Physics 1C **Name:** ______________________
Spring 2017
Mid-term Exam 2
May 23, 2017
Time Limit: 90 Minutes ID number: ______________________

Exam Notes:

- This is a closed books, closed notes exam. **No cheat sheets, please!**
- Show all work, clearly and in order. **Circle or otherwise indicate your final answers.**
- Make sure to **include units** in your answers, when numerical values are given.
- Always take a few moments to **double-check that your responses make sense.**
- Good luck!

Grade Table (for grader use only)

Part	Points	Score
A	16	
B	14	
C	12	
D	14	
E	14	

Potentially useful equations and constants:

Maxwell's equations:

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q}{\epsilon_0} \ , \quad \oint \vec{B}\cdot d\vec{A} = 0 \ , \quad \oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt} \ , \quad \oint \vec{B}\cdot d\vec{l} = \mu_0 I + \mu_0\epsilon_0\frac{d\Phi_E}{dt}$$

Energy in capacitor: $\frac{Q^2}{2C}$, Energy in inductor: $\frac{1}{2}LI^2$, *Inductance*: $L = \frac{\Phi_B}{i}$

L-R and L-C circuit in series: $\tau = \frac{L}{R}$, $\omega_0 = \sqrt{\frac{1}{LC}}$

L-R-C circuit in series: $Z = \sqrt{R^2 + (X_L - X_C)^2}$, $\tan\phi = \frac{X_L - X_C}{R}$

Speed of light in vacuum: $c = \frac{1}{\sqrt{\epsilon_0\mu_0}} = 3\times 10^8$ m/s

$$\epsilon_0 = 8.85\times 10^{-12}\ \mathrm{C^2/N\cdot m^2}\ ,\ \mu_0 = 4\pi\times 10^{-7}\ \mathrm{T\cdot m/A}$$

Poynting vector and *Intensity*: $\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$, $I = S_{avg}$

Part A

1. (2 points) Starting from zero, an electric current is established in a circuit made of a battery of emf $\mathcal{E}$, a resistor of resistance R and an inductor of inductance L. The electric current eventually reaches its steady-state value. What would be the effect of using an inductor with a larger inductance in this circuit?

   A. The steady-state value of the current would be larger, but it would take the same amount of time to reach it.

   B. The steady-state value of the current would be the same, but it would take more time to reach it.

   C. The steady-state value of the current would be the same, but it would take the same amount of time to reach it.

   D. The steady-state value of the current would be larger, but it would take more time to reach it.

   E. The steady-state value of the current would be the same, but it would take less time to reach it.

2. (2 points) A capacitor, initially charged, and an inductor form an $L - C$ circuit. When the electric current in the circuit is equal to zero how much energy is stored on each device?

   A. Inductor: zero energy. Capacitor: zero energy

   B. Inductor: Half of the maximum energy. Capacitor: Half of the maximum energy

   C. Inductor: maximum energy. Capacitor: zero energy

   D. Inductor: zero energy. Capacitor: maximum energy

   E. Inductor: maximum energy. Capacitor: maximum energy

3. (2 points) The average energy dissipated in an inductor, when part of a series $R-L-C$ circuit, is

   A. zero.

   B. proportional to the inductance.

   C. proportional to both the inductance and frequency.

   D. proportional to the frequency.

   E. none of the above.

4. (2 points) A 12V DC battery is connected to the *primary* coil of a *transformer*. If the *primary* coil consists of 5 turns, the voltage across the *secondary* coil with 10 turns is:

   A. greater than 12V
   B. equal to 12V
   C. less than 12V
   D. zero

5. (2 points) An electric field is created by a(n) __________ , and an electromagnetic wave is generated by a(n) __________ .

   A. static charge / constant current
   B. static charge / constant magnetic field
   C. changing magnetic field / static charge
   D. static charge / accelerating charge
   E. constant magnet / accelerating charge

6. (2 points) Which one of the following lists is a correct representation of electromagnetic waves from longer wavelength to shorter wavelength?

   A. radio waves, infrared, microwaves, UV, visible, X-rays, gamma rays
   B. radio waves, UV, X-rays, microwaves, infrared, visible, gamma rays
   C. radio waves, microwaves, visible, X-rays, infrared, UV, gamma rays
   D. radio waves, microwaves, infrared, visible, UV, X-rays, gamma rays
   E. radio waves, infrared, X-rays, microwaves, UV, visible, gamma rays

7. (2 points) When light goes from one material into another material having a higher index of refraction,

   A. its speed, wavelength, and frequency all decrease.
   B. its speed and wavelength decrease, but its frequency stays the same.
   C. its speed decreases but its wavelength and frequency both increase.
   D. its speed decreases but its frequency and wavelength stay the same.
   E. its speed increases, its wavelength decreases, and its frequency stays the same.

8. (2 points) A ray of light traveling in a material of index of refraction n_1 strikes the boundary with another transparent material of index of refraction n_2. If there is no transmitted ray and all of the light intensity is reflected, what can you conclude about the indices of refraction of these two materials?

   A. $n_1 \geq n_2$
   B. $n_1 > n_2$
   C. $n_1 = n_2$
   D. $n_1 \leq n_2$
   E. $n_1 < n_2$

Part B

1. (6 points) At $t = 0$, let $Q = Q_0$, and $I = 0$ in an LC circuit composed of only an inductor and a capacitor.

   (a) (3 points) At the first moment when the energy is shared equally by the inductor and the capacitor, what is the charge on the capacitor?

   Conservation of Energy: $\Delta E = 0 \Rightarrow E_{C0} = E_C + E_L$,
   but $E_C = E_L$, so $E_{C0} = 2E_C \Rightarrow \frac{Q_0^2}{2C} = 2\frac{Q^2}{2C} \Rightarrow Q = \frac{\sqrt{2}}{2}Q_0$

   (b) (3 points) How much time has elapsed in terms of the period T?

   Given the initial conditions, $Q(t) = Q_0 \cos \omega t$. So, when $Q(t) = \frac{\sqrt{2}}{2}Q_0 \Rightarrow \cos \omega t = \frac{\sqrt{2}}{2} \Rightarrow t = T/8$ (remember, $\omega = 2\pi/T$).

2. (8 points) In the figure below, $\mathcal{E} = 1$ V, $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, $R_3 = 30\ \Omega$, and $L = 2$ H. Find the values of i_1, i_2

   (a) (2 points) immediately after switch S is closed,

   Immediately after switch S is closed, there is no current through the inductor, so $i_3 = 0$. Then, $i_1 = i_2 = \frac{\mathcal{E}}{R_1+R_2} = 33.3$ mA.

   (b) (2 points) a long time later,

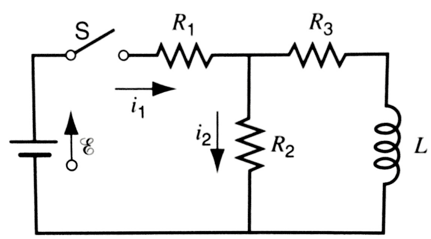
   Current through inductor is now constant, so there is no voltage drop across it. So, effectively we have R_2, R_3 in parallel with R_1 in series and
   $R_{total} = R_1 + (1/R_2 + 1/R_3)^{-1} = 22\Omega$.
   So, $i_1 = \frac{\mathcal{E}}{R_{total}} = 45.4$ mA, and $i_2 = \frac{V_2}{R_2} = \frac{\mathcal{E}-i_1R_1}{R_2} = 27.3$ mA.

   (c) (2 points) immediately after switch S is opened again,

   $i_1 = 0$ and i_2 is finite due to the inductor's emf, so just after the switch is opened $i_2 = i_3^0 = 45.4 - 27.3 = 18.1$ mA

   (d) (2 points) a long time later.

   $i_1 = i_2 = 0$



Part C

1. (12 points) In a certain RLC circuit, operating at 60 Hz, the maximum voltage across the inductor is twice the maximum voltage across the resistor, while the maximum voltage across the capacitor is the same as the maximum voltage across the resistor.

   (a) (4 points) What is the phase relationship between the voltage across the RLC circuit and the current running through it?

   It is given that $X_L = 2R, X_C = R$. So, $\tan\phi = \frac{X_L - X_C}{R} = \frac{2R-R}{R} = 1 \Rightarrow \phi = \pi/4$

   (b) (4 points) If the AC source is actually a US wall power plug ($V_{rms} = 120$ V), what should be the circuit's resistance in order to obtain a maximum current of 300 mA?

   The impedance of the circuit is $Z = \sqrt{R^2 + (X_L - X_C)^2} \Rightarrow Z = \sqrt{2}R$,
   and $Z = \frac{V_0}{I_0} \Rightarrow \sqrt{2}R = \frac{\sqrt{2}V_{rms}}{I_0} \Rightarrow R = 400\Omega$.

2. (4 points) In a certain RLC circuit, when the AC voltage source has a particular angular frequency ω, the peak voltage across the inductor is 6 times greater that the peak voltage across the capacitor. Determine ω in terms of the resonant frequency ω_0 of this circuit.

   $V_{L0} = 6V_{C0} \Rightarrow X_L I_0 = 6X_C I_0 \Rightarrow \omega^2 = 6\frac{1}{LC} \Rightarrow \omega = \sqrt{6}\omega_0$

Part D

1. (14 points) He-Ne lasers are very commonly used in physics demonstrations. They produce light of wavelength ∼630 nm with typical electric field amplitude of ∼1 kV/m.

   (a) (2 points) What is the frequency of this laser beam?
   $c = \lambda f \Rightarrow f = c/\lambda \simeq 476$ THz

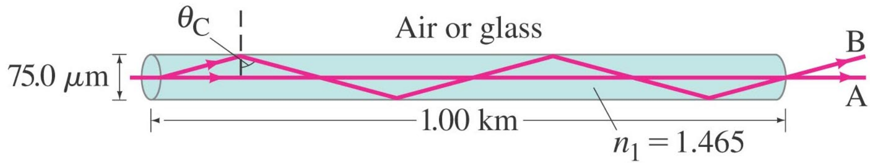
   (b) (3 points) If $E = E_{max} \sin(kx + \omega t)$ and $\vec{E} \parallel \hat{y}$, what are the values of k and ω?
   $k = \frac{2\pi}{\lambda} \simeq 10^7 \text{m}^{-1}$, and $\omega = 2\pi f \simeq 300 \times 10^{13}$ rad/s.

   (c) (4 points) Write the vector equation for $\vec{B}(\vec{r}, t)$, the magnetic field of the laser beam.
   $B_{max} = E_{max}/c = 3.33\mu\text{T}$ and $\vec{B} \parallel -\hat{k}$ (remember, $\vec{S} \parallel \vec{E} \times \vec{B}$). So:
   $\vec{B}(x, t) = -3.33\mu\text{T} \sin\left[(10^7\text{m}^{-1})\,\text{x} + (300 \times 10^{13}\text{rad/s})\,\text{t}\right]\hat{\text{k}}$

   (d) (2 points) Does the laser beam transport any energy? If yes, in what direction?
   Yes, in the direction of wave propagation, -x. (remember, $\vec{S} \parallel \vec{E} \times \vec{B}$)

   (e) (3 points) What is the intensity of this laser beam?
   $I = S_{avg} = \frac{E_0 B_0}{2\mu_0} = 1325 \text{ W/m}^2$

Part E



1. (14 points) Two light rays A and B of identical lasers travel down a cylindrical optical fiber of diameter $d = 75\mu$m, length $\ell = 1$km, and index of refraction $n_1 = 1.465$. Ray A travels a straight path down the fiber's axis, whereas ray B propagates down the fiber by repeated reflections at the critical angle each time it impinges on the fiber's boundary.

   (a) (2 points) What is the speed of light in the optical fiber?

   $v = c/n_1 \simeq 2.05 \times 10^8$ m/s

   (b) (2 points) If the light has a wavelength of 440 nm in the optical fiber, what is the actual wavelength of our laser (measured in air)?

   $\lambda_0 = n_1\lambda \simeq 645$ nm

   (c) (6 points) Determine the extra time it takes for ray B to travel down the entire fiber in comparison with ray A, when the fiber is surrounded by air.

   The critical angle for total internal reflection is $\sin\theta_c = n_2/n_1$.
   And the extra time traveled by ray B is $\Delta t = t_B - t_A = \frac{\ell_B}{v} - \frac{\ell_A}{v}$ (1).
   But ray B is traveling in the optical fiber at an angle θ_c compared to ray A at all times, so $\ell_B = \frac{\ell_A}{\sin\theta_c}$ (2).
   Combining (1), (2) and keeping in mind that $v = c/n_1$, we get:
   $\Delta t = \frac{\ell}{c} n_1 \left(\frac{n_1}{n_2} - 1\right) \simeq 2.3 \times 10^{-6}$ s.

   (d) (2 points) If, instead, the optical fiber is surrounded by a cylindrical glass "cladding" ($n_2 = 1.460$), how do you expect the time calculated in (c) to change? Explain.

   For glass, $n_2 = 1.46 > n_{air} = 1$, so the critical angle ($\sin\theta_c = n_2/n_1$) will become larger which means Δt decreases.

   (e) (2 points) Would you expect your answer in (c) to be different if a green laser was used instead of a red one? Explain.

   Yes, because the fiber optic is expected to be "dispersive" in general, i.e. index of refraction depends on wavelength/frequency. Different color laser means different frequency which should alter n_1, which in turn leads to a change in θ_c and thus Δt.