{S}\cdot\hat{n}\right| \right\rangle$$
$$\mathcal{P}_{\text{rad}} = \eta\frac{\mathcal{I}}{c} \quad \text{with} \quad \eta = 1 \text{ or } 2$$

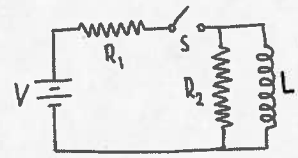
Dispersion Relation

$$v_n = \frac{c}{n} = \nu\frac{\lambda}{n} = \nu\lambda_n$$

Law of Reflection, Snell's Law, and Malus' Law

$$\theta_I = \theta_R$$
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$\mathcal{I} = \mathcal{I}_0 \cos^2\theta$$

1. The switch, S, in the circuit below has been open for a very long time. The DC source has a voltage, V. At $t = 0$, the switch is closed.



(a) What are the currents running through R_1, R_2, and L immediately after the switch is closed AND as $t \to \infty$? Provide qualitative explanations to support your answers. **[10]**

- When the switch is closed, the inductor will choke the current trying to run through it by generating a back emf, so that $I_L(t=0)=0$. As a result, the entire current outputted by the battery will flow through R_1 and R_2. In other words, at $t=0$, the circuit is just a series-resistor circuit with equivalent resistance $R \equiv R_1 + R_2$. So, $I_1(t=0) = I_2(t=0) = V/R$.
- When $t \to \infty$, the inductor no longer sees a changing current so that $V_L = L\, dI_L/dt = 0$. Thus the inductor acts like a short since it has no resistance. Thus, all the current will flow through L when it reaches the junction. Thus, $I_2(t\to\infty) = 0$ and $I_L(t\to\infty) = I_1(t\to\infty) = V/R_1$.

(b) Using Kirchhoff's laws for the instantaneous currents and voltages in this two-loop circuit, find an expression for the current running through the inductor at any time $t \geq 0$. **[15]**

(i) For the loop including V, R_1, and L: $V - I_1 R_1 - L\frac{dI_L}{dt} = 0. \Rightarrow I_1 = \frac{1}{R_1}\left(V - L\frac{dI_L}{dt}\right)$.

(ii) For the loop including R_2 and L: $L\frac{dI_L}{dt} - I_2 R_2 = 0$.

(iii) The junction rule says: $I_1 = I_2 + I_L$.

Defined: $\frac{1}{R'} \equiv \frac{1}{R_1} + \frac{1}{R_2}$

→ Plugging (iii) into (ii) for $I_2 \Rightarrow L\frac{dI_L}{dt} - (I_1 - I_L)R_2 = 0$.

→ Solving for I_1 in (i) and plugging into this

$$\Rightarrow L\frac{dI_L}{dt} - \left[\frac{1}{R_1}\left(V - L\frac{dI_L}{dt}\right) - I_L\right]R_2 = 0.$$

$$L\frac{dI_L}{dt} + R_2 I_L + \frac{R_2}{R_1}L\frac{dI_L}{dt} - \frac{R_2}{R_1}V = 0.$$

$$L\left(1 + \frac{R_2}{R_1}\right)\frac{dI_L}{dt} = -R_2\left(I_L - \frac{V}{R_1}\right)$$

$$\frac{dI_L}{I_L - V/R_1} = -\frac{R_2}{L\left(1 + \frac{R_2}{R_1}\right)}dt$$

$$\Rightarrow \int_0^{I_L}\frac{dI_L'}{I_L' - V/R_1} = -\frac{R_2 R_1}{L(R_1 + R_2)}\int_0^t dt'$$

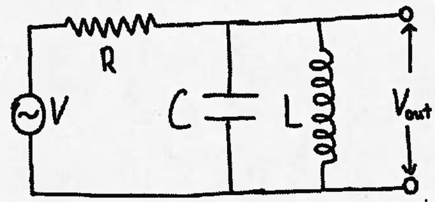
$$\ln\left(\frac{I_L - V/R_1}{-V/R_1}\right) = -\frac{R_2 R_1}{L(R_1+R_2)}t = -\frac{t}{L\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} \equiv -\frac{t}{L/R'}$$

$$I_L - V/R_1 = -\frac{V}{R_1}e^{-R't/L}$$

$$\Rightarrow I_L(t) = \frac{V}{R_1}\left(1 - e^{-t/\tau'}\right)$$

with $\tau' \equiv \frac{L}{R'} = L\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$.

2. Consider the AC circuit shown below. The AC source outputs a voltage $V(t) = V_0 \cos(\omega t)$, which is, of course, the real part of its complex counterpart $\tilde{V}(t) = V_0 e^{i\omega t}$. The inductor has inductance L, the capacitor has capacitance C, and the resistor has resistance R. This circuit's output, V_{out} (indicated by the terminals) will be fed to another circuit. Currently, there is no load hooked up to the output, so that the load has infinite resistance.



(a) Find the equivalent impedance (i.e., magnitude of the equivalent complex impedance) for this circuit. **[10]**

L & C are in parallel: $\frac{1}{\tilde{Z}_{LC}} = \frac{1}{\tilde{Z}_L} + \frac{1}{\tilde{Z}_C} = \frac{1}{i\omega L} + \frac{1}{1/i\omega C} = -\frac{i}{\omega L} + i\omega C = i\left(\omega C - \frac{1}{\omega L}\right)$

$$\Rightarrow \tilde{Z}_{LC} = \frac{1}{i\left(\omega C - \frac{1}{\omega L}\right)} = \frac{i}{\frac{1}{\omega L} - \omega C} = \frac{i\omega L}{1-\omega^2 LC}.$$

LC & R are in series: $\tilde{Z} = \tilde{Z}_{LC} + \tilde{Z}_R = i\frac{\omega L}{1-\omega^2 LC} + R = R + i\frac{\omega L}{1-\omega^2 LC}$

$$\Rightarrow |\tilde{Z}| = \sqrt{\tilde{Z}\tilde{Z}^*} = \sqrt{R^2 + \left(\frac{\omega L}{1-\omega^2 LC}\right)^2}.$$

(b) Find the current magnitude and the phase (relative to the voltage outputted by the source) for the equivalent circuit. **[8]**

Write: $\tilde{Z} = |\tilde{Z}| e^{i\phi}$, with $|\tilde{Z}| = \sqrt{R^2 + \left(\frac{\omega L}{1-\omega^2 LC}\right)^2}$ and $\phi = \tan^{-1}\left(\frac{\operatorname{Im}\{\tilde{Z}\}}{\operatorname{Re}\{\tilde{Z}\}}\right) = \tan^{-1}\left(\frac{\omega L/R}{1-\omega^2 LC}\right)$

So: $\tilde{I} = \frac{\tilde{V}}{\tilde{Z}} = \frac{V_0 e^{i\omega t}}{|\tilde{Z}| e^{i\phi}} = \frac{V_0}{|\tilde{Z}|} e^{i(\omega t - \phi)}.$

Thus, the magnitude is: $|\tilde{I}| = \frac{V_0}{|\tilde{Z}|} = \frac{V_0}{\sqrt{R^2 + \left(\frac{\omega L}{1-\omega^2 LC}\right)^2}}$

And phase is: $\phi = \tan^{-1}\left(\frac{\omega L/R}{1-\omega^2 LC}\right).$

(c) **Find the voltage drops across, AND the currents running through, the capacitor and the inductor. [20]**

From Kirchhoff's loop rule, for the loop containing V, R, and C, we have: $\tilde{V} = \tilde{V}_R + \tilde{V}_C$

" " " " C and L, " " : $\tilde{V}_L = \tilde{V}_C$

Thus:
$$\tilde{V}_C = \tilde{V} - \tilde{V}_R = \tilde{I}\tilde{Z} - \tilde{I}\tilde{Z}_R$$
$$= \tilde{I}(\tilde{Z} - \tilde{Z}_R) = \tilde{I}\tilde{Z}_{LC} = \frac{V_0}{|\tilde{Z}|} e^{i(\omega t - \phi)} \frac{i\omega L}{1-\omega^2 LC}$$
$$= \frac{V_0}{\sqrt{R^2 + \frac{\omega^2 L^2}{(1-\omega^2 LC)^2}}} \frac{1}{\sqrt{\left(\frac{1-\omega^2 LC}{\omega L}\right)^2}} e^{i(\omega t - \phi)} e^{i\pi/2}$$
$$= \frac{V_0}{\sqrt{\left(\frac{R}{\omega L}\right)^2 (1-\omega^2 LC)^2 + 1}} e^{i(\omega t - (\phi - \pi/2))}$$

$$V_L = V_C = \mathrm{Re}\{\tilde{V}_C\} = \frac{V_0}{\sqrt{\left(\frac{R}{\omega L}\right)^2 (1-\omega^2 LC)^2 + 1}} \cos(\omega t - \phi + \pi/2) = -\frac{V_0}{\sqrt{\left(\frac{R}{\omega L}\right)^2 (1-\omega^2 LC)^2 + 1}} \sin(\omega t - \phi)$$

For the current:

- $$\tilde{I}_C = \frac{\tilde{V}_C}{\tilde{Z}_C} = \frac{V_0}{\sqrt{\left(\frac{R}{\omega L}\right)^2 (1-\omega^2 LC)^2 + 1}} i\omega C e^{i[\omega t - (\phi - \pi/2)]} = \frac{\omega C V_0}{\sqrt{\left(\frac{R}{\omega L}\right)^2 (1-\omega^2 LC)^2 + 1}} e^{i\pi/2} e^{i[\omega t - (\phi - \pi/2)]}$$

$$\Rightarrow \tilde{I}_C = \frac{\omega C V_0}{\sqrt{\left(\frac{R}{\omega L}\right)^2 (1-\omega^2 LC)^2 + 1}} e^{i(\omega t - (\phi - \pi))} \Rightarrow I_C = \frac{\omega C V_0}{\sqrt{\left(\frac{R}{\omega L}\right)^2 (1-\omega^2 LC)^2 + 1}} \cos[\omega t - \phi + \pi]$$
$$= -\frac{\omega C V_0}{\sqrt{\left(\frac{R}{\omega L}\right)^2 (1-\omega^2 LC)^2 + 1}} \cos(\omega t - \phi).$$

- $$\tilde{I}_L = \frac{\tilde{V}_L}{\tilde{Z}_L} = \frac{V_0}{\sqrt{\left(\frac{R}{\omega L}\right)^2 (1-\omega^2 LC)^2 + 1}} \left(\frac{-i}{\omega L}\right) e^{i[\omega t - (\phi - \pi/2)]} = \frac{V_0/\omega L}{\sqrt{\left(\frac{R}{\omega L}\right)^2 (1-\omega^2 LC)^2 + 1}} e^{i(\omega t - \phi + \pi/2)} e^{-i\pi/2}$$

$$\Rightarrow \tilde{I}_L = \frac{V_0/\omega L}{\sqrt{\left(\frac{R}{\omega L}\right)^2 (1-\omega^2 LC)^2 + 1}} e^{i(\omega t - \phi)} \Rightarrow I_C = \frac{V_0/\omega L}{\sqrt{\left(\frac{R}{\omega L}\right)^2 (1-\omega^2 LC)^2 + 1}} \cos(\omega t - \phi).$$

(d) Now, consider the voltage drop across the output of the circuit, which has the form

$$V_{out}(t) = V_{out}\cos(\omega t + \delta) = \Re\left\{V_{out}e^{i(\omega t+\delta)}\right\},$$

where $V_{out} = V_{out}(\omega)$ and δ is some phase determined from your analysis. The power will be proportional to $V_{out}^2(\omega)$. We will treat the angular frequency of the circuit, ω, as a free parameter.

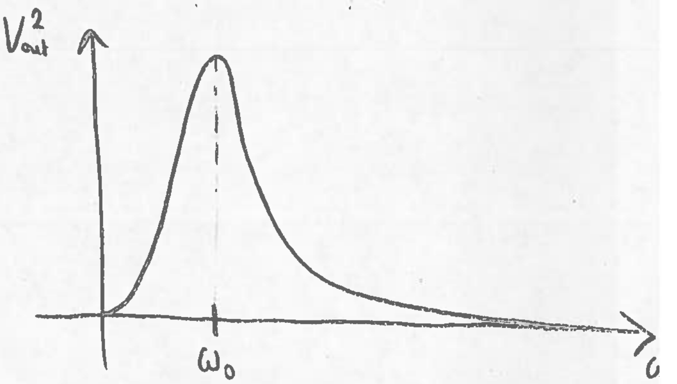
i. Using your analysis from the previous parts, write V_{out}^2 in terms of ω, V_0, the RL time constant, $\tau \equiv L/R$, and the LC natural angular frequency, $\omega_0^2 \equiv 1/(LC)$. Also, sketch V_{out}^2 as a function of ω. **[12]**

We have $\tilde{V}_{out}(t) = \tilde{V}_c(t) = \dfrac{V_0}{\sqrt{\left(\frac{R}{\omega L}\right)^2(1-\omega^2 LC)^2+1}}\, e^{i(\omega t-(\phi-\pi/2))}$

$$\Rightarrow V_{out}(\omega) = \frac{V_0}{\sqrt{\left(\frac{R}{\omega L}\right)^2(1-\omega^2LC)^2+1}} \Rightarrow V_{out}^2 = \frac{V_0^2}{\left(\frac{R}{\omega L}\right)^2(1-\omega^2LC)^2+1}$$

Substituting, we have: $V_{out}^2 = \dfrac{V_0^2}{\frac{1}{\tau^2\omega^2}\left(1-\frac{\omega^2}{\omega_0^2}\right)^2+1} = V_0^2\dfrac{\tau^2\omega^2}{\left(1-\frac{\omega^2}{\omega_0^2}\right)^2+\tau^2\omega^2}$.

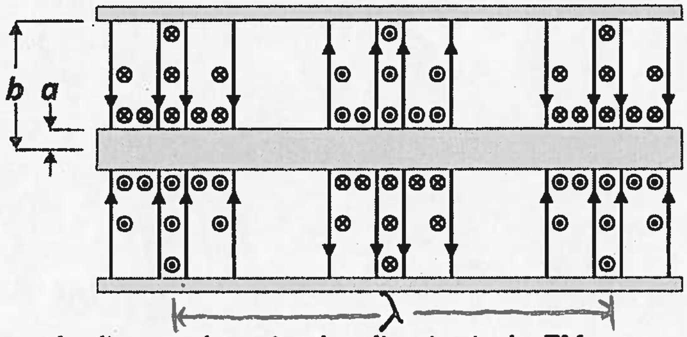
This is a like a Lorentzian, as it has a peak value with all other values being significantly smaller. The peak frequency is the natural LC oscillation frequency, and the peak's sharpness depends on the amount of damping: $\beta = \frac{R}{2L} = \frac{1}{2\tau}$. The smaller β, the sharper the resonance.



ii. This circuit is a type of band-pass filter when considering the outputted signal. Provide an explanation as to what the meaning of such a filter is based on your analysis. **[5]**

Note that the outputted power is maximal at ω_0, so that if the load was some resistor, it would see a maximal current at ω_0. This is because the LC impedance at resonance is large, which is why a lot more current would be outputted to the load. It acts as a band-pass filter since it would let in huge currents in the neighborhood of the resonance, so that the load circuit could only be fed with a narrow band of frequencies if desired. The narrowness of the band will again depend on how sharp the peak of the power profile is. Thus, this filter acts as a combination high- and low-pass filter.

3. A sinusoidal electromagnetic (EM) wave is traveling in a coaxial waveguide consisting of an inner wire surrounded by an outer shell. The radius of the inner conductor is a and the radius of the outer conductor is b. In the cross-sectional view of the coaxial waveguide along its length below, the radial electric field lines and the azimuthal magnetic field lines are shown at one instant of time.



(a) With respect to the diagram above, in what direction is the EM wave traveling AND how can you determine the wavelength? Defend your answers. **[8]**

- The direction must be along $\vec{E}\times\vec{B}$, which according to the diagram would be in the rightward direction upon using the right-hand rule for cross products at any of the field regions.
- The wavelength can be determined from looking at, for instance, successive identical regions of field strength and direction (i.e., regions of identical phase). The extent of the wavelength is shown above.

(b) The electric and magnetic fields in the region between the conductors have the form

$$E = \frac{\mathcal{E}_0}{r}\cos(kz - \omega t)$$

$$B = \frac{\mathcal{B}_0}{r}\cos(kz - \omega t)$$

where $\mathcal{E}_0$ and $\mathcal{B}_0$ are constants and r is the radial distance from the axis of the wire ($a < r < b$).

i. What is the relationship between $\mathcal{E}_0$, $\mathcal{B}_0$, k, and ω? **[5]**

We must have $\frac{E}{B} = c \Rightarrow \frac{\mathcal{E}_0}{\mathcal{B}_0} = c$

However, $c = \nu\lambda = \frac{\omega}{2\pi}\frac{2\pi}{k} = \frac{\omega}{k}$

Thus: $\frac{\mathcal{E}_0}{\mathcal{B}_0} = \frac{\omega}{k}$.

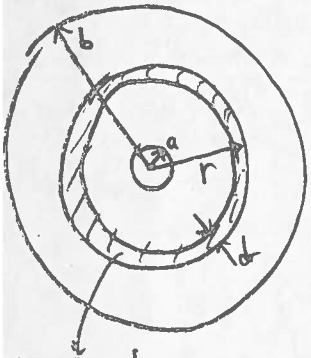
ii. Find both the instantaneous and average power that comes out of the coaxial waveguide at its far end, where $z = L$. **[20]**

The wave travels along $+z$. So, the Poynting vector at some radial position at $x = L$ will be:

$$\vec{S} = \hat{z}\left|\frac{\vec{E}\times\vec{B}}{\mu_0}\right| = \hat{z}\,\frac{\mathcal{E}_0 B_0}{\mu_0}\,\frac{1}{r^2}\cos^2(kL-\omega t)$$

The wave comes out in a plane perpendicular to the axis. However, we must integrate $\vec{S}\cdot\hat{n} = \vec{S}\cdot\hat{z}$ over the field region to find the total power outputted.

So:



$dA = 2\pi r\,dr.$

$$P(t) = \int_S \vec{S}\cdot\hat{n}\,dA = \frac{\mathcal{E}_0 B_0}{\mu_0}\cos^2(kL-\omega t)\int_a^b \frac{1}{r^2}\,2\pi r\,dr = \frac{\mathcal{E}_0 B_0}{\mu_0}\cos^2(kL-\omega t)\,2\pi \ln(r)\Big|_{r=a}^{r=b}$$

$$\Rightarrow \quad P(t) = \frac{2\pi\,\mathcal{E}_0 B_0}{\mu_0}\ln\left(\frac{b}{a}\right)\cos^2(kL-\omega t).$$

To find the average power, we must time average:

$$\langle P\rangle = \frac{2\pi\,\mathcal{E}_0 B_0}{\mu_0}\ln\left(\frac{b}{a}\right)\langle\cos^2(kL-\omega t)\rangle = \frac{2\pi\,\mathcal{E}_0 B_0}{\mu_0}\ln\left(\frac{b}{a}\right)\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\frac{1}{2}\left[1+\cos(2(kL-\omega t))\right]dt$$

$$\Rightarrow \quad \langle P\rangle = \frac{\pi\,\mathcal{E}_0 B_0}{\mu_0}\ln\left(\frac{b}{a}\right) \quad \text{since } \langle\cos^2(t)\rangle_t = \frac{1}{2}.$$

iii. Suppose that at the far end of the waveguide, the wave is absorbed by a perfect absorber. What is the average force exerted by the wave on the absorber? **[7]**

Since this is an absorber, we have that $P_{rad} = \frac{I}{c}$, where I is the intensity. The intensity is the power per unit area, and the radiation pressure is the radiation force per unit area. So:

$$F_{rad} = P_{rad}\,a = \frac{I\,a}{c} = \frac{\langle P\rangle}{c}$$

$$\Rightarrow \quad F_{rad} = \frac{\pi\,\mathcal{E}_0 B_0 \ln\left(\frac{b}{a}\right)}{\mu_0 c}.$$

4. A small, underwater pool light, which emits white light, is located a distance h below the surface of the pool and is very far away from the edges of the pool. The index of refraction of the water is n and the index of refraction of the air above is $n_0 = 1 < n$.

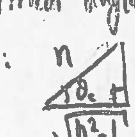
(a) Qualitatively explain why the emitted light would form a bright circle at the surface of the water, as viewed from outside the water, AND obtain an expression for the radius of this circle. **[13]**

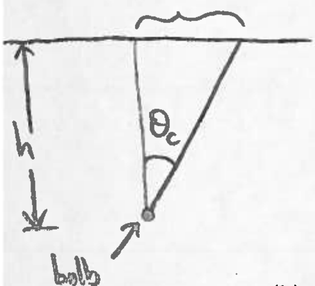
The light from the bulb will emit in all directions radially away from the bulb. Thus, the incident light at the surface from the water will range in all angles, with right above the bulb having an angle $\theta = 0$ relative to the normal, and infinitely far away horizontally having an angle $\theta = \pi/2$. However, since the light is going from water (high index) to air (low index), it can undergo total internal reflection (TIR). The critical angle is given by:

$h/\tan\theta_c \equiv R$ (radius of circle)

$$\sin\theta_c = \frac{n_0}{n} = \frac{1}{n}$$

$\Rightarrow$ This defines a triangle:





For $\theta > \theta_c$, there will be no transmission. For $\theta < \theta_c$, there will be transmission (partial). By circular symmetry, the critical angle traces out a cone, whose cross-section is the circle of light. The radius is:

$$R = \frac{h}{\tan\theta_c} = h\,\frac{\cos\theta_c}{\sin\theta_c} = h\,\frac{\sqrt{n^2-1}/n}{1/n} = \sqrt{n^2-1}\,h.$$

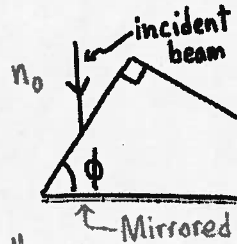
(b) If you were able to view from the perspective of the small light, what would you see when viewing towards the surface of the water? Explain your reasoning. **[5]**

Since light rays will partially transmit out if $\theta < \theta_c$, then you will be able to see the world beyond the water surface only if your viewing angle relative to the normal is less than θ_c (i.e., if you view through the bright circle). For any $\theta > \theta_c$, the light rays can't escape, which means you can't see outside for $\theta > \theta_c$. However, at these angles, you will see a reflection of the pool's surface since light is scattering off of objects from the bulb.

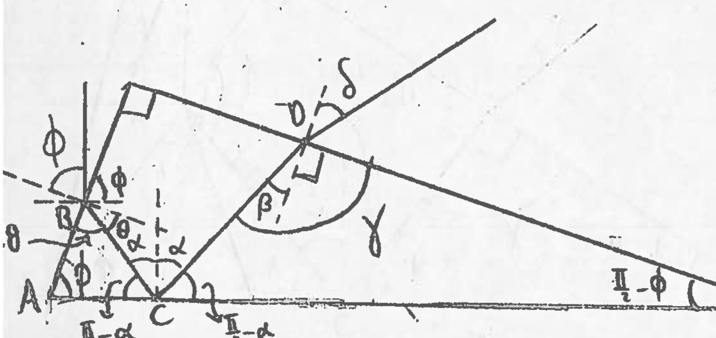
(c) Now, factor in the dispersion of white light to describe the colors seen at the edge of the bright circle when viewed from outside the water. Explain the reasoning behind your description. **[7]**

We know that the critical angle is smaller for a higher index of refraction. Since the index of red is smaller than that for violet, then violet light will have a smaller critical angle than red. This means that violet will undergo TIR first if we sweep the angle of incidence from $0 \to \pi/2$. Thus, if violet is extinguished from the transmitted light first in the angle sweep, then the light will look more bluish/greenish on the inner edge (as viewed from the air) since there is an absence of violet. As more colors undergo TIR, the outgoing beam will look more reddish towards the edge of the bright circle. Beyond that, all colors undergo TIR, so that there is darkness beyond that edge.

5. A light-ray traveling in air, with refractive index $n_0 = 1.00$, is incident on one face of a right-angled prism, with refractive index $n = 1.50$. The ray is incident vertically on the left-hand leg of the prism, as shown below. The longest side of the prism (i.e., the hypotenuse) is mirrored. The angle between the left-hand leg and the mirrored surface is labeled ϕ. Assuming $\phi = 60.0°$, determine the angle (relative to the normal) made by the outgoing ray from the right-hand leg of the prism. **[25]**



Draw a picture to decipher all angles.



- From ΔABC:

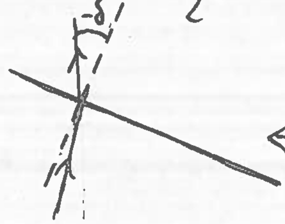
$$\pi = \phi + \frac{\pi}{2} - \theta + \frac{\pi}{2} - \alpha.$$
$$\pi = \pi + \phi - \theta - \alpha.$$
$$\Rightarrow \alpha = \phi - \theta$$

- From ΔCDE (with one leg defined by the reflected ray):

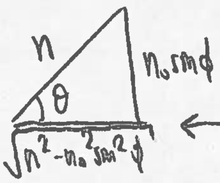
$$\pi = \frac{\pi}{2} - \alpha + \frac{\pi}{2} - \phi + \gamma$$
$$\pi = \pi - \alpha - \phi + \gamma$$
$$\Rightarrow \gamma = \alpha + \phi = \phi - \theta + \phi = 2\phi - \theta.$$

- The incident ray on the right leg makes an angle β, which is given by:

$$\beta = \gamma - \frac{\pi}{2} = 2\phi - \theta - \frac{\pi}{2}.$$



From Snell's Law applied to the left leg:



$$n_0 \sin\phi = n \sin\theta$$
$$\Rightarrow \sin\theta = \frac{n_0 \sin\phi}{n} \Rightarrow \cos\theta = \frac{\sqrt{n^2 - n_0^2 \sin^2\phi}}{n}$$

From Snell's Law applied to the right leg:

$$n \sin\beta = n_0 \sin\delta$$

We want δ. So:

$$\sin\delta = \frac{n}{n_0}\sin\beta = \frac{n}{n_0}\sin\left(2\phi - \theta - \frac{\pi}{2}\right)$$
$$\sin\delta = \frac{n}{n_0}\left[\sin(2\phi-\theta)\cos\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{2}\right)\cos(2\phi-\theta)\right]$$
$$\sin\delta = -\frac{n}{n_0}\cos(2\phi-\theta)$$
$$= -\frac{n}{n_0}\left[\cos(2\phi)\cos\theta + \sin(2\phi)\sin\theta\right]$$
$$= -\frac{n}{n_0}\frac{1}{n}\left[\sqrt{n^2 - n_0^2\sin^2\phi}\,\cos(2\phi) + n_0 \sin\phi \sin(2$$
$$\Rightarrow \delta = \sin^{-1}\left[-\frac{\sqrt{n^2 - n_0^2\sin^2\phi}}{n_0}\cos(2\phi) - \sin\phi\sin(2\phi)\right].$$
$$\delta = -7.91°$$

$\Rightarrow$ Just means it exits on the other side of the normal, so that α is smaller than portrayed, and θ is bigger than portrayed.