Name................................

Student ID.......................

FINAL

Physics 1C - Dantchev - Summer Quarter 2013

Good luck!

Choose 7 out of 8 problems.

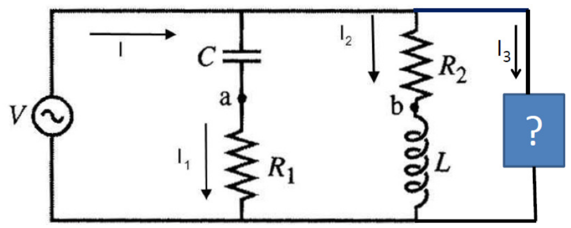
All 8 problems will be graded but ...
only the 7 with highest scores will be taken into account.

Indicate your reasoning clearly and include all essential calculations on the pages given. If your work is not entirely on the given problem's page, indicate exactly on what page it continues, or is located. Use the other side of the page if you need that much extra space. It is in your interest to have the answer clearly stated at the end of the solution of a given problem.

Start with the problem that is simplest for you, solve it and go to the next one. *First* solve those parts of the problems that you find *easy*. *After* that concentrate on more difficult parts.

Problem **1**

Consider the circuit shown in the figure, where $V = V_0 \cos(\omega t)$ with $V_0 = 60$ V.



It is known that $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, $L = 1\ mH$, $C = 100\ \mu F$, $f = 60\ Hz$. The box contains an RLC series circuit whose elements we do not know. Measurements outside the box reveal that $I_3(t) = (3\ A)\cos(\omega t - 50^0)$.

1. Find the current $I_1 = I_{1,0}\cos(\omega t - \phi_1)$ in resistor R_1;
2. Find the current $I_2 = I_{2,0}\cos(\omega t - \phi_2)$ in resistor R_2;
3. Find the total current $I = I_0 \cos(\omega t - \phi)$ drawn from the source;
4. What is the impedance Z_B of the "black box" circuit (BBC)?
5. What is the power factor of the BBC?
6. Must there be a resistor in the BBC? Justify your answer. Could you find the value of R_B?
7. Must there be an inductor in the BBC? Justify your answer.
8. What average power is delivered to the box by the generator?
9. Determine the power P dissipated in the whole circuit.

Solution of Problem 1

1. For the capacitor branch one has $V_0\cos(\omega t) = Q/C + I_1 R_1$ with C and R_1 connected in series. The current through the capacitor and the resistor is the same, $I_1 = I_{1,0}\cos(\omega t - \phi_1)$. Thus $I_{1,0} = V_0/Z_1 \simeq 2.12\ A$, where $Z_1 = \sqrt{R_1^2 + X_C^2} \simeq 28.35\ \Omega$, with $X_C = 1/(\omega C) \simeq 26.53\ \Omega$. Then $\tan\phi_1 = -X_C/R_1 \simeq -2.65$, i.e. $\phi_1 = -69.34°$.

2. For the inductor branch we have R_2 and L connected in series, $V_0\cos(\omega t) = I_2 R_2 + L\frac{dI_2}{dt}$. For the current I_2 one has $I_2 = I_{2,0}\cos(\omega t - \phi_2)$ where $I_{2,0} = V_0/Z_2 \simeq 3.00\ A$, with $Z_2 = \sqrt{R_2^2 + X_L^2} \simeq 20.00\ \Omega$, and $X_L = \omega L \simeq 0.38\ \Omega$. Then $\tan\phi_2 = X_L/R_2 \simeq 0.02$, i.e. $\phi_2 = 1.08°$.

3. From the Kirchhoff's rule $I = I_1 + I_2 + I_3$. We use the phasor diagram technique , i.e. $\vec{I} = \vec{I_1} + \vec{I_2} + \vec{I_3}$. Thus, for $t = 0$, $I_x = I_{1,0}\cos(-\phi_1) + I_{2,0}\cos(-\phi_2) + I_{3,0}\cos(-\phi_3)$ and $I_y = I_{1,0}\sin(-\phi_1) + I_{2,0}\sin(-\phi_2) + I_{3,0}\sin(-\phi_3)$, where $I_0 = \sqrt{I_{0,x}^2 + I_{0,y}^2}$ and $\tan(\phi) = I_y/I_x$. Numerically one has $I_x = 5.68\ A$, $I_y = 0.37\ A$, i.e., $I_0 = 5.69$A, and $\tan(\phi) = 0.065$, i.e. $\phi = 3.74°$.

4. We know that $V = V_0\cos(\omega t)$ and $I(t) = I_{3,0}\cos(\omega t - \phi_3)$ with $V_0 = 60$ V and $I_{3,0} = 3$ A, $\phi_3 = 50^0$. Thus, $Z_B = V_0/I_{3,0} = 60/3 = 20\ \Omega$.

5. The power factor of the BBC is $\cos(\phi_3) = \cos(50^0) = 0.64$.

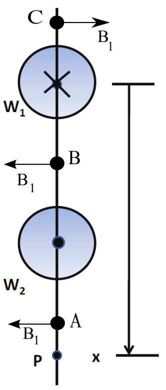
6. "Yes", since $\cos(\phi_3) = R_B/Z_B \neq 0$. One has $R_B = Z_B\cos(\phi_3) = 12.86\ \Omega$.

7. $\tan\phi_3 = \frac{X_L - X_C}{R_B}$. Since $\phi_3 = 50^0 > 0$ one has $X_L > X_C$ with $X_L > 0$, i.e. $X_L = \omega L_B > 0$. Thus, there must be an inductor in the BBC.

8. $\overline{P_B} = I_{3,\text{rms}} V_{\text{rms}}\cos\phi_3 = \frac{1}{2} I_{3,0} V_0 \cos\phi_3 = 57.85\ W$.

9. Power is dissipated only in the resistors R_1, R_2 and R_B. Thus

$$\overline{P} = \frac{1}{2}I_{1,0}^2 R_1 + \frac{1}{2}I_{2,0}^2 R_2 + \frac{1}{2}I_{3,0}^2 R_B \simeq 170.34\ W.$$



Problem **2**

Two long straight parallel wires, separated by $d = 1\ m$ (measured center-to-center), are perpendicular to the plane of the page as shown in the figure. Wire W_1 carries a current of $i_1 = 15\ A$ into the page. What must be the current i_2 (give the magnitude and direction) in wire W_2 for the resultant magnetic field $\mathbf{B}_P$ at point P to be zero? What will be the force per unit length (give the magnitude and direction) between the wires then? Consider the cases when the coordinate x is: a) $x = 1.5\ m$; b) $x = 0.75\ m$; c) $x = -0.75\ m$. (The coordinate x is measured from the *center* of the wire W_1. The positive x direction is downwards; $\mu_0 = 4\pi \times 10^{-7}\ T.m/A$)

Let $D = 30$ mm is the diameter of wire W_1. For the case $i_2 = 0$ determine the magnetic field $\mathbf{B}$: i) at the surface of the wire W_1; ii) at the center of the wire W_1; iii) inside the wire W_1, 5 mm below the surface; iv) outside the wire W_1, 25 mm from the surface. v) Make a sketch how the magnetic field changes with the distance from the center of the wire W_1.

Solution of Problem 2

a) Let $\vec{B}_1$ is the magnetic field due to the wire 1, while $\vec{B}_2$ is the one due to the wire 2. For the points A, B and C with coordinates (not to scale) $x = 1.5$ m, $x = 0.75$ m and $x = -0.75$ m, correspondingly, the direction of $\vec{B}_1$ is shown on the figure. Thus, for $x = 1.5$ m (point A) i_2 has to be out of the page. Furthermore, since

$$\frac{\mu_0}{2\pi}\frac{i_1}{x} = \frac{\mu_0}{2\pi}\frac{i_2}{x-1},$$

one obtains that $i_2 = \frac{x-1}{x} i_1 = 5$ A. The wires then repel each other with a force per unit length $F/L = \frac{\mu_0}{2\pi}\frac{i_1 i_2}{d} = 1.5 \times 10^{-5}$ N/m.

b) For $x = 0.75$ (point B) i_2 is into the page. Then W_1 and W_2 attract each other. One has

$$\frac{\mu_0}{2\pi}\frac{i_1}{x} = \frac{\mu_0}{2\pi}\frac{i_2}{1-x} \Rightarrow i_2 = \frac{1-x}{x} i_1 = 5\text{A}.$$

The force per unit length $F/L = \frac{\mu_0}{2\pi}\frac{i_1 i_2}{d} = 1.5 \times 10^{-5}$ N/m.

c) For $x = -0.75$ (point C) i_2 shall again be out of the page and

$$\frac{\mu_0}{2\pi}\frac{i_1}{|x|} = \frac{\mu_0}{2\pi}\frac{i_2}{1+|x|} \Rightarrow i_2 = \frac{1+|x|}{|x|} i_1 = 35\text{A}.$$

The wires then again repel each other with a force per unit length $F/L = \frac{\mu_0}{2\pi}\frac{i_1 i_2}{d} = 1.05 \times 10^{-4}$ N/m.

i) $D/2 = R = 15$ mm. From the Ampere's law $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$ one has that

$$B(2\pi R) = \mu_0 i_1 \Rightarrow B = \frac{\mu_0}{2\pi}\frac{i_1}{R} = 2 \times 10^{-4}\text{T}.$$

ii) $I_{\text{encl}} = 0$ for this case $\Rightarrow B = 0$.

iii) Choosing the Ampere's loop to be a circle centered at the center of W_1, one obtains

$$B(2\pi r) = \mu_0 i_1 \frac{\pi r^2}{\pi R^2},$$

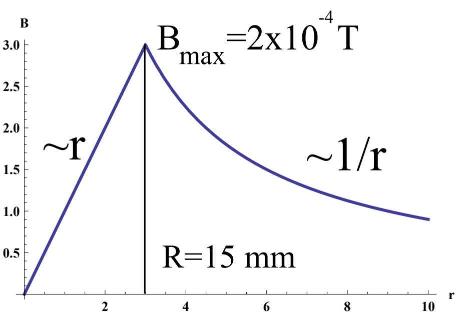
where $r = R - 5 = 10$ mm. Thus,

$$B = \frac{\mu_0}{2\pi} i_1 \frac{r}{R^2} = 1.33 \times 10^{-4}\text{T}.$$

iv) In this case $B(2\pi r) = \mu_0 i_1$, where $r = R + 25$ mm $= 40$ mm. Then

$$B = \frac{\mu_0}{2\pi}\frac{i_1}{r} = 7.5 \times 10^{-5}\text{T}.$$

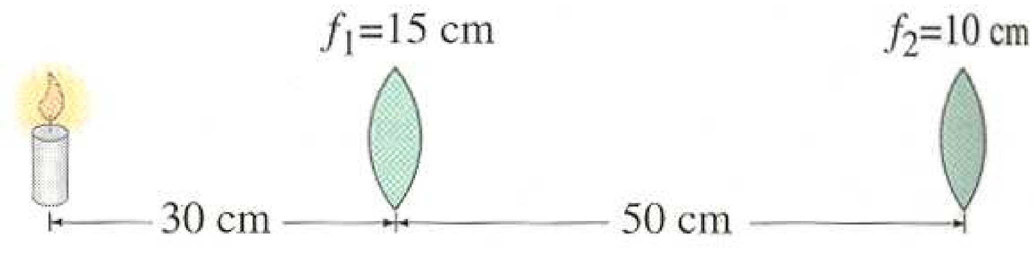
v)



Problem **3**

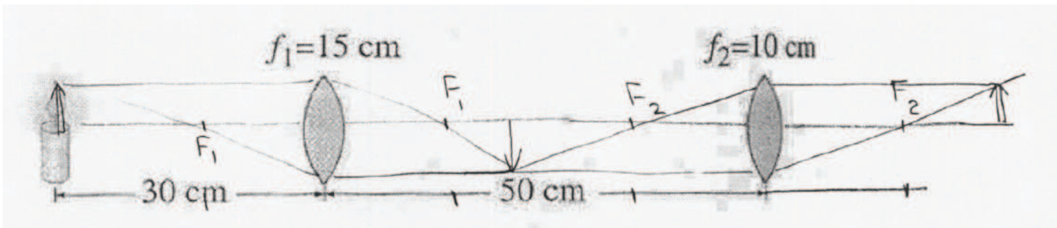
Two thin converging lenses of focal lengths $f_1 = 15$ *cm* and $f_2 = 10$ *cm* are separated by 50 *cm*. A lighted candle is placed 30 *cm* in front of the first lens.

a) Find the position of the final image. Make a sketch.
b) Find the lateral magnification of the system.
c) Is the final image upright or inverted?
d) Answer the above questions if the distance between the lenses were 20 *cm*.



Solution of Problem 3

a) Since $\frac{1}{s_1} + \frac{1}{s_1'} = \frac{1}{f_1}$, with $f_1 = 15$ cm and $s_1 = 30$ cm one obtains $s_1' = 30$ cm to the right of the first lens, 20 cm in front of the second one, i.e. $s_2 = 20$ cm. From $\frac{1}{s_2} + \frac{1}{s_2'} = \frac{1}{f_2}$ one obtains $s_2' = 20$ cm to the right of the second lens.



b) $m = m_1 m_2$; $m_1 = -s_1'/s_1 = -30/30 = -1$ and $m_2 = -s_2'/s_2 = -20/20 = -1$. Thus, $m = (-1)^2 = 1 > 0$.
c) Since $m > 0$, the image is upright, the same size as the object - due to $m = 1$.
d) In this case s_1' [see *a)*] will be 10 cm to the right of the second lens, i.e. $s_2 = -10$ cm. From $\frac{1}{s_2} + \frac{1}{s_2'} = \frac{1}{f_2}$ one obtains $s_2' = 5$ cm to the right of the second lens. The magnification of the system is $m = m_1 m_2$, where $m_1 = -s_1'/s_1 = -30/30 = -1$ (again), but $m_2 = -s_2'/s_2 = -5/(-10) = 1/2$. Thus $m = (-1)(1/2) = -1/2$ and, therefore, the final image will be inverted ($m < 0$) and reduced ($|m| < 1$).

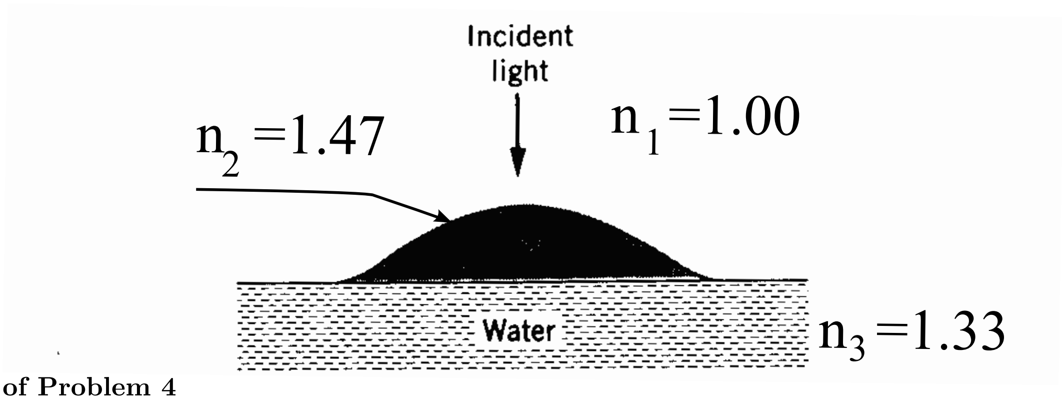
Problem 4

An oil drop ($n = 1.47$) floats on a water ($n = 1.33$) surface and is observed from above by reflected light.

a) Will the outer (thinnest) region of the drop correspond to a bright, or a dark region? Justify your answer.

b) How thick is the oil film where one observes the second green region ($\lambda = 546$ nm) from the outside of the drop?

c) In the lab an experiment has been performed and it has been reported that when light with wavelength $\lambda = 600\ nm$ is incident normally to the middle of the drop the reflected light is minimum, while for $\lambda = 500\ nm$ it is maximum. Asking himself what can then the smallest possible thickness at the middle of the drop be, a clever student concluded that there must be a mistake while performing the experiment. He reported that to the professor and the experiment has been repeated. Explain why was the student sure that there must be a mistake in the experimental data reported?



Solution of Problem 4

a) Since $n_2 > n_1$, the ray reflected from the air-oil interface will undergo a phase change of π. For the ray reflected from the oil-water interface one has $n_2 > n_3$, thus this ray will not suffer a phase change of π. Therefore, the condition for observing interference maximum (constructive interference) between these two rays is

$$2d\,n_2 = (m + \frac{1}{2})\lambda. \tag{1}$$

When $d = 0$ this constraint is not obeyed, thus the outer (thinnest) region of the drop will be dark.

b) One will obtain the first green region from Eq. (1) with $m = 0$ in it. Then, the second green region will be the one with $m = 1$. Thus, with $\lambda = 546\ nm$, Eq. (1) becomes

$$2d\, \times 1.47 = (1 + \frac{1}{2})\ 546,$$

which leads to $d = 279$ nm.

c) The condition for having minimum reflected light at the middle of the drop is

$$2d\,n_2 = m_{\min}\lambda_{\min}, \tag{2}$$

while for having maximum it is given by Eq. (1), i.e.

$$2d\,n_2 = \left(m_{\max} + \frac{1}{2}\right)\lambda_{\max}. \tag{3}$$

It is given that $\lambda_{\min} = 600\ nm$, while for maximum one has $\lambda_{\max} = 500\ nm$. From Eqs. (2) and (3) one derives

$$m_{\min}\lambda_{\min} = \left(m_{\max} + \frac{1}{2}\right)\lambda_{\max}, \tag{4}$$

or, plugging there the particular values of $\lambda_{\min}$ and $\lambda_{\max}$

$$12m_{\min} = 10m_{\max} + 5, \tag{5}$$

where $m_{\min}$ and $m_{\max}$ are integer numbers. Since $10m_{\max} + 5$ is an odd integer, while $12m_{\min}$ is an even integer, this equation has no solutions. Therefore, it shall be a mistake in the experimental data reported.

Problem 5

An airplane flying at a distance of 11.3 km from a radio transmitter receives a signal of intensity 7.83 $\mu W/m^2$. Calculate

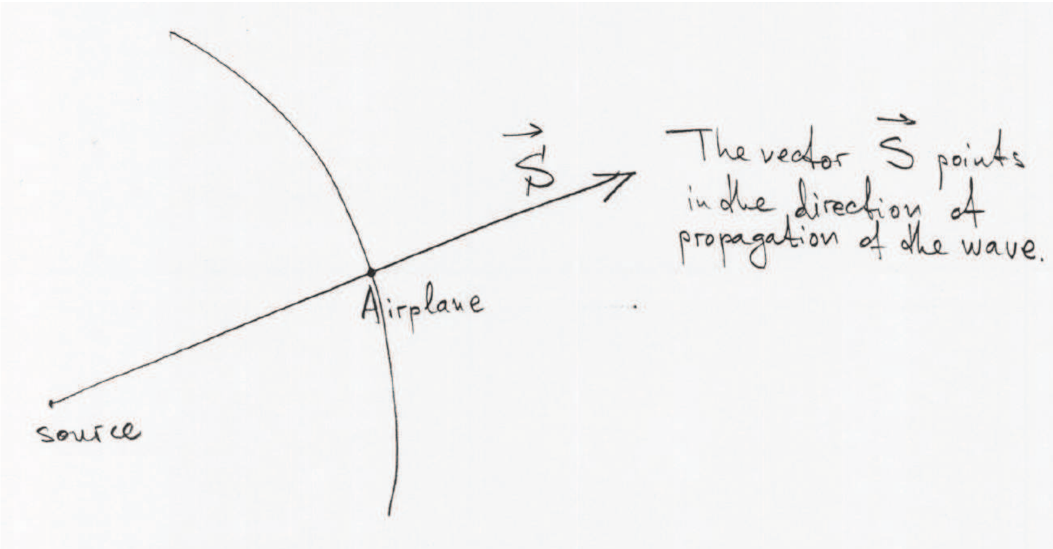
a) the amplitude of the electric field E_m at the airplane due to this signal.
b) the maximum value of the magnetic field B_m at the airplane.
c) the total power radiated by the transmitter, assuming the transmitter to radiate uniformly in all directions.
d) what is the average value of the Poynting vector S and its direction at the airplane? Make a sketch.

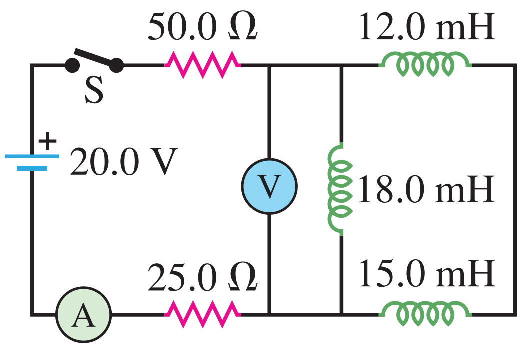
Solution of Problem 5

a)

$$I = 7.83 \times 10^{-6}\ \text{W/m}^2 = \frac{1}{2\mu_0 c} E_m^2 \Rightarrow E_m = \sqrt{2\mu_0 c I} = 76.8\ \text{mV/m}.$$

b) $B_m = E_m/c = 256\ \text{pT} = 2.56 \times 10^{-10}\text{T}$.
c) $P = 4\pi r^2 I$, where $r = 11.3$ km $\Rightarrow P = 12.56\ kW$.
d) $\overline{S} = I = 7.83\ \mu\text{W/m}^2$.





Problem **6**

In the circuit shown in the figure, the switch has been open for a long time and is suddenly closed. Neither the battery nor the inductors have any appreciable resistance.
(a) What do the ammeter and voltmeter read just after S is closed?
(b) What do the ammeter and the voltmeter read after S has been closed a very long time?
(c) What do the ammeter and the voltmeter read 0.115 ms after S is closed?

Solution of Problem 6

This is the homework problem 30.61. Read, please, the corresponding solution from there.

Problem 7

Laser light of wavelength 632.8 nm falls normally on a slit that is 0.0250 mm wide. The transmitted light is viewed on a distant screen where the intensity at the center of the central bright fringe is 8.50 W/m^2.

(a) Find the maximum number of totally dark fringes on the screen, assuming the screen is large enough to show them all.

(b) At what angle does the dark fringe that is most distant from the center occur?

(c) What is the maximum intensity of the bright fringe that occurs immediately before the dark fringe in part (b)? Approximate the angle at which this fringe occurs by assuming it is midway between the angles to the dark fringes on either side of it.

Solution of Problem 7

This is the homework problem 36.53. Read, please, the corresponding solution from there.

Problem 8

In the figure given below $a = 27\ cm$ and $b = 36\ cm$. It is also known that the current in the long straight wire, which is insulated from the loop, changes with the time t according to the law

$$i(t) = 5(t^2 - 15t + 56)(t - 4),$$

where i is in amperes and t is in seconds. The direction of the current shown in the picture assumes that $i > 0$.

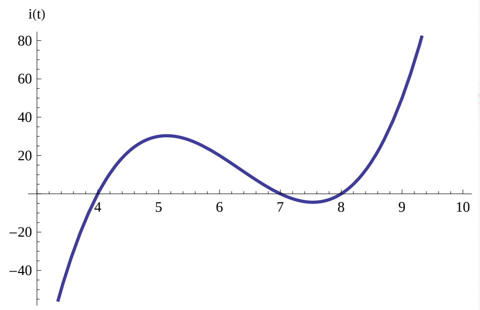
a) What is the direction of the magnetic field at points P and Q as a function of time? Assume that $t \geq 0$.

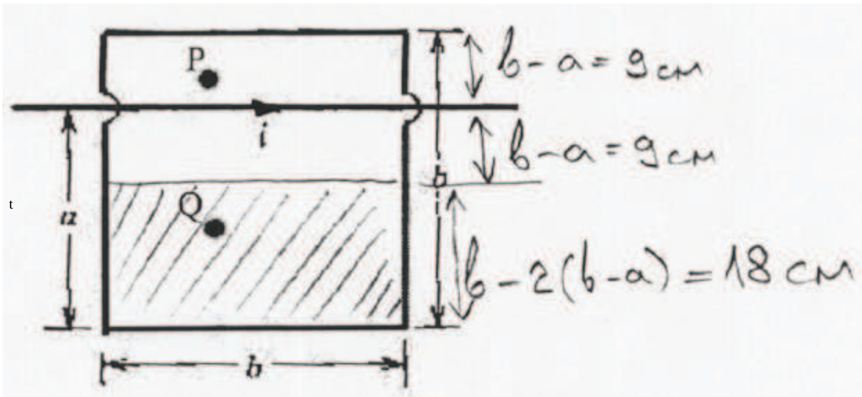
b) What is the flux of the magnetic field Φ_B through the square loop as a function of t?

c) Find the induced electromotive force $\mathcal{E}$ in the square loop as a function of t.

d) At which moments t does the emf $\mathcal{E}$ change its direction?

e) Find the emf in the square loop at $t = 3.0\ s$. What is the direction of this emf? Make a sketch.





Solution of Problem 8

a) $i(t) = 0$ for $t = 4, 7, 8$ s, $i(0) < 0$ for $t = 0$ and, thus, $i(t) < 0$ for $t \in [0, 4)$; $i(t) > 0$ for $t \in [4, 7)$, $i(t) < 0$ for $t \in [7, 8)$ and, finally, $i(t) > 0$ for $t \in [8, \infty)$. At the point P the magnetic field is out of the page for $i > 0$ and into the page for $i < 0$. At the point Q the magnetic field is into the page for $i > 0$ and out of the page for $i < 0$.

b) $B = \frac{\mu_0}{2\pi}\frac{i}{r}$, where r is the distance from the wire. The fluxes in the both symmetrical regions (on the both sides of the wire) will cancel each other. Defining the normal to the plane of the square loop to point into the page, we obtain for the flux

$$\Phi_B(t) = \int_{b-a}^{a} \frac{\mu_0}{2\pi}\frac{i(t)}{r}(bdr) = \frac{\mu_0}{2\pi} i(t) b \ln\frac{a}{b-a} = -\frac{\mu_0}{2\pi} b \ln\left(\frac{b}{a} - 1\right) i(t) = 3.955 \times 10^{-7}(t^2 - 15t + 56)(t - 4).$$

c)

$$\mathcal{E}(t) = -\frac{d\Phi_B(t)}{dt} = -3.955 \times 10^{-7}(3t^2 - 38t + 116).$$

d) We look for $\mathcal{E}(t) = 0$. Solving, we obtain $t_1 = \frac{1}{3}(19 - \sqrt{13}) = 5.13$ and $t_2 = \frac{1}{3}(19 + \sqrt{13}) = 7.53$.

e) $\mathcal{E}(t = 3) = -3.955 \times 10^{-7}(3t^2 - 38t + 116)|_{t=3} = -11.5 \times 10^{-6}$ V. Since $\mathcal{E}(t = 3) < 0$ the direction of $\mathcal{E}$ has to be in the direction opposite to that one given by the right-hand rule - if one grasps the normal to the square with his right hand his fingers will point in the positive direction of $\mathcal{E}$, i.e. $\mathcal{E}$ at $t = 3$ shall be in the counterclockwise direction looking down the page.