Name...............................

Student ID.......................

FINAL

Physics 1C - Dantchev - Summer Quarter 2011

Good luck!

Choose 7 out of 8 problems.

All 8 problems will be graded but ...
only the 7 with highest scores will be taken into account.

Indicate your reasoning clearly and include all essential calculations on the pages given. If your work is not entirely on the given problem's page, indicate exactly on what page it continues, or is located. Use the other side of the page if you need that much extra space. It is in your interest to have the answer clearly stated at the end of the solution of a given problem.

Start with the problem that is simplest for you, solve it and go to the next one. *First* solve those parts of the problems that you find *easy*. *After* that concentrate on more difficult parts.

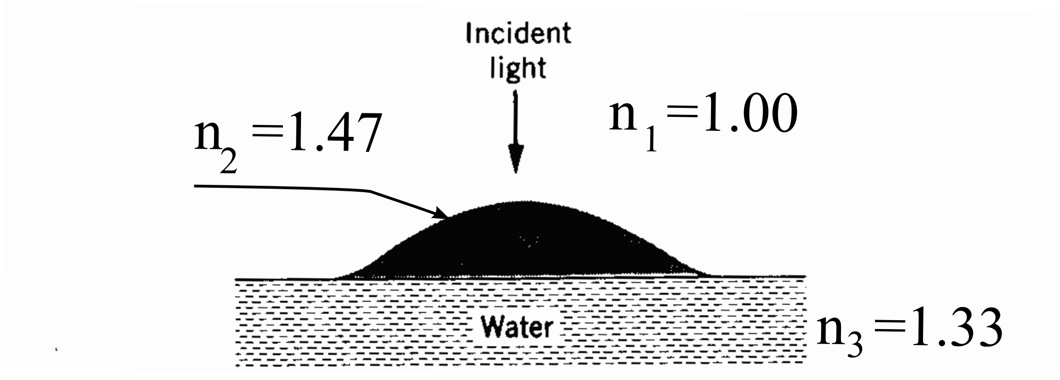
Problem 1

An oil drop ($n = 1.47$) floats on a water ($n = 1.33$) surface and is observed from above by reflected light.

a) Will the outer (thinnest) region of the drop correspond to a bright, or a dark region? Justify your answer.

b) How thick is the oil film where one observes the second green region ($\lambda = 546$ nm) from the outside of the drop?

c) In the lab an experiment has been performed and it has been reported that when light with wavelength $\lambda = 600\ nm$ is incident normally to the middle of the drop the reflected light is minimum, while for $\lambda = 500\ nm$ it is maximum. Asking himself what can then the smallest possible thickness at the middle of the drop be, a clever student concluded that there must be a mistake while performing the experiment. He reported that to the professor and the experiment has been repeated. Explain why was the student sure that there must be a mistake in the experimental data reported?



Solution of Problem 1

a) Since $n_2 > n_1$, the ray reflected from the air-oil interface will undergo a phase change of π. For the ray reflected from the oil-water interface one has $n_2 > n_3$, thus this ray will not suffer a phase change of π. Therefore, the condition for observing interference maximum (constructive interference) between these two rays is

$$2d\,n_2 = (m + \frac{1}{2})\lambda. \tag{1}$$

When $d = 0$ this constraint is not obeyed, thus the outer (thinnest) region of the drop will be dark.

b) One will obtain the first green region from Eq. (1) with $m = 0$ in it. Then, the second green region will be the one with $m = 1$. Thus, with $\lambda = 546\ nm$, Eq. (1) becomes

$$2d\ \times 1.47 = (1 + \frac{1}{2})\ 546,$$

which leads to $d = 279$ nm.

The TA has been advised to accept as correct also the answer that one will get by plugging $m = 2$ in Eq. (1). The result in this case will be $d = 464\ nm$.

c) The condition for having minimum reflected light at the middle of the drop is

$$2d\,n_2 = m_{\min}\lambda_{\min}, \tag{2}$$

while for having maximum it is given by Eq. (1), i.e.

$$2d\,n_2 = \left(m_{\max} + \frac{1}{2}\right)\lambda_{\max}. \tag{3}$$

It is given that $\lambda_{\min} = 600\ nm$, while for maximum one has $\lambda_{\max} = 500\ nm$. From Eqs. (2) and (3) one derives

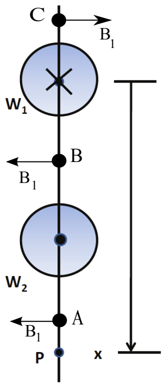
$$m_{\min}\lambda_{\min} = \left(m_{\max} + \frac{1}{2}\right)\lambda_{\max}, \tag{4}$$

or, plugging there the particular values of $\lambda_{\min}$ and $\lambda_{\max}$

$$12m_{\min} = 10m_{\max} + 5, \tag{5}$$

where $m_{\min}$ and $m_{\max}$ are integer numbers. Since $10m_{\max} + 5$ is an odd integer, while $12m_{\min}$ is an even integer, this equation has no solutions. Therefore, it shall be a mistake in the experimental data reported.

Problem **2**



Two long straight parallel wires, separated by $d = 1\ m$ (measured center-to-center), are perpendicular to the plane of the page as shown in the figure. Wire W_1 carries a current of $i_1 = 15\ A$ into the page. What must be the current i_2 (give the magnitude and direction) in wire W_2 for the resultant magnetic field $\mathbf{B}_P$ at point P to be zero? What will be the force per unit length (give the magnitude and direction) between the wires then? Consider the cases when the coordinate x is: a) $x = 1.5\ m$; b) $x = 0.75\ m$; c) $x = -0.75\ m$. (The coordinate x is measured from the *center* of the wire W_1. The positive x direction is downwards; $\mu_0 = 4\pi \times 10^{-7}\ T.m/A$)

Let $D = 30$ mm is the diameter of wire W_1. For the case $i_2 = 0$ determine the magnetic field $\mathbf{B}$: i) at the surface of the wire W_1; ii) at the center of the wire W_1; iii) inside the wire W_1, 5 mm below the surface; iv) outside the wire W_1, 25 mm from the surface. v) Make a sketch how the magnetic field changes with the distance from the center of the wire W_1.

Solution of Problem 2

a) Let $\overrightarrow{B}_1$ is the magnetic field due to the wire 1, while $\overrightarrow{B}_2$ is the one due to the wire 2. For the points A, B and C with coordinates (not to scale) $x = 1.5$ m, $x = 0.75$ m and $x = -0.75$ m, correspondingly, the direction of $\overrightarrow{B}_1$ is shown on the figure. Thus, for $x = 1.5$ m (point A) i_2 has to be out of the page. Furthermore, since

$$\frac{\mu_0}{2\pi}\frac{i_1}{x} = \frac{\mu_0}{2\pi}\frac{i_2}{x-1},$$

one obtains that $i_2 = \frac{x-1}{x} i_1 = 5$ A. The wires then repel each other with a force per unit length $F/L = \frac{\mu_0}{2\pi}\frac{i_1 i_2}{d} = 1.5 \times 10^{-5}$ N/m.

b) For $x = 0.75$ (point B) i_2 is into the page. Then W_1 and W_2 attract each other. One has

$$\frac{\mu_0}{2\pi}\frac{i_1}{x} = \frac{\mu_0}{2\pi}\frac{i_2}{1-x} \Rightarrow i_2 = \frac{1-x}{x} i_1 = 5\text{A}.$$

The force per unit length $F/L = \frac{\mu_0}{2\pi}\frac{i_1 i_2}{d} = 1.5 \times 10^{-5}$ N/m.

c) For $x = -0.75$ (point C) i_2 shall again be out of the page and

$$\frac{\mu_0}{2\pi}\frac{i_1}{|x|} = \frac{\mu_0}{2\pi}\frac{i_2}{1+|x|} \Rightarrow i_2 = \frac{1+|x|}{|x|} i_1 = 35\text{A}.$$

The wires then again repel each other with a force per unit length $F/L = \frac{\mu_0}{2\pi}\frac{i_1 i_2}{d} = 1.05 \times 10^{-4}$ N/m.

i) $D/2 = R = 15$ mm. From the Ampere's law $\oint \overrightarrow{B} \cdot d\overrightarrow{l} = \mu_0 I_{\text{encl}}$ one has that

$$B(2\pi R) = \mu_0 i_1 \Rightarrow B = \frac{\mu_0}{2\pi}\frac{i_1}{R} = 2 \times 10^{-4}\text{T}.$$

ii) $I_{\text{encl}} = 0$ for this case $\Rightarrow B = 0$.

iii) Choosing the Ampere's loop to be a circle centered at the center of W_1, one obtains

$$B(2\pi r) = \mu_0 i_1 \frac{\pi r^2}{\pi R^2},$$

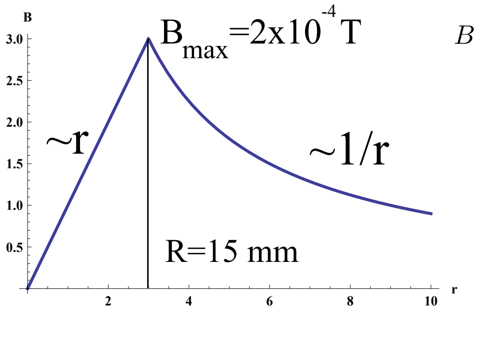
where $r = R - 5 = 10$ mm. Thus,

$$B = \frac{\mu_0}{2\pi} i_1 \frac{r}{R^2} = 1.33 \times 10^{-4}\text{T}.$$

iv) In this case $B(2\pi r) = \mu_0 i_1$, where $r = R + 25$ mm $= 40$ mm. Then

$$B = \frac{\mu_0}{2\pi}\frac{i_1}{r} = 7.5 \times 10^{-5}\text{T}.$$

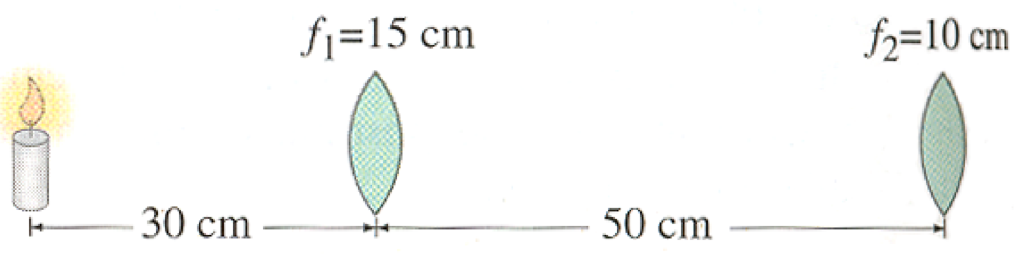
v)



Problem **3**

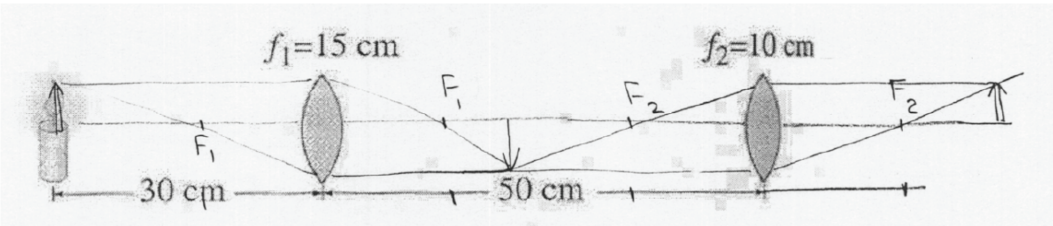
Two thin converging lenses of focal lengths $f_1 = 15$ *cm* and $f_2 = 10$ *cm* are separated by 50 *cm*. A lighted candle is placed 30 *cm* in front of the first lens.

a) Find the position of the final image. Make a sketch.
b) Find the lateral magnification of the system.
c) Is the final image upright or inverted?
d) Answer the above questions if the distance between the lenses were 20 *cm*.



Solution of Problem 3

a) Since $\frac{1}{s_1} + \frac{1}{s_1'} = \frac{1}{f_1}$, with $f_1 = 15$ cm and $s_1 = 30$ cm one obtains $s_1' = 30$ cm to the right of the first lens, 20 cm in front of the second one, i.e. $s_2 = 20$ cm. From $\frac{1}{s_2} + \frac{1}{s_2'} = \frac{1}{f_2}$ one obtains $s_2' = 20$ cm to the right of the second lens.



b) $m = m_1 m_2$; $m_1 = -s_1'/s_1 = -30/30 = -1$ and $m_2 = -s_2'/s_2 = -20/20 = -1$. Thus, $m = (-1)^2 = 1 > 0$.

c) Since $m > 0$, the image is upright, the same size as the object - due to $m = 1$.

d) In this case s_1' [see *a)*] will be 10 cm to the right of the second lens, i.e. $s_2 = -10$ cm. From $\frac{1}{s_2} + \frac{1}{s_2'} = \frac{1}{f_2}$ one obtains $s_2' = 5$ cm to the right of the second lens. The magnification of the system is $m = m_1 m_2$, where $m_1 = -s_1'/s_1 = -30/30 = -1$ (again), but $m_2 = -s_2'/s_2 = -5/(-10) = 1/2$. Thus $m = (-1)(1/2) = -1/2$ and, therefore, the final image will be inverted ($m < 0$) and reduced ($|m| < 1$).

Problem 4

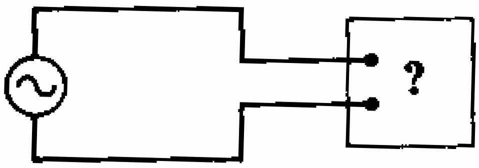
The figure shows an AC generator connected to a "black box" through a pair of terminals. The box contains an RLC series circuit whose elements we do not know. Measurements outside the box reveal that

$$\mathcal{E}(t) = (60\ V)\cos(\omega t)$$

and

$$i(t) = (3\ A)\cos(\omega t - 50^0).$$

a) What is the impedance Z of this circuit?
b) What is the power factor?
c) Must there be a resistor in the circuit? Justify your answer. Could you find the value of R?
d) Must there be an inductor in the circuit? Justify your answer.
e) What average power is delivered to the box by the generator?



Solution of Problem 4

a) Let us write $\mathcal{E}$ as $\mathcal{E} = V_m \sin(\omega t)$ and $i(t) = i_m \sin(\omega t - \phi)$ with $V_m = 60$ V and $i_m = 3$ A, $\phi = 50^0$. Thus, $Z = V_m/i_m = 60/3 = 20\ \Omega$.
b) The power factor is $\cos(\phi) = \cos(50^0) = 0.64$.
c) "Yes", since $\cos\phi = R/Z \neq 0$. One has $R = Z\cos(\phi) = 12.86\ \Omega$.
d) $\tan\phi = \frac{X_L - X_C}{R}$. Since $\phi = 50^0 > 0$ one has $X_L > X_C$ with $X_L > 0$, i.e. $X_L = \omega L > 0$. Thus, there must be an inductor in the circuit.
e) $\overline{P} = i_{\rm rms} V_{\rm rms}\cos\phi = \frac{1}{2} i_m V_m \cos\phi = 57.85\ W$.

Problem 5

An airplane flying at a distance of 11.3 km from a radio transmitter receives a signal of intensity 7.83 $\mu W/m^2$. Calculate

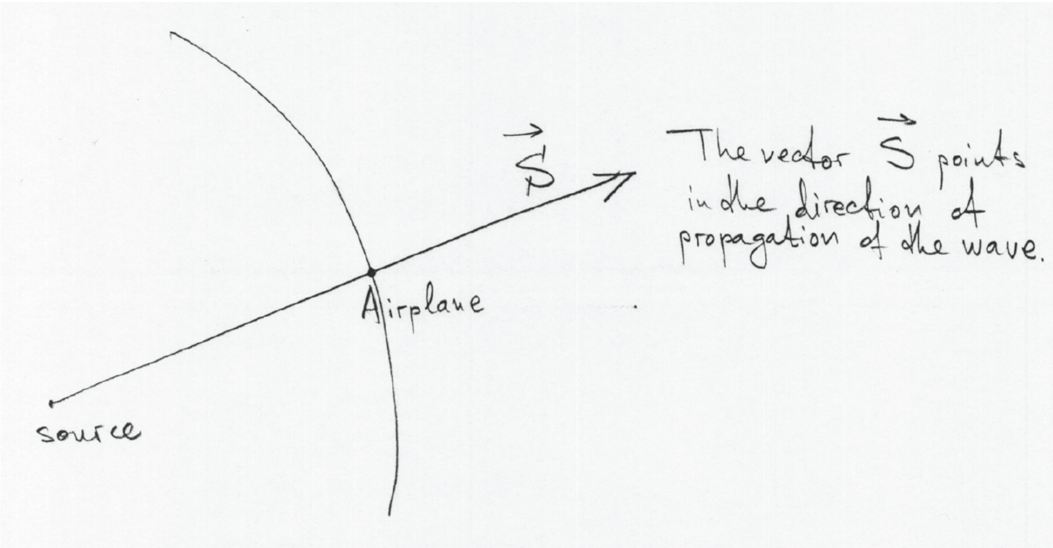
a) the amplitude of the electric field E_m at the airplane due to this signal.
b) the maximum value of the magnetic field B_m at the airplane.
c) the total power radiated by the transmitter, assuming the transmitter to radiate uniformly in all directions.
d) what is the average value of the Poynting vector S and its direction at the airplane? Make a sketch.

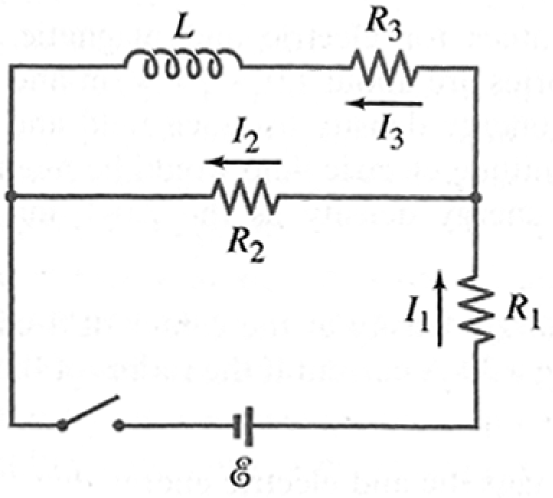
Solution of Problem 5

a)

$$I = 7.83 \times 10^{-6}\ \mathrm{W/m^2} = \frac{1}{2\mu_0 c} E_m^2 \Rightarrow E_m = \sqrt{2\mu_0 c I} = 76.8\ \mathrm{mV/m}.$$

b) $B_m = E_m/c = 256\ \mathrm{pT} = 2.56 \times 10^{-10}\mathrm{T}$.
c) $P = 4\pi r^2 I$, where $r = 11.3$ km $\Rightarrow P = 12.56\ kW$.
d) $\overline{S} = I = 7.83\ \mu\mathrm{W/m^2}$.





Problem 6

In the circuit shown in the figure $\mathcal{E} = 120\ V$, $R_1 = 10\ \Omega$, $R_2 = 20\Omega$ and $R_3 = 30\ \Omega$, while $L = 2.5\ H$. Find the values of the currents I_1, I_2 and I_3
a) immediately after the switch is closed;
b) a long time later;
c) immediately after the switch is opened *again*;
d) at a time $t = 0.05\ s$ after the switch is opened;
e) a long time after the switch is opened.

Solution of Problem 6

a) Immediately after the switch is closed the induced emf in the inductor blocks the flow of current in it, i.e. $I_3(t = 0) = 0$. From the loop containing the battery and the resistors R_1 and R_2 we then have $I_1 = I_2 = \mathcal{E}/(R_1+R_2) \simeq 4.0\ A$.
b) A long time later the current in the inductor will not change with time, i.e. the induced emf in it will be zero. Then one has

$$I_1 = I_2 + I_3, \tag{6}$$

and from the Kirchhoff's loop rule for the lower loop one now has

$$\mathcal{E} - I_1 R_1 - I_2 R_2 = 0, \tag{7}$$

while for the upper loop, which contains the inductor and the resistors R_2 and R_3 the corresponding equation reads

$$0 + I_3 R_3 - I_2 R_2 = 0. \tag{8}$$

Solving the above set of 3 equations for the 3 variables I_1, I_2 and I_3 leads to

$$I_1 = \mathcal{E}\frac{R_2 + R_3}{R_1R_2 + R_2R_3 + R_3R_1}, \qquad I_2 = \mathcal{E}\frac{R_3}{R_1R_2 + R_2R_3 + R_3R_1}, \qquad \text{and} \qquad I_3 = \mathcal{E}\frac{R_2}{R_1R_2 + R_2R_3 + R_3R_1}.$$

Numerically the above leads to $I_1 = 5.45\ A$, $I_2 = 3.27\ A$ and $I_3 = 2.18\ A$.
c) Immediately after the switch is opened again the current in R_1 will drop to 0, i.e. $I_1 = 0$. The current I_3 in the LR circuit starts to decay exponentially from its initial value (see point **b)** above) $I_3 = \mathcal{E}\frac{R_2}{R_1R_2+R_2R_3+R_3R_1} \simeq 2.18\ A$. From the junction equation (with $I_1 = 0$) one has $I_2 = -I_3 \simeq -2.18\ A$.
d) As already stated in point **c**, the current I_3 in the LR circuit decays exponentially according to the law $I_3(t) = I_3(0)e^{-t/\tau_L}$, where $I_3(0) \simeq 2.18A$ and $\tau_L = L/(R_2 + R_3) \simeq 0.05\ s$. Thus, at $t = 0.05$ one has $I_3(t = 0.05) = I_3(0)/e \simeq 0.80\ A$. For I_2 one again has $I_2 = -I_3$, while $I_1 = 0$ (see point **c)** above).
e) A long time after the switch is opened all currents will be zero, i.e. $I_1 = I_2 = I_3 = 0$.

Problem **7**

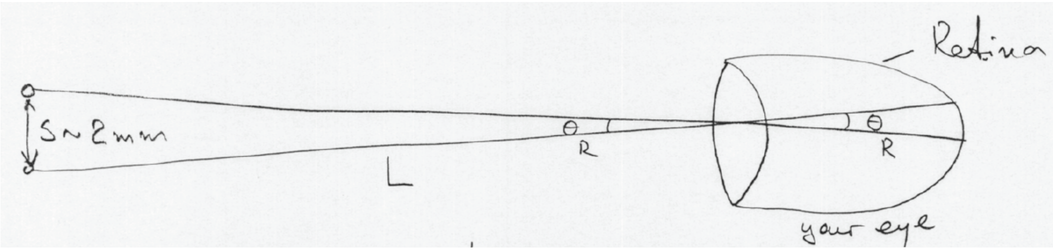
The paintings of Georges Seurat consist of closely spaced small dots (~ 2 *mm* in diameter) of pure pigment. The illusion of color mixing occurs because the observer's eyes diffracts light entering them.

a) Calculate the minimum distance an observer must stand from such a painting to achieve the desired blending of color. Take the wavelength of the light to be 500 *nm* and the diameter of the pupil to be 4.5 *mm*.

b) A security guard is coming in the exhibition hall and is turning on an additional light source. Because of that the pupil of the observer's eye shrinks to 4.0 *mm*. Shall the observer go closer to the painting, or shall he go away from it, in order to keep the blending of colors already achieved? What shall be then his distance from the painting?

Solution of Problem 7

a)



From Rayleigh's criterion one has $D \sin \theta_R = 1.22\lambda$, where $\lambda = 500$ nm and D is the diameter of the pupil. Since $D \gg \lambda$ one has $\sin \theta_R \simeq \theta_R$. Obviously, see the figure,

$$\frac{s}{L} = \tan \theta_R \simeq \theta_R = 1.22\frac{\lambda}{D}.$$

Solving for L, we obtain

$$L = \frac{sD}{1.22\lambda} \simeq 14.75\ m$$

for $\lambda = 500$ nm and $D = 4.5$ mm.

b) In this case one again has

$$L = \frac{sD}{1.22\lambda}$$

but now $D = 4$ mm. Performing the calculation, one obtains $L = 13.11$ m, i.e. the observer shall come closer to the painting.

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Georges Seurat (1859 – 1891) is a French neo-impressionist painter; his painting "The Lighthouse at Honfleur" is in "The National Gallery of Art", Washington, D.C.

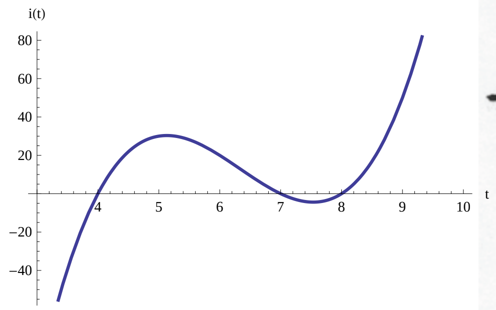
Problem 8

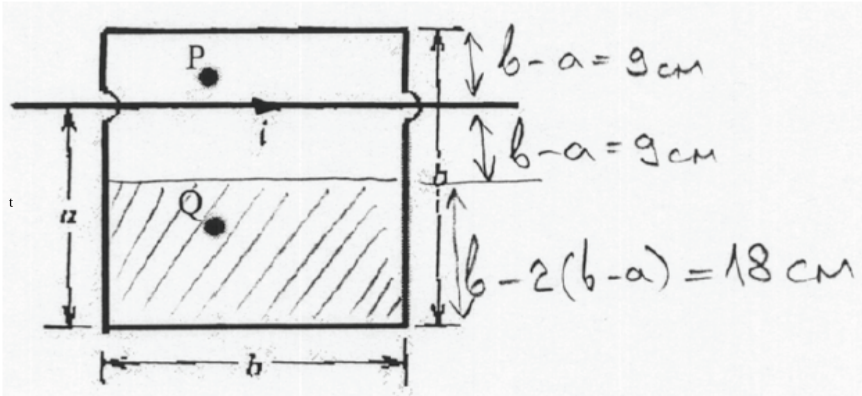
In the figure given below $a = 27\ cm$ and $b = 36\ cm$. It is also known that the current in the long straight wire, which is insulated from the loop, changes with the time t according to the law

$$i(t) = 5(t^2 - 15t + 56)(t - 4),$$

where i is in amperes and t is in seconds. The direction of the current shown in the picture assumes that $i > 0$.

a) What is the direction of the magnetic field at points P and Q as a function of time? Assume that $t \geq 0$.
b) What is the flux of the magnetic field Φ_B through the square loop as a function of t?
c) Find the induced electromotive force $\mathcal{E}$ in the square loop as a function of t.
d) At which moments t does the emf $\mathcal{E}$ change its direction?
e) Find the emf in the square loop at $t = 3.0\ s$. What is the direction of this emf? Make a sketch.





Solution of Problem 8

a) $i(t) = 0$ for $t = 4, 7, 8$ s, $i(0) < 0$ for $t = 0$ and, thus, $i(t) < 0$ for $t \in [0, 4)$; $i(t) > 0$ for $t \in [4, 7)$, $i(t) < 0$ for $t \in [7, 8)$ and, finally, $i(t) > 0$ for $t \in [8, \infty)$. At the point P the magnetic field is out of the page for $i > 0$ and into the page for $i < 0$. At the point Q the magnetic field is into the page for $i > 0$ and out of the page for $i < 0$.

b) $B = \frac{\mu_0}{2\pi}\frac{i}{r}$, where r is the distance from the wire. The fluxes in the both symmetrical regions (on the both sides of the wire) will cancel each other. Defining the normal to the plane of the square loop to point into the page, we obtain for the flux

$$\Phi_B(t) = \int_{b-a}^{a} \frac{\mu_0}{2\pi}\frac{i(t)}{r}(bdr) = \frac{\mu_0}{2\pi} i(t) b \ln\frac{a}{b-a} = -\frac{\mu_0}{2\pi} b \ln\left(\frac{b}{a} - 1\right) i(t) = 3.955 \times 10^{-7}(t^2 - 15t + 56)(t - 4).$$

c)

$$\mathcal{E}(t) = -\frac{d\Phi_B(t)}{dt} = -3.955 \times 10^{-7}(3t^2 - 38t + 116).$$

d) We look for $\mathcal{E}(t) = 0$. Solving, we obtain $t_1 = \frac{1}{3}(19 - \sqrt{13}) = 5.13$ and $t_2 = \frac{1}{3}(19 + \sqrt{13}) = 7.53$.

e) $\mathcal{E}(t = 3) = -3.955 \times 10^{-7}(3t^2 - 38t + 116)|_{t=3} = -11.5 \times 10^{-6}$ V. Since $\mathcal{E}(t = 3) < 0$ the direction of $\mathcal{E}$ has to be in the direction opposite to that one given by the right-hand rule - if one grasps the normal to the square with his right hand his fingers will point in the positive direction of $\mathcal{E}$, i.e. $\mathcal{E}$ at $t = 3$ shall be in the counterclockwise direction looking down the page.