Mid-term Exam 2: PHYSICS 1C (Winter 2018)

Time: 2:00PM – 3:00PM, February 26, 2018, Instructor: Prof. Zhongbo Kang

Student Name: ____________________

Student I.D. Number: ____________________

Exam Version: **A**

Note:

- The exam is closed book, and closed notes.
- You can use a calculator. However, using a smart phone is NOT allowed.
- Two pages of physical equations are provided.
- Remember to write down each step of your calculations.

Score Sheet:

Problem 1 (10 points): 10

Problem 2 (10 points): 9

Problem 3 (10 points): 10

Problem 4 (10 points): 10

Problem 5 (10 points): 10

Total (50 points): 49

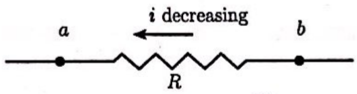
Two constants for your reference: $\epsilon_0 = 8.85\times10^{-12}\ C^2/N\cdot m^2$, $\mu_0 = 4\pi\times10^{-7}\ T\cdot m/A$

Problem 1 (10 pts):

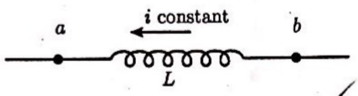
10

(a) (3 pts) In the following three situations, please determine the sign of potential difference V_{ab} between point a and b? For each of them, choose your answers from:

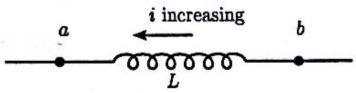
a. $V_{ab} > 0$ b. $V_{ab} < 0$ c. $V_{ab} = 0$ d. V_{ab} cannot be determined



Your choice: B



Your choice: C



Your choice: B

(b) (3 pts) You plan to take your hair dryer to Europe, where the electric outlets put out 240 V instead of 120 V as seen in the US. The dryer puts out 1600 W at 120 V. Denote the resistance of your dryer as R_1 when operated at 120 V, and the resistance as R_2 for your dryer appear to have when operated at 240 V. Values of (R_1, R_2) = D Ω

a. (9.0, 9.0) b. (36.0, 9.0) c. (36.0, 36.0) d. (9.0, 36.0)

$P = IV = 120I = 1600$

$I_1 = \frac{1600}{120} = \frac{40}{3} = 13.3$

$R_1 = \frac{V_1}{I_1} = \frac{120}{13.3} = 9$

$V_1 I_1 = V_2 I_2$

$13.3(120) = 240 I_2$

$I_2 = 6.17$

$R_2 = \frac{V_2}{I_2} = \frac{240}{6.17} = 36$

(c) (4 pts) An electromagnetic plane wave propagates in the vacuum. Its electric field $\vec{E}(x,t) = E_{\max}\cos(kx + \omega t)\hat{j}$, please determine the direction of the Poynting vector.

$-X$

Your choice: C

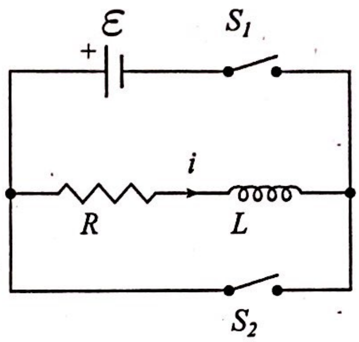
a. $+y$ b. $-y$ c. $-x$ d. $+x$ e. $-z$ f. $+z$

g. not enough information, cannot be determined

9

Problem 2 (10 pts)

For a *R-L* circuit as shown in the figure, the voltage $\varepsilon = 12.0$ V, the resistance $R = 100\ \Omega$, and the inductance $L = 0.01$ H. Suppose both switches are open to begin with, and then at some initial time $t = 0$, we close switch S_1 (leave S_2 still open). The current i is shown in the figure.



4 (a) (4 pts) Please obtain an expression of the current i as a function of time t. If one denotes the value of the current as I_0 when $t \to \infty$, please determine I_0.

3 (b) (4 pts) Please calculate the instantaneous power P_L in the inductor as a function of t. At what value of t, P_L is a maximum?

2 (c) (2 pts) When the current becomes I_0, one then resets our stopwatch to redefine the initial time, we open switch S_1 but close switch S_2 at $t = 0$. Obtain an expression of the current i as a function of time t.

$\mathcal{E}=12V$ $R=100\Omega$ $L=0.01H$ $\tau=\frac{L}{R}$ $\frac{t}{\tau}=\frac{R}{L}t$

a) $i = I_0(1-e^{-\frac{t}{\tau}}) = \frac{\mathcal{E}}{R}(1-e^{-\frac{R}{L}t}) = \frac{12}{100}(1-e^{-\frac{100}{0.01}t}) = \boxed{0.12(1-e^{-10{,}000t})}$

$I_0 = \frac{\mathcal{E}}{R} = \boxed{0.12A}$

b) $P = iV = \cancel{0.12(1-e^{-10{,}000t})}V$ ok

$V = L\frac{di}{dt} = L(0.12(1+10{,}000e^{-10{,}000t}))$

$\boxed{P_L = 0.12(1-e^{-10{,}000t})(0.0012(1+10{,}000e^{-10{,}000t}))}$

$\cancel{P_L = max\ @\ t = 10^{-4}s = \tau}$ $-10{,}000e^{-10{,}000t} = 0$

c) $i = I_0 e^{-\frac{t}{\tau}} = \boxed{0.12e^{-10{,}000t}}$

Problem 3 (10 pts)

In an ac series circuit, we have $R = 300\ \Omega$, $L = 60$ mH, $C = 0.50\ \mu$F, voltage amplitude $V = 50$ V, and $\omega = 10{,}000$ rad/s.

(a) (2 pts) Find the impedance Z and the voltage amplitude across each circuit element.

(b) (5 pts) Find expressions for the time dependence of the instantaneous current i and the instantaneous voltages across the resistor (v_R), inductor (v_L), capacitor (v_C), and the source (v).

(c) (3 pts) Calculate the power factor and the average power delivered to the entire circuit, and to each circuit element.

$R = 300\,\Omega \quad L = 0.06\,H \quad C = 0.5\times10^{-6}\,F \quad V_{max} = 50V \quad \omega = 10{,}000\,rad/s$

a) $X_L = \omega L = 10{,}000(0.06) = 600\,\Omega$ $\qquad Z = \sqrt{R^2 + (X_L - X_C)^2}$

$X_C = \frac{1}{\omega C} = \frac{1}{10{,}000(0.5\times10^{-6})} = 200\,\Omega$

$Z = \sqrt{300^2 + (400)^2} = \boxed{500\,\Omega}$

$I_{max} = \frac{V_{max}}{Z} = \frac{50}{500} = 0.1\,A$

$V_{R,max} = I_{max}R = 0.1(300) = \boxed{30V}$ $\qquad V_{C,max} = I_{max}X_C = 0.1(200) = \boxed{20V}$

$V_{L,max} = I_{max}X_L = 0.1(600) = \boxed{60V}$

b) $i = I_{max}\cos\omega t = \boxed{0.1\cos 10{,}000t}$ $\qquad v_C = V_{C,max}\cos(\omega t - \frac{\pi}{2}) = \boxed{20\cos(10{,}000t - \frac{\pi}{2})}$

$v_R = V_{R,max}\cos\omega t = \boxed{30\cos 10{,}000t}$ $\qquad v_L = V_{L,max}\cos(\omega t + \frac{\pi}{2}) = \boxed{60\cos(10{,}000t + \frac{\pi}{2})}$

$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{400}{300}\right) = 0.927$ $\qquad v = V_{max}\cos(\omega t + \phi) = \boxed{50\cos(10{,}000t + 0.927)}$

c) $\boxed{\cos\phi = 0.6}$

$P_{av} = \frac{1}{2}VI\cos\phi = \frac{1}{2}(50)(0.1)(0.6) = \boxed{1.5\,W}$ $\qquad \boxed{P_{av,C} = 0}$ $\quad \boxed{P_{av,L} = 0}$

$\boxed{P_{av,R} = P_{av} = 1.5W}$

Problem 4 (10 pts)

A capacitor has two circular plates with radius 3.5 cm. For questions (a) and (b), assuming the capacitor is filled with air. For question (c), the capacitor is completely filled with a dielectric of permittivity 4.0×10^{-11} F/m. The dielectric is ideal and nonmagnetic, and the conduction current in the dielectric is zero.

(a) (4 pts) At a particular instant, the conduction current in the wires is 0.670 A. What is the displacement current density in the air space between the plates? What is the rate at which the electric field between the plates is changing?

10

(b) (3 pts) What is the induced magnetic field between the plates at a distance of 2.00 cm from the axis?

(c) (3 pts) For $t > 0$, the electric flux through the dielectric is $(8200\ \mathrm{V \cdot m/s^3})\, t^4$. At what time does the displacement current in the dielectric equal 30 μA?

$r = 0.035m$

a) $I_c = 0.670\,A$ $\quad i_d = 0.670A$ $\quad J = \frac{i_d}{A} = \frac{0.670A}{0.0038m^2} = \boxed{174.1 \frac{A}{m^2}}$

$A = \pi r^2 = 0.0038m^2$ $\quad \frac{d\Phi}{dt} = \frac{d}{dt}(EA) = A\frac{dE}{dt}$ $\quad i_d = \varepsilon_0 \frac{d\Phi}{dt} = \varepsilon_0 A \frac{dE}{dt}$ $\quad \frac{i_d}{\varepsilon_0 A} = \frac{dE}{dt} = \frac{0.67}{\varepsilon_0(0.0038)} = \boxed{1.97 \times 10^{13} \frac{V}{m \cdot s}}$

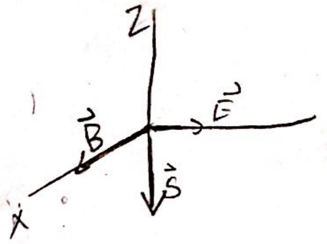
b) $\oint \vec{B} \cdot d\vec{l} = \mu_0 i$ $\quad 2\cancel{\pi} r B = \mu_0 J \cancel{\pi} r^{\cancel{2}}$ $\quad B = \frac{\mu_0 J r}{2} = \frac{\mu_0}{2}(174.1)(0.02) = \boxed{2.19 \times 10^{-6}\, T}$

$r = 0.02m$

c) $\Phi_E = 8200t^4$ $\quad \varepsilon = 4 \times 10^{-11}$

$i_d = \varepsilon \frac{d\Phi_E}{dt} = (4 \times 10^{-11})(4 \cdot 8200t^3) = 1.31 \times 10^{-6} t^3 = 30 \times 10^{-6} A$

$t = \sqrt[3]{\frac{30 \times 10^{-6}}{1.31 \times 10^{-6}}} = \boxed{2.84s}$



Problem 5 (10 pts)

An electromagnetic wave traveling in $-z$ direction in vacuum has a wavelength $\lambda = 6\times10^{-7}$ m. The magnetic field associated with such an electromagnetic wave is along $+x$ direction and has an amplitude $B_{max} = 6\times10^{-8}$ T. Note for the vectors below, your expression should reflect the direction.

(a) (3 pts) Please derive the expression for the wave corresponding to the magnetic field $\vec{B}$.

(b) (3 pts) Please derive the expression for the wave corresponding to the electric field $\vec{E}$.

(c) (2 pts) Please derive the expression for the Poynting vector $\vec{S}$.

(d) (2 pts) Find the instantaneous values of the total energy density u.

$v = -z \quad \lambda = 6\times10^{-7} \quad \vec{B} \to +x \quad B_{max} = 6\times10^{-8}\,T$

a) $\vec{B} = B_{max}\cos(kz+\omega t)\hat{i} = \boxed{6\times10^{-8}\cos(1.05\times10^{7}z + 3.14\times10^{15}t)\hat{i}}$

$k = \frac{2\pi}{\lambda} = 1.05\times10^{7}\,m^{-1} \quad \omega = 2\pi f = 3.14\times10^{15}\,rad/s$

$c = \lambda f \quad f = \frac{c}{\lambda} = 5\times10^{14}\,Hz$

b) $\vec{E} = E_{max}\cos(kz+\omega t)\hat{j} = \boxed{18\cos(1.05\times10^{7}z + 3.14\times10^{15}t)\hat{j}}$

$E_{max} = cB_{max} = (3\times10^{8})(6\times10^{-8}) = 18\,V/m$

c) $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = \boxed{-0.859\cos^2(1.05\times10^{7}z + 3.14\times10^{15}t)\hat{k}}$

$\vec{E}\times\vec{B} = -(6\times10^{-8})(18)\cos^2(1.05\times10^{7}z + 3.14\times10^{15}t)\hat{k}$

d) $u = \varepsilon_0 E^2 = \varepsilon_0 18^2\cos^2(1.05\times10^{7}z + 3.14\times10^{15}t)$

$\boxed{u = 2.87\times10^{-9}\cos^2(1.05\times10^{7}z + 3.14\times10^{15}t)}$