Physics 1C
Fall 2016
Mid-term Exam
October 18, 2016
Time Limit: 90 Minutes

Name: ____________________________

ID number: ____________________________

Exam Notes:

- This is a closed books, closed notes exam. **No cheat sheets, please!**
- Show all work, clearly and in order. **Circle or otherwise indicate your final answers.**
- Make sure to **include units** in your answers, when numerical values are given.
- Always take a few moments to **double-check that your responses make sense.**
- Good luck!

Grade Table (for grader use only)

Part	Points	Score
A	18	
B	15	
C	16	
D	15	
E	16	

Potentially useful equations:

Maxwell's equations:

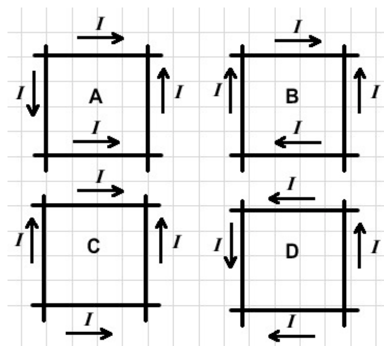
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$
$$\oint \vec{B} \cdot d\vec{A} = 0$$
$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

Biot-Savart law: $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2}$

$\int \frac{dx}{x} = \ln x$

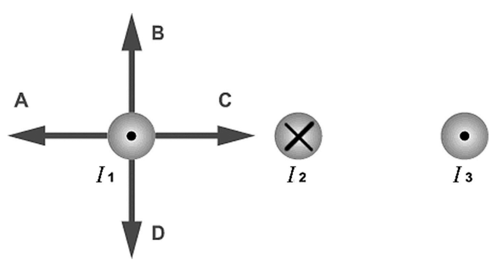
Part A

1. (2 points) The figure shows four different sets of insulated wires that cross each other at right angles without actually making electrical contact. The magnitude of the current is the same in all the wires, and the directions of current flow are as indicated. For which (if any) configuration will the magnetic field at the center of the square formed by the wires be equal to zero?



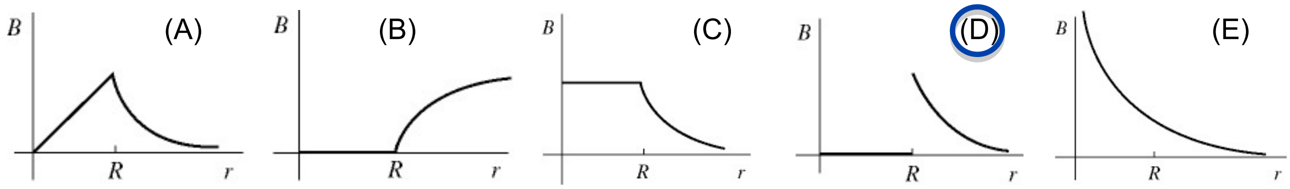
   A. A
   B. B
   C. C
   D. D
   E. The field is not equal to zero in any of these cases.

2. (2 points) The figure shows three long, parallel current-carrying wires. The magnitudes of the currents are equal and their directions are as shown. Which of the arrows drawn near the wire carrying current 1 correctly indicates the direction of the magnetic force acting on that wire?

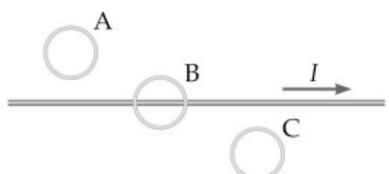


   A. A
   B. B
   C. C
   D. D
   E. The field is not equal to zero in any of these cases.

3. (2 points) A very long, hollow, thin-walled conducting cylindrical shell (like a pipe) of radius R carries a current along its length uniformly distributed throughout the thin shell. Which one of the graphs shown in the figure most accurately describes the magnitude B of the magnetic field produced by this current as a function of the distance r from the central axis?



4. (2 points) The long straight wire in the figure carries a current I that is decreasing with time at a constant rate. The circular loops A, B, and C all lie in a plane containing the wire. The induced emf in each of the loops A, B, and C is such that



A. no emf is induced in any of the loops.

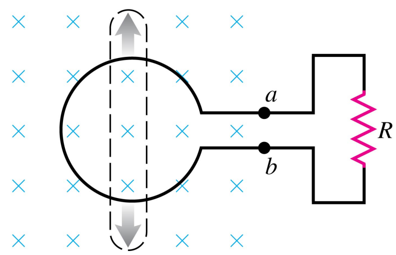
B. a counterclockwise emf is induced in all the loops.

C. loop A has a clockwise emf, loop B has no induced emf, and loop C has a counterclockwise emf.

D. loop A has a counter-clockwise emf, loop B has no induced emf, and loop C has a clockwise emf.

E. loop A has a counter-clockwise emf, loops B and C have clockwise emfs.

5. (2 points) A flexible loop of wire lies in a uniform magnetic field of magnitude B directed into the plane of the picture. The loop is pulled as shown, reducing its area. The induced current flows



A. downward through resistor R and is proportional to B.

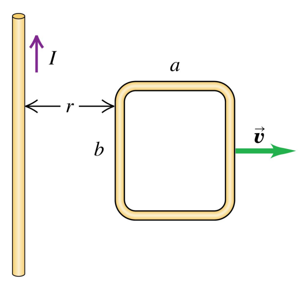
B. upward through resistor R and is proportional to B.

C. downward through resistor R and is proportional to B^2.

D. upward through resistor R and is proportional to B^2.

E. None of the above is true.

6. (2 points) The loop of wire is being moved to the right at constant velocity. A constant current I flows in the long, straight wire in the direction shown. The current induced in the loop is



A. clockwise and proportional to I.

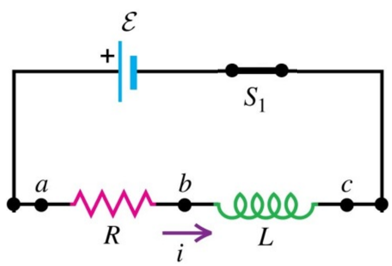
B. counterclockwise and proportional to I.

C. clockwise and proportional to I^2.

D. counterclockwise and proportional to I^2.

E. zero.

7. (2 points) An inductor (inductance L)and a resistor (resistance R) are connected to a source of emf as shown. When switch S_1 is closed, a current begins to flow. The final value of the current is



A. directly proportional to RL.

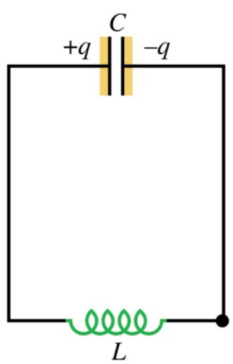
B. directly proportional to R/L.

C. directly proportional to L/R.

D. directly proportional to $1/(RL)$.

E. independent of L.

8. (2 points) An inductor (inductance L) and a capacitor (capacitance C) are connected as shown. If the values of both L and C are doubled, what happens to the time required for the capacitor charge to oscillate through a complete cycle?



A. It becomes four times longer.

B. It becomes twice as long.

C. It is unchanged.

D. It becomes half as long.

E. It becomes one-fourth as long.

9. (2 points) Which of the following statements about inductors are correct? There may be more than one correct choice.

   A. When it is connected in a circuit, an inductor always resists having current flow through it.

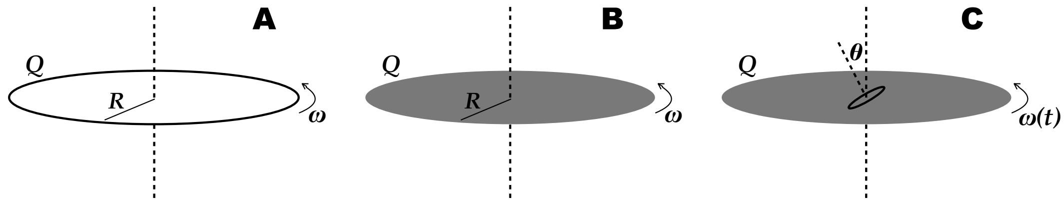
   B. Inductors store energy by building up charge.

   C. When an inductor and a resistor are connected in series with a DC battery, the current in the circuit is reduced to zero in one time constant.

   D. An inductor always resists any change in the current through it.

   E. When an inductor and a resistor are connected in series with a DC battery, the current in the circuit is zero after a very long time.

Part B



1. (15 points)

   (a) (4 points) A ring of radius R has charge Q uniformly distributed along its circumference. If the ring rotates at an angular frequency ω about its axis (see Fig.A), what is the magnetic field (*magnitude* and *direction*) at the center of the rotating ring?

   Check Lecture 3, Slide 7

   The rotating (moving) charge is creating a magnetic field, and the uniform linear charge distribution is $\lambda = \frac{Q}{2\pi R}$. Each length element carrying charge $dq = \lambda d\ell$ generates field (Biot-Savart law):

   $d\vec{B} = \frac{\mu_0}{4\pi}\frac{dq\vec{v}\times\hat{r}}{R^2} = \frac{\mu_0}{4\pi}\frac{dqv}{R^2}\hat{k}$. And $v = \omega R$, so the total magnetic field is:

   $\vec{B} = \frac{\mu_0}{4\pi}\int\frac{dq\omega}{R}\hat{k} = \frac{\mu_0}{4\pi}\int\frac{\lambda d\ell\omega}{R}\hat{k} = \frac{\mu_0}{4\pi}\frac{Q\omega}{2\pi R^2}\int_0^{2\pi R}d\ell\hat{k} \Rightarrow \vec{B} = \frac{\mu_0}{4\pi}\frac{Q\omega}{R}\hat{k}$

   Alternative solution: We know that the field created by a current loop is:

   $\vec{B} = \frac{\mu_0 I}{2R}\hat{k}$,

   and here $I = \frac{Q}{T}$, where T is the period of rotation, or equivalently $I = \frac{\omega Q}{2\pi}$. So:

   $\vec{B} = \frac{\mu_0}{4\pi}\frac{Q\omega}{R}\hat{k}$

   (b) (5 points) Now, the same charge Q is distributed uniformly over the surface of a thin plastic disc of the same radius R (see Fig.B). Again, the disc rotates at an angular frequency ω about its axis. What is the magnetic field at the center of the rotating disc?

   We can treat the disk as a series of thin rings of thickness dr and radius from 0 to R, carrying charge $dq = \sigma dA$. Then using the result from (a):

   $\vec{B} = \int_0^R \frac{\mu_0}{4\pi}\frac{\omega dq}{r}\hat{k} = \frac{\mu_0\omega}{4\pi}\int_0^R\frac{1}{r}\sigma dA\hat{k} = \frac{\mu_0\omega}{4\pi}\int_0^R\frac{1}{r}\frac{Q}{\pi R^2}2\pi r dr\hat{k} \Rightarrow \vec{B} = \frac{\mu_0\omega Q}{2\pi R}\hat{k}$

   Alternative solution: As in (a), we can use the known result for the current loop and replace $I = \frac{dq}{T} = \frac{\omega}{2\pi}\sigma dA$.

   (c) (2 points) What would have to change in your calculation in part (b) if the charge Q was distributed on the disc surface according to a surface charge density $\sigma(r)$ instead, where r is the distance from the disc's center? Briefly explain.

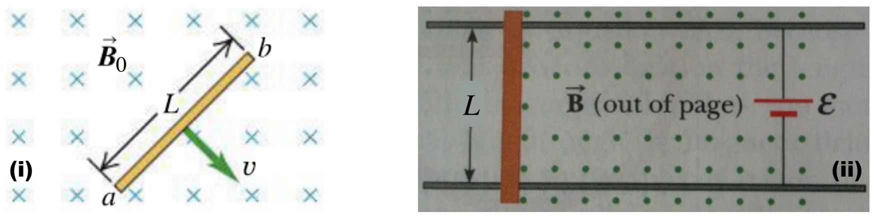
   If the charge distribution is not uniform, we can not assume $\sigma = \frac{Q}{\pi R^2}$ but instead we need to include the functional form of $\sigma(r)$ in the integral and calculate.

   (d) (4 points) Assume that in the scenario of part (b), a small conducting ring of area S is placed near the center of the disc with its axis tilted from the disc axis by an angle θ, as shown. If the disc's rotation starts speeding up at some $t = 0$ so that $\omega(t) = \omega_0 + kt^2$, what is the emf in the ring (ignore the ring's self-inductance)?

   $\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(BA\cos\theta\right) = -\frac{d}{dt}\left(\frac{\mu_0\omega Q}{2\pi R}A\cos\theta\right) = -\frac{\mu_0 Q}{2\pi R}A\cos\theta\frac{d\omega}{dt} = -\frac{\mu_0 Q}{\pi R}A\cos\theta kt$

Part C

1. (16 points) A conducting rod of length L, mass m, and resistance R can move freely in a region of space where there is a magnetic field $\vec{B_0}$ (Fig.(i)).



(a) (3 points) If the rod moves with constant velocity $\vec{v}$ as shown in fig.(i), what's the induced emf between the ends of the rod, if any?

The velocity and field are perpendicular to one another, so: $\mathcal{E} = Bvl$.

(b) (4 points) Assume, now, that the rod in (a) pivots about one of its ends instead with constant angular speed ω, and the plane of rotation remains perpendicular to $\vec{B_0}$. What's the induced emf between the ends of the rod, if any?

Check Lecture 5, Slide 8. This is a simplified version of Faraday's dynamo!

$d\mathcal{E} = \left(\vec{v} \times \vec{B}\right) \cdot d\vec{l}$ for each length element of the rod. Since $\vec{v} \perp \vec{B}$ and $v = \omega r$, the total emf is:

$\mathcal{E} = \int_0^L \omega r B dr = B\omega \frac{L^2}{2}$.

(c) (6 points) The same rod is then placed in a different magnetic field $\vec{B}$ and on parallel frictionless rails, as shown in Fig.(ii). A battery maintains a constant $\mathcal{E}$ between the rails. Assuming that the bar starts from rest, derive an expression for the rod's speed as a function of time.

Check HW problem 29.56

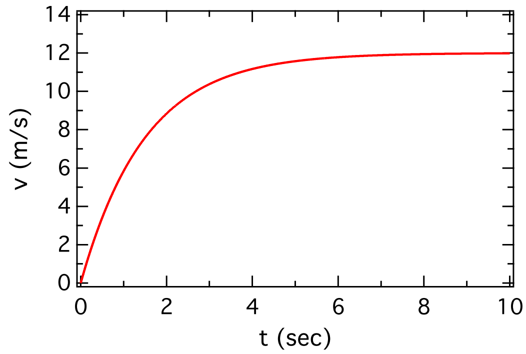
Newton's 2nd law: $\Sigma F = ma = m\frac{dv}{dt}$,

where the net force here is the Lorentz force due to current in the rod $F = BIL$. The total current is due to the combination of the battery's emf $\mathcal{E}_B$ and the induced emf on the rod. So $I = \frac{\mathcal{E}_B - BvL}{R}$, and plugging this in Newton's law, we get:

$B\left(\frac{\mathcal{E}_B - BvL}{R}\right) L = m\frac{dv}{dt} \Rightarrow \int \frac{dv}{\mathcal{E}_B - BvL} = \frac{BL}{mR}\int dt \Rightarrow v(t) = \frac{\mathcal{E}_B}{BL}\left(1 - e^{-B^2L^2t/mR}\right)$

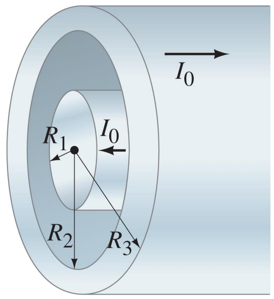
(d) (3 points) Plot a *speed* vs. *time* diagram, assuming that the battery is $\mathcal{E} = 12\text{V}$, $B = 2\text{T}$, $L = 0.5\text{m}$, $m = 0.5\text{kg}$, and $R = 3\Omega$.

In this case, the speed is $v(t) = 12\left(1 - e^{-2t/3}\right)\text{m/s}$, which is as shown below. The *terminal velocity* is 12m/s.



Part D

1. (15 points) A coaxial cable consists of a solid inner conductor of radius R_1, surrounded by a concentric cylindrical tube of inner radius R_2 and outer radius R_3 (see below). The conductors carry equal and opposite total current I_0 with current densities $J_1(r) = C_1 r$ and $J_2(r) = C_2 r$ for the inner and outer conductors, respectively, where r is the distance from the center of the cable. The constants C_1 and C_2 are not known.



(a) (5 points) Determine the magnetic field in terms of I_0 for $0 \le r \le 2R_3$.

Check HW problem 28.75

The constants C_1 and C_2 need to be calculated fist. Assuming the direction of current in the inner conductor as positive, it is:

inner: $I_0 = \int J_1(r)dA = \int C_1 r 2\pi r dr = 2\pi C_1 \int_0^{R_1} r^2 dr = 2\pi C_1 \frac{R_1^3}{3} \Rightarrow C_1 = \frac{3I_0}{2\pi R_1^3}$

outer: $-I_0 = \int J_2(r)dA = 2\pi C_2 \int_{R_2}^{R_3} r^2 dr = 2\pi C_1 \frac{\left(R_3^3 - R_2^3\right)}{3} \Rightarrow C_2 = -\frac{3I_0}{2\pi\left(R_3^3 - R_2^3\right)}$.

To find the magnetic field at a distance r from the cable's center, we need to apply Ampere's law in the different "regions" of the cable. We assume circular paths perpendicular to the conductors. Then:

For $0 \le r \le R_1$: $\oint \vec{B} d\vec{l} = \mu_0 I_{\text{encl}} \Rightarrow B 2\pi r = \mu_0 I_{\text{encl}}$, where the enclosed current is $I_{\text{encl}} = \int J_1(r)dA = \int C_1 r 2\pi r dr = 2\pi C_1 r^3/3$.

So, $B = \frac{\mu_0 C_1 r^2}{3} = \frac{\mu_0 I_0 r^2}{2\pi R_1^3}$.

For $R_1 \le r \le R_2$: Between the wires the current enclosed is $I_{\text{encl}} = I_0$, so $B = \frac{\mu_0 I_0}{2\pi r}$.

For $R_2 \le r \le R_3$: Inside the outer wire the current enclosed is the current from the inner wire and a portion of the current from the outer wire, so:

$I_{\text{encl}} = I_0 + \int J_2(r)dA = I_0 + \int_{R_2}^{r} C_2 r 2\pi r dr = I_0 = I_0 + 2\pi C_2 \left(r^3 - R_2^3\right)/3 = \frac{I_0\left(R_3^3 - r^3\right)}{\left(R_3^3 - R_2^3\right)}$,

so $B = \frac{\mu_0 I_0\left(R_3^3 - r^3\right)}{2\pi r\left(R_3^3 - R_2^3\right)}$.

For $R_3 \le r \le 2R_3$: Outside the outer wire, $I_{\text{encl}} = I_0 - I_0$, so $B = 0$.

(b) (4 points) Consider the case where the coaxial cable is build with two concentric thin cylindrical shells instead, with radii R_1 and R_2 respectively. If they carry uniformly distributed equal and opposite current I_0, sketch a plot of magnetic field vs. distance from the center r, for $0 \leq r \leq 2R_2$.

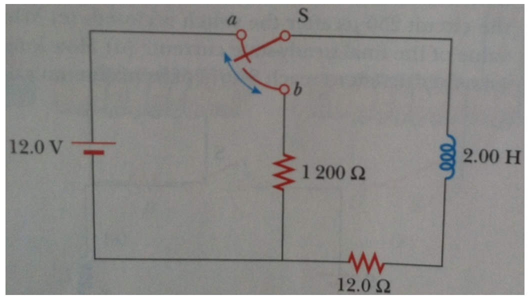
Again, we need to apply Ampere's law as above.
For $0 \leq r \leq R_1$: $I_{\text{encl}} = 0$, so $B = 0$.
For $R_1 \leq r \leq R_2$: $I_{\text{encl}} = I_0$, so $B = \frac{\mu_0 I_0}{2\pi r}$.
For $R_2 \leq r \leq 2R_2$: $I_{\text{encl}} = 0$, so $B = 0$.

(c) (6 points) For a length ℓ of the coaxial cable in part (b), what is the total energy stored in its magnetic field? (*Hint*: The cable has some self-inductance!)

The total energy stored is $U = 1/2LI_0^2$, where $L = \frac{\Phi_B}{I_0}$. So, we need to find the magnetic flux through a cross-section between the two conductors. This is $\Phi_B = \int BdA$ and since B changes with distance r, we can divide the area into strips of width dr. Then
$\Phi_B = \int BdA = \int B\ell dr = \int_{R_1}^{R_2} \frac{\mu_0 I_0}{2\pi r} \ell dr = \frac{\mu_0 I_0 \ell}{2\pi} \ln{(R_2/R_1)}$.
So, $U = \frac{\mu_0 I_0^2 \ell}{4\pi} \ln{(R_2/R_1)}$.

Part E

1. (8 points) One application of an $R - L$ circuit is the generation of time-varying high voltage from a low voltage source as shown below.



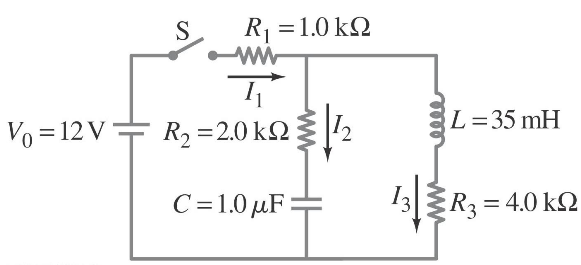
   (a) (4 points) After the switch has been in position a for a long time, it is thrown quickly to b. Compute the initial voltage across each resistor and across the inductor.

   Just after the switch is changed, the current in the circuit is the same as just before since the inductor is opposing the change, so $I_0 = \frac{\mathcal{E}}{R} = 1$A.
   So, the voltages across the resistors are $V_{12\Omega} = I_0 R = 12$V and $V_{1200\Omega} = I_0 R = 1200V$. And applying kirchhoff's loop rule: $V_L = -1212$V.

   (b) (4 points) How much time elapses before the voltage across the inductor drops to 12 V?

   As time elapses, the initial value of the current I_0 decays exponentially as $I(t) = I_0 e^{-\frac{R_{tot}}{L}t}$. The voltage across the inductor is $V_L = -L\frac{dI}{dt} = I_0 R_{tot} e^{-\frac{R_{tot}}{L}t}$. So, when $V_L = 12$V, the time elapsed is: $t = \frac{2}{1212}\ln\left(\frac{1212}{12}\right)$s.

2. (8 points) For the $R-L-C$ circuit shown below, at time $t = 0$, the switched is closed.



   (a) (5 points) After a sufficiently long time, steady currents flow through the resistors. Determine these three currents.

   After sufficiently long time, the capacitor is fully charged so no current goes through it and the current through the inductor is constant. So
   $I_2 = 0$ and $I_1 = I_3 = \frac{V_0}{R_1+R_3} = 2.4$mA.

(b) (3 points) If the switch is opened again, after the situation described in (a) has been reached, do you expect the charge on the capacitor to oscillate with time? Briefly explain.

Generally, the charge in the RLC circuit will tend to oscillate, but if the resistance is relatively large compared to the inductance and capacitance, the system will be overdamped and no oscillation will occur. In other words, all the energy stored initially in the capacitor will be very quickly dissipated in the resistors in the latter case.