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Student ID.....

## MIDTERM II

### Physics 1C - Dantchev - Summer Quarter 2015

Good luck!

Indicate your reasoning clearly and include all essential calculations on the pages given. If your work is not entirely on the given problem's page, indicate exactly on what page it continues, or is located. Use the other side of the page if you need that much extra space. It is in your interest to have the answer clearly stated at the end of the solution of a given problem.

Start with the problem that is simplest for you, solve it and go to the next one. First solve those parts of the problems that you find easy. After that concentrate on more difficult parts.

## Problem 1

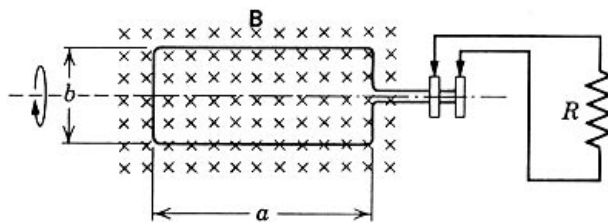
As a new electrical engineer for a given power company, you are assigned the project of designing a commercial alternating current generator based on a rectangular loop of  $N$  turns, each of length  $a = 2\text{ m}$  and width  $b = 0.5\text{ m}$ , that is rotating in a uniform magnetic field  $B = 1\text{ mT}$  perpendicular to the axis of the rotation of the loop (see the figure).

A) What should be the number of turns in such a generator if it is to be used in the US, where it has to supply  $\mathcal{E}_{\max} = 170\text{ V}$  at  $f = 60\text{ Hz}$ . Here  $\mathcal{E}_{\max}$  is the maximal magnitude of the produced emf. (The well-known value of  $\mathcal{E}_{\text{rms}} = 120\text{ V}$  in a commercial power supply (in the US) is the so-called root-mean-square value of  $\mathcal{E}$ . It is connected to  $\mathcal{E}_{\max}$  by  $\mathcal{E}_{\max} = \sqrt{2}\mathcal{E}_{\text{rms}}$ .)

B) Find  $N$  if you have to design such a generator for Europe, where it has to supply  $\mathcal{E}_{\max} = 340\text{ V}$  at  $f = 50\text{ Hz}$ .

C) How will the supplied emf  $\mathcal{E}$  vary with time in the both cases? Make a sketch.

D) How does the flux  $\Phi_B$  through the loop depend on the time? Make a sketch.



### Solution of Problem 1

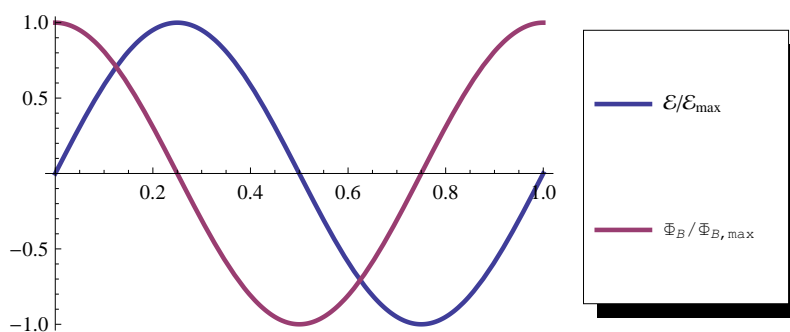
A)  $\mathcal{E}_{\max} = \omega(NBA)$ ,  $A = ab$ ,  $\omega = 2\pi f \Rightarrow \mathcal{E}_{\max} = 2\pi fNBab$ . Thus

$$N_{\text{USA}} = \frac{\mathcal{E}_{\max}^{\text{USA}}}{2\pi fNBab} = 451 \text{ turns.}$$

B)

$$N_{\text{Europe}} = \frac{\mathcal{E}_{\max}^{\text{Europe}}}{2\pi fNBab} = 1082 \text{ turns.}$$

C) and D) In both cases  $\mathcal{E}$  and  $\Phi_B$  will vary sinusoidally with time  $\mathcal{E} = \mathcal{E}_{\max} \sin(2\pi ft)$ ,  $\Phi_B = \Phi_{B,\max} \cos(2\pi ft)$ , where  $f = 60\text{ Hz}$  in USA and  $f = 50\text{ Hz}$  in Europe. Here  $\Phi_{B,\max} = Bab$ .

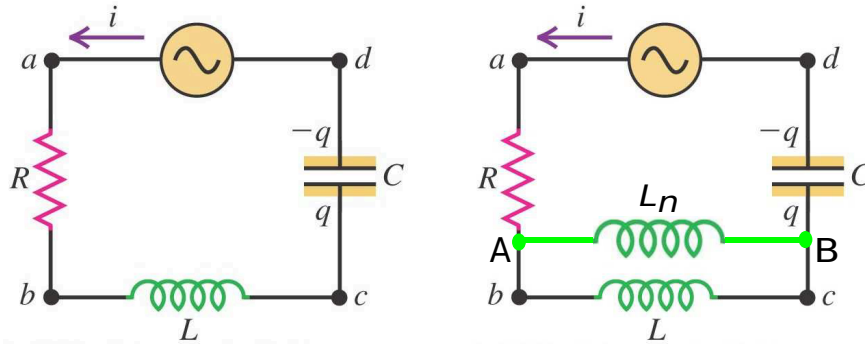


## Problem 2

In the  $L - R - C$  series circuit shown in the left figure it is known that  $v = V \cos(\omega t)$ , where  $f = \omega/(2\pi) = 60$  Hz, the voltage amplitude of the source is  $V = 120$  V,  $R = 80.0$   $\Omega$ , and the reactance of the capacitor is  $X_C = 480$   $\Omega$ . The voltage amplitude across the capacitor is  $V_C = 360$  V.

For the circuit in the left figure: 1. What is the current amplitude  $I$  in the circuit? 2. What is the impedance  $Z$  of the circuit? 3. What two values can the reactance of the inductor have? For which of these two values is the angular frequency less than the resonance angular frequency? Explain your answer. 4. Determine the average power  $P$  dissipated in the circuit. 5. Determine the possible values of the phase angle  $\Phi$  of the current  $i = I \cos(\omega t - \Phi)$  in the circuit.

A student makes a connection between the points A and B (right figure) through an inductor  $L = 1$  H and measures the current  $i$ . He establishes that the current  $i$  differs from the one on the left figure, i.e., he measures different  $I$  and  $\Phi$ . Can you find 6) what are the possible values of  $I$  and 7) of  $\Phi$  he measures?



### Solution of Problem 2

1. One has  $V_C = IX_C$ , thus  $I = V_C/X_C = 0.750$  A.

2. One has  $Z = \sqrt{R^2 + (X_L - X_C)^2}$ , with  $Z = VI$ . One knows  $V$  from the problem and  $I$  from 1). Thus,  $Z = V/I = 160$   $\Omega$ .

3. Since  $Z = \sqrt{R^2 + (X_L - X_C)^2}$  one has  $Z^2 - R^2 = (X_L - X_C)^2$ . Thus,  $X_L - X_C = \pm\sqrt{Z^2 - R^2}$ , i.e.  $X_L = X_C \pm \sqrt{Z^2 - R^2}$ . Thus, it is possible that  $X_L = 619$   $\Omega$ , or  $X_L = 341$   $\Omega$ . At the resonant frequency  $X_L = X_C$ , with  $X_C$  fixed at its given value of  $X_C = 480$   $\Omega$ . Obviously, since  $X_L = \omega L$  and  $341 < 480$ ,  $X_L = 341$   $\Omega$  corresponds to the frequency smaller than the resonant ones.

4. One has  $P = \frac{1}{2}IV \cos \phi = \frac{1}{2}IVR/Z = \frac{1}{2}I^2R = 22.5$  W

5. One has  $\tan \phi = \frac{X_L - X_C}{R} = \pm\sqrt{Z^2 - R^2}/R = \pm 1.74$ . Thus,  $\phi = \pm 60^\circ$ .

6. and 7.

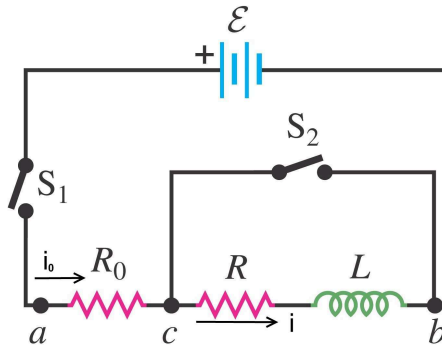
The two inductors are connected in parallel and are equivalent to a single inductor with  $L_{\text{eff}}^{-1} = L^{-1} + L_n^{-1}$  (see problem 30.78 from the homework), i.e.,  $X_{L_{\text{eff}}}^{-1} = X_L^{-1} + X_{L_n}^{-1}$ . For the corresponding  $R - L_{\text{eff}} - C$  circuit one has  $\tan \Phi = (X_{L_{\text{eff}}} - X_C)/R$  and  $I = V/\sqrt{R^2 + (X_{L_{\text{eff}}} - X_C)^2}$ . With  $X_L = 619$   $\Omega$  one obtains  $X_{L_{\text{eff}}} = 234.297$   $\Omega$  and, thus  $\tan \Phi = -3.071$ , i.e.,  $\Phi = -71.965^\circ$ . With  $X_L = 341$   $\Omega$  one obtains  $X_{L_{\text{eff}}} = 179.047$   $\Omega$  and, thus  $\tan \Phi = -3.7626$ , i.e.,  $\Phi = -75.114^\circ$ .

### Problem 3

In the circuit shown in the figure  $\mathcal{E} = 36.0 \text{ V}$ ,  $R_0 = 50.0 \Omega$ ,  $R = 150 \Omega$  and  $L = 4.00 \text{ H}$ .

Initially, both the switch  $S_1$  and the switch  $S_2$  are left open. a) Find the values of the currents  $i_0$  through  $R_0$  and  $i$  through  $R$ , as well as the potential differences  $v_{ab}$  and  $v_{cb}$ , immediately after the switch  $S_1$  is closed. b) Find the expressions for  $i_0(t)$ ,  $i(t)$ ,  $v_{ac}(t)$  and  $v_{cb}(t)$  as a function of time  $t$ . c) With the switch  $S_2$  still open what will be the values of  $i_0$ ,  $i$ ,  $v_{ab}$  and  $v_{cb}$  after  $S_1$  has been closed for a very long time, so that the currents have reached their steady final values?

After the currents have reached their steady values with  $S_1$  closed and  $S_2$  open, the switch  $S_2$  has been closed too. d) What are  $i_0$ ,  $i$ ,  $v_{ab}$  and  $v_{cb}$  just after  $S_2$  is closed. e) What are  $i_0$ ,  $i$ ,  $v_{ab}$  and  $v_{cb}$  a long time after  $S_2$  is closed.



#### Solution of Problem 3

a) Once the switch  $S_1$  is closed one obtains a complete circuit with  $\mathcal{E}$ ,  $R_0$ ,  $R$  and  $L$  connected in series. Thus,  $i = i_0$ . At the moment  $t = 0$  the inductor blocks the circuit and, therefore,  $i(t = 0) = i_0(t = 0) = 0$ . In this case  $v_{ac}(0) = i_0(0)R_0 = 0$ . From the loop rule  $\mathcal{E} = v_{ab} = v_{ac} + v_{cb}$ . Thus,  $v_{cb}(0) = \mathcal{E} - v_{ac}(0) = \mathcal{E}$ .

b) At any instant  $i(t) = i_0(t)$ . The two resistors are equivalent to one with resistance  $R_{\text{tot}} = R_0 + R = 200 \Omega$ . The circuit is then equivalent to the  $L - R$  one studied in lecture with  $R$  replaced by  $R_{\text{tot}}$ . Thus, one has

$$i(t) = i_0(t) = \frac{\mathcal{E}}{R_{\text{tot}}} [1 - \exp(-t/\tau_L)], \quad \text{where} \quad \tau_L = \frac{L}{R_{\text{tot}}} = 0.020 \text{ s}.$$

In addition  $v_{ab} = \mathcal{E}$ , while

$$v_{ac}(t) = R_0 i_0(t) = \frac{\mathcal{E} R_0}{R_{\text{tot}}} [1 - \exp(-t/\tau_L)], \quad v_{cb}(t) = \mathcal{E} - v_{ac}(t) = \mathcal{E} \left\{ 1 - \frac{R_0}{R_{\text{tot}}} [1 - \exp(-t/\tau_L)] \right\}$$

c) When  $t \rightarrow \infty$ , from b) one obtains  $i(\infty) = i_0(\infty) = \mathcal{E}/R_{\text{tot}} = 0.180 \text{ A}$ ;  $v_{ac}(\infty) = R_0 i_0(\infty) = 9.0 \text{ V}$ ;  $v_{cb}(\infty) = \mathcal{E} - v_{ac}(\infty) = 36 - 9 = 27 \text{ V}$ ;  $v_{ab} = 36 \text{ V}$ .

d) Immediately after the switch  $S_2$  is closed, the inductor maintains the current through  $R$ , thus  $i = 0.180 \text{ A}$  (see c)). Since  $c$  and  $b$  are now connected with conductor of zero resistance  $v_{cb} = 0$ . From the internal loop containing only  $R_0$  and  $\mathcal{E}$ , one has  $\mathcal{E} - i_0 R_0 = 0$ , thus  $i_0 = \mathcal{E}/R_0 = 36/50 = 0.720 \text{ A}$  and  $v_{ac} = i_0 R_0 = 36 \text{ V}$ .

e) After a long time  $i = 0$ ,  $v_{cb} = 0$ ,  $i_0 = \mathcal{E}/R_0 = 36/50 = 0.720 \text{ A}$ ,  $v_{ac} = i_0 R_0 = 36 \text{ V}$ .

## Problem 4

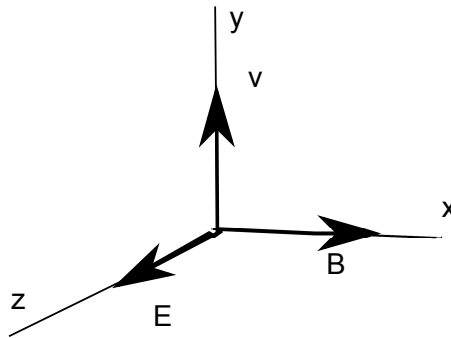
A plane electromagnetic sinusoidal wave of wave length  $\lambda = 6 \text{ m}$  travels in free space in the  $y$  direction. At some point and at some instant, the magnetic field  $\mathbf{B}$  has its maximum value of  $3.5 \times 10^{-8} \text{ T}$ . The magnetic field is along the  $x$  axis.

- Determine the frequency  $f$  and the period  $T$  of the wave.
- Calculate the maximum magnitude and determine the direction of the electric field  $\mathbf{E}$ . Make a sketch.
- Write expressions for the space-time variation of the electric and magnetic field components for the plane wave. Make a sketch.
- Find the intensity  $I$  of the wave ( $\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m}/\text{A}$ ).
- What (average) force does this radiation exert on a totally reflecting surface with area  $1 \text{ km}^2$  perpendicular to the direction of propagation?

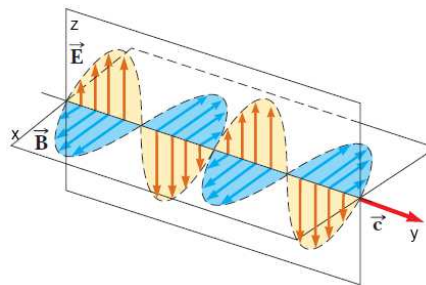
### Solution of Problem 4

a)  $\lambda f = c \Rightarrow f = c/\lambda = 0.5 \times 10^8 \text{ s}^{-1}$ .  
 $T = 1/f = 2 \times 10^{-8} \text{ s}$ .

b)  $E_{\text{max}} = cB_{\text{max}} = 10.5 \text{ V/m}$ . Since the wave propagates along the  $y$  axis, which shall coincide with the direction of  $\vec{E} \times \vec{B}$ , and since  $\vec{B}$  is along the  $x$  axis, the  $\vec{E}$  field shall along the  $z$  axis.



c) One has  $\vec{E} = \vec{k} \cos(ky - \omega t)$ ,  $\vec{B} = \vec{i} \cos(ky - \omega t)$ , with  $k = 2\pi/\lambda$ , and  $\omega = 2\pi f$ .



d)  $I = \bar{S} = \frac{1}{2\mu_0} E_{\text{max}} B_{\text{max}} = 0.146 \text{ W/m}^2$ .

e) For a totally reflecting surface  $p_{\text{rad}} = 2I/c$ . The radiation force is  $F = Ap_{\text{rad}} = 9.7 \times 10^{-4} \text{ N}$ .