Physics 1C **Name:** ____________________

Spring 2017

Mid-term Exam 1

April 27, 2017

Time Limit: 90 Minutes ID number: ____________________

Exam Notes:

- This is a closed books, closed notes exam. **No cheat sheets, please!**
- Show all work, clearly and in order. **Circle or otherwise indicate your final answers.**
- Make sure to **include units** in your answers, when numerical values are given.
- Always take a few moments to **double-check that your responses make sense.**
- Good luck!

Grade Table (for grader use only)

Part	Points	Score
A	16	
B	12	
C	13	
D	16	
E	13	

Potentially useful equations and constants:

Biot-Savart law: $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2}$, Lorentz force: $d\vec{F} = Id\vec{l}\times\vec{B}$ (or, $\vec{F} = q\vec{v}\times\vec{B}$)

$\mu_0 = 4\pi\times10^{-7}\ \mathrm{T\cdot m/A}$

Maxwell's equations:

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q}{\epsilon_0}$$
$$\oint \vec{B}\cdot d\vec{A} = 0$$
$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$
$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I + \mu_0\epsilon_0\frac{d\Phi_E}{dt}$$

Motional *emf*: $d\mathcal{E} = \left(\vec{v}\times\vec{B}\right)\cdot d\vec{l}$

Part A

1. (2 points) A proton is moving at an angle of 80° to a uniform magnetic field. What is the relationship between the direction of the force on the proton and the direction of the magnetic field?

   A. parallel to the magnetic field

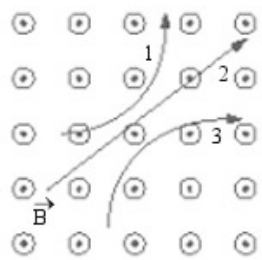
   B. at an angle of 10° to the magnetic field

   C. at an angle of 80° to the magnetic field

   D. at an angle of 180° to the magnetic field

   E. perpendicular to the magnetic field

2. (2 points) Three particles travel through a region of space where a uniform magnetic field is out of the page, as shown below. The electric charge of each of the three particles is, respectively,



   A. 1 is neutral, 2 is negative, and 3 is positive.

   B. 1 is neutral, 2 is positive, and 3 is negative.

   C. 1 is positive, 2 is neutral, and 3 is negative.

   D. 1 is positive, 2 is negative, and 3 is neutral.

   E. 1 is negative, 2 is neutral, and 3 is positive.

3. (2 points) Two long parallel wires placed side-by-side on a horizontal table carry identical size currents in opposite directions. The wire on your right carries current toward you, and the wire on your left carries current away from you. From your point of view, the magnetic field at the point exactly midway between the two wires

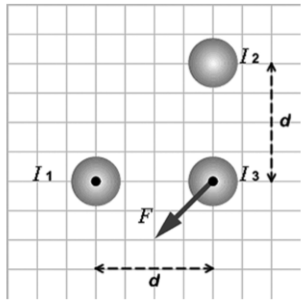
   A. points toward you.

   B. points away from you.

   C. is zero.

   D. points up.

   E. points down.



4. (2 points) The figure shows three long, parallel, current-carrying wires. The current directions are indicated for currents I_1 and I_3. The arrow labeled F represents the net magnetic force acting on current I_3. The three currents have equal magnitudes. What is the direction of the current I_2?

   A. vertically upward

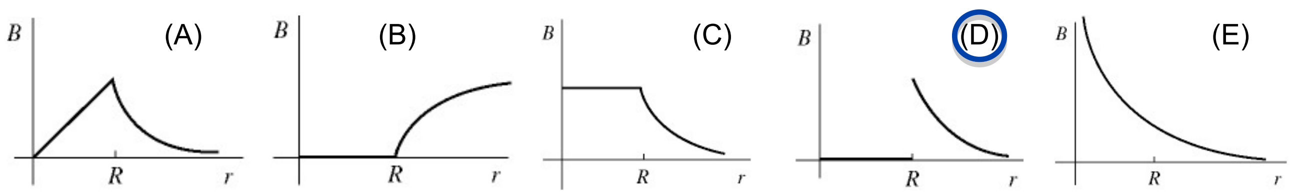
   B. vertically downward

   C. into the picture (in the direction opposite to that of I_1 and I_3)

   D. horizontal to the right

   E. out of the picture (in the same direction as I_1 and I_3)

5. (2 points) A very long, hollow, thin-walled conducting cylindrical shell (like a pipe) of radius R carries a current along its length uniformly distributed throughout the thin shell. Which one of the graphs shown in the figure most accurately describes the magnitude B of the magnetic field produced by this current as a function of the distance r from the central axis?



6. (2 points) Magnetic flux depends upon

   A. the magnetic field.

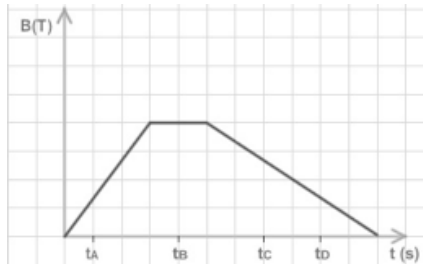
   B. the orientation of the area with respect to the field.

   C. the area involved.

   D. none of the above

   E. all of the above

7. (2 points) The figure below shows the time evolution of a uniform magnetic field. Four particular instants labeled t_A to t_D are also identified on the graph. The field passes through a circular coil whose normal is parallel to the direction of the field. At what time does the current induced in the coil have the largest value?



A. The current is the same at all these times.

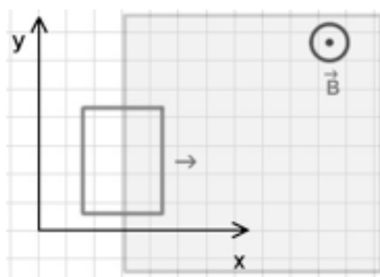
B. t_A

C. t_B

D. t_C

E. t_D

8. (2 points) A metallic frame moving along the positive direction enters a region of space with a uniform magnetic field pointing in the positive z-direction as shown below. In what direction should a force be applied to the frame to keep it moving at a constant speed while it is entering the field?



A. positive y

B. negative y

C. positive z

D. negative x

E. positive x

Part B

1. (12 points) A positron, i.e., a positively charged electron with charge e and mass m_e, is accelerated by an electric field acquiring final kinetic energy E. It is then projected into a uniform magnetic field $\vec{B}$ with its velocity vector making an angle ϕ with $\vec{B}$ (see figure below). Derive expressions for the following characteristics of the positron's helical path

   (a) (5 points) the period,

   (Check HW problem 27.25)

   The positron has kinetic energy $E = 1/2mv^2$, so its speed is $v = \sqrt{2E/m}$. Due to the component of the velocity perpendicular to the magnetic field, $v_\perp = v \sin\phi$, there is a magnetic force acted on the positron equal to $F_m = qv_\perp B$. This force is responsible for the circular motion of the particle in the perpendicular direction, and its period is:

   $T = \frac{2\pi r}{v_\perp}$ (1)

   But since the magnetic force acts as the centripetal force: $m\frac{v_\perp^2}{r} = qv_\perp B \Rightarrow r = \frac{mv_\perp}{qB}$.

   So, replacing in (1) above: $T = \frac{2\pi m}{qB}$

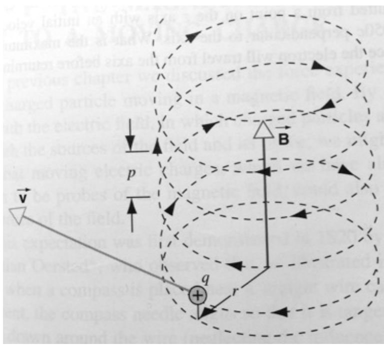
   (b) (4 points) the radius r, and

   We found above that $r = \frac{mv_\perp}{qB}$, and replacing $v_\perp$ we get: $r = \frac{m}{qB}\sqrt{\frac{2E}{m}} \sin\phi$

   (c) (3 points) the pitch p (=the "height" of one complete helix turn).

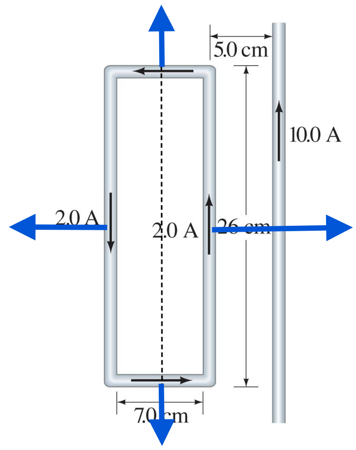
   In the parallel to the magnetic field direction, there is no force acted on the positron so it moves with constant velocity $v_\parallel = v\cos\phi$.

   So, $p = v_\parallel T = \sqrt{\frac{2E}{m}} \cos\phi \frac{2\pi m}{qB}$



Part C

1. (13 points) A rectangular loop of wire carries a 2-A current and lies in a plane which also contains a very long straight wire carrying a 10-A current as shown below. [*Hint*: The magnetic field strength away from a infinite straight conductor is $B = \frac{\mu_0 I}{2\pi r}$]



(a) (2 points) Draw on the figure the direction of the magnetic force on each side of the loop, if any.

The magnetic field due to the straight conductor points out-of-the-page, so the forces on the rectangular loop are as shown above (remember, $\vec{F} = I\vec{l} \times \vec{B}$)

(b) (5 points) What is the net force acted on the loop due to the straight wire (*magnitude* and *direction*)?

The forces on the horizontal sides cancel one another out. On each vertical side, the magnitude of the magnetic force is $F = I_{loop}LB$, where the magnetic field at distance r from the staright wire is $B = \frac{\mu_0 I_{wire}}{2\pi r}$. Hence, on each vertical side $F = \frac{\mu_0 I_{wire} I_{loop} L}{2\pi r}$. So, the force is stronger on the right side of the loop which is closer to the wire and the net force is:
$F_{net} = \frac{\mu_0 I_{wire} I_{loop} L}{2\pi d_1} - \frac{\mu_0 I_{wire} I_{loop} L}{2\pi d_2}$, where d_1, d_2 are the distances of the right and left sides, respectively.
Thus, $F_{net} = \frac{\mu_0 I_{wire} I_{loop} L}{2\pi}\left(\frac{1}{d_1} - \frac{1}{d_2}\right) = 1.2 \times 10^{-5}$N, towards the wire.

(c) (2 points) What is the loop's magnetic dipole moment (*magnitude* and *direction*)?

The current flows counter-clockwise, so per right-hand rule convention, the magnetic moment points out-of-the-page, and it is:
$\vec{\mu} = IA\hat{k} = (0.0364 \text{ A·m}^2)\hat{k}$

(d) (2 points) What is the net torque on the loop due to the straight wire?

The net torque is zero because $\vec{\mu} \parallel \vec{B}$! The loop is at equilibrium, none of the forces acted on it tend to rotate it.

(e) (2 points) By what angle about its long axis (dashed line in figure) would you rotate the loop to maximize the net torque?

To maximize the torque, we need to bring $\vec{\mu}$ perpendicular to $\vec{B}$, so rotate by 90°.

Part D

1. (16 points) A long, straight, solid cylindrical conductor of radius R_1 is oriented with its axis in the z-direction, and it carries a uniformly distributed total current I_0.

Check HW problems 28.43, 28.75

In all cases below, current flows in the z-direction, so the magnetic field at any point of distance r from the center will be pointing in the azimuthal $(\hat{\theta})$ direction. To find the strength of the field, we will consider circular amperean loops of radius r around the center.

(a) (4 points) Determine the magnetic field in terms of I_0 inside $(0 \leq r \leq R_1)$ and outside $(R_1 \leq r)$ the cylinder.

For $0 \leq r \leq R_1$: Applying Ampere's law $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}} \Rightarrow B 2\pi r = \mu_0 I_{\text{encl}}$, where the enclosed current is

$I_{\text{encl}} = J A_{\text{encl}} = J \pi r^2 = \frac{I_0}{\pi R_1^2} \pi r^2$.

So, $B = \frac{\mu_0 I_0 r}{2\pi R_1^2}$

For $R_1 \leq r$: Same as above but now the current enclosed is $I_{\text{encl}} = I_0$, so $B = \frac{\mu_0 I_0}{2\pi r}$.

(b) (4 points) Assume now that the current density is not uniform anymore but depends on the distance r from the cylinder's center as $\vec{J}_1(r) = C_1 r \hat{k}$, for $0 \leq r \leq R_1$. What is now the magnetic field in terms of the total current I_0 inside $(0 \leq r \leq R_1)$ and outside $(R_1 \leq r)$ the cylinder?

For $0 \leq r \leq R_1$: Again, applying Ampere's law $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}} \Rightarrow B 2\pi r = \mu_0 I_{\text{encl}}$, where the enclosed current is now

$I_{\text{encl}} = \int_0^r \vec{J}_1(r) \cdot d\vec{A} = \int_0^r C_1 r 2\pi r dr = 2\pi C_1 r^3/3$,

and the constant C_1 can be related to I_0 since

$I_0 = \int_0^{R_1} \vec{J}_1(r) \cdot d\vec{A} = \int_0^{R_1} C_1 r 2\pi r dr = 2\pi C_1 \int_0^{R_1} r^2 dr = 2\pi C_1 \frac{R_1^3}{3} \Rightarrow C_1 = \frac{3 I_0}{2\pi R_1^3}$.

So, $B = \frac{\mu_0 C_1 r^2}{3} = \frac{\mu_0 I_0 r^2}{2\pi R_1^3}$.

For $R_1 \leq r$: Same as above but now the current enclosed is $I_{\text{encl}} = I_0$, so $B = \frac{\mu_0 I_0}{2\pi r}$.

(c) (5 points) Imagine that the cylinder of part (b) is surrounded by a concentric cylindrical tube of inner radius R_2 and outer radius R_3 (see figure below). If the current density in this tube is given by $\vec{J}_2(r) = -C_2 r \hat{k}$ with a total current I_0, determine the magnetic field in terms of I_0 for $R_2 \leq r \leq R_3$.

For $R_2 \leq r \leq R_3$: Inside the outer cylinder, the current enclosed is the current from the inner cylinder and a portion of the current from the outer cylinder, so:

$I_{\text{encl}} = I_0 + \int_{R_2}^r \vec{J}_2(r) \cdot d\vec{A} = I_0 - \int_{R_2}^r C_2 r 2\pi r dr = I_0 - 2\pi C_2 \left(r^3 - R_2^3\right)/3$,

and the constant C_2 can be related to I_0 since

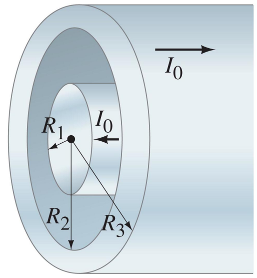
$I_0 = \int_{R_2}^{R_3} J_2(r) dA = 2\pi C_2 \int_{R_2}^{R_3} r^2 dr = 2\pi C_2 \frac{\left(R_3^3 - R_2^3\right)}{3} \Rightarrow C_2 = \frac{3 I_0}{2\pi \left(R_3^3 - R_2^3\right)}$.

So $B = \frac{\mu_0 I_0 \left(R_3^3 - r^3\right)}{2\pi r \left(R_3^3 - R_2^3\right)}$.

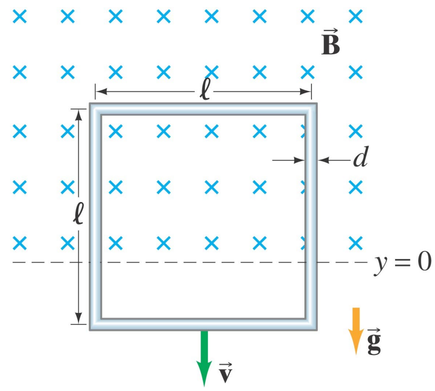
(d) (3 points) For the coaxial cable of part (c) (see figure), determine the magnetic field in terms of I_0 for $R_3 \leq r$.

For $R_3 \leq r$: Outside the outer cylinder, $I_{\text{encl}} = I_0 - I_0$, so $B = 0$.

Note that the constants C_1 and C_2 are not known.



Part E



1. (13 points) In a certain region of space near Earth's surface, a uniform horizontal magnetic field of magnitude B exists above a level defined to be $y = 0$. Below $y = 0$, the field abruptly becomes zero (see Figure). A vertical square wire loop has resistance R, uniformly distributed mass m, diameter d, and side length ℓ. It is initially at rest with its lower horizontal side above $y = 0$ and is then allowed to fall under gravity, with its plane perpendicular to the direction of the magnetic field.

   (a) (2 points) Find the magnitude of the *emf* induced in the loop, before its lower horizontal side reaches $y = 0$.

   As the loop moves in the uniform magnetic field, the magnetic flux remains constant. So $\frac{d\Phi_B}{dt} = 0$ and thus $\mathcal{E} = 0$.

   (b) (2 points) Repeating the same experiment but this time releasing the loop from rest with its lower horizontal side at $y = 0$, in what direction does current flow, if any? Briefly explain.

   As the loop moves out of the magnetic field, the inwards magnetic flux is decreasing. The induced emf is trying to oppose this decrease, thus inducing a clockwise current.

   (c) (6 points) While the loop is still partially immersed in the magnetic field (as it falls into the zero-field region), determine the magnetic "drag" force that acts on it at the moment when its speed is v.

   Due to the induced clockwise current in the upper horizontal side of the loop, there will be a magnetic force $\vec{F}_M = I\vec{\ell} \times \vec{B}$. In other words, the magnetic force here will be upwards with magnitude $F_M = I\ell B$, where the current is given by $I = \mathcal{E}/R = Bv\ell/R$. So, $F_M = B^2 v\ell^2/R$.

   (d) (3 points) If you know that the loop reaches a terminal velocity v_T before its upper horizontal side exits the field, find an expression for v_T.

   The loop's velocity reaches its terminal, constant value v_T when the net force in the direction of motion is zero:
   $\Sigma F_y = 0 \Rightarrow mg = F_M \Rightarrow v_T = \frac{mgR}{B^2\ell^2}$.