Name: ______________________________

Student ID number: ______________________________

Discussion section: ________________

1	
2	
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MIDTERM 2
PHYSICS 1C, SPRING 2014

MAY 22, 2014

READ THE FOLLOWING CAREFULLY:

▷ Closed book. Calculators are allowed. No electronic devices that can transmit/recieve information should be out as you take the exam. One 3″×5″ index card with notes is allowed.

▷ This exam consists of 10 pages (including this one) with problems numbered 1 through 3 (number 1 has four parts a-d); make sure you have been given all pages/problems.

▷ You have 60 minutes to complete the exam.

▷ Make sure to write your name at the top of each page of this exam. Use the space provided on the exam pages to do your work. You may use the back of the pages also, but please mark clearly which problem you are working on (and also state underneath that problem that you have done work on the back of the page).

▷ **You must justify your answers to each question**. Simply giving the correct answer without proper justification (can be brief) will not result in full credit. If you can not figure out how to complete a particular computation, a written statement of the concepts involved and qualitative comments on what you think the answer should be may be assigned partial credit.

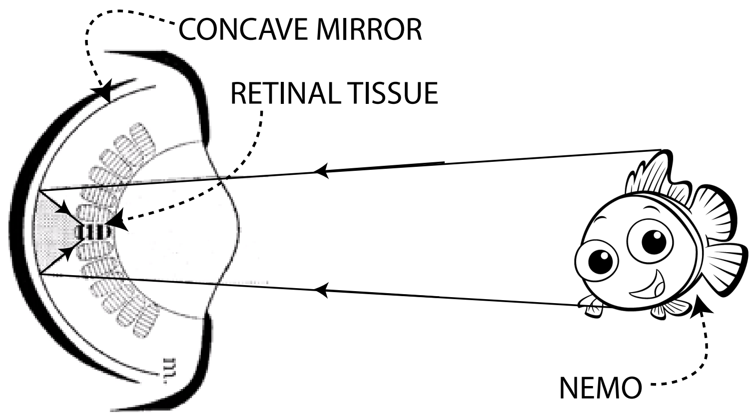
▷ You must show a photo ID when turning in your exam.

▷ Mistakes in grading: If you find a mistake in the grading of your exam, alert the instructor within one week of the exams being returned (this will occur in discussion section following the exam date). DO NOT write on the returned graded exam.

▷ In case you need any of this information: The permeability of free space (μ_o) is $4\pi \times 10^{-7}$ m kg s^{-2} A^{-2}, the permittivity of free space (ϵ_o) is 8.85×10^{-12} m^{-3} kg^{-1} s^4 A^2, the speed of light in vacuum is 2.998×10^8 m/s, resistivity of copper is 1.68×10^{-8} Ωm, the charge on an electron is -1.6×10^{-19}C, and my 11 year old son can run 5km in 23 minutes.

[1.] Short answer conceptual questions.

(a) (8 pts) Scallops have eyes that use concave mirrors to focus light onto retinal tissue as shown below. The scallop is focused on a fish a certain distance away. If the fish moves further away from the scallop, how should the scallop adjust the radius of curvature of its mirror eye in order to keep the fish image focused on its retina? Explain.



The scallop should increase the radius of curvature of its mirror. As the fish moves away, the object distance s increases. The scallop wants to keep the image distance, s' the same (focused on the retinal tissue). So, from :

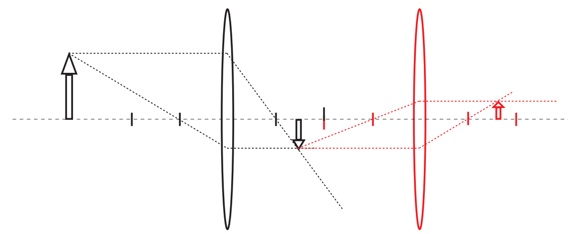
$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$$

We see that if p increases while q is constant (making the left-hand side of the equation smaller), R must increase to reduce the right-hand side also. Another way to say it: because the fish is further away, it takes less focusing power to make the rays converge on the retina (they come in at shallower angles). So a less curved mirror is needed, meaning larger radius of curvature.

(b) (10 pts) Polarized sunglasses can help block light reflected off of horizontal surfaces at shallow angles (e.g. off of roads, car hoods, windshields, surface of lakes). Explain how they work and why they can preferentially block reflected light.

> Polarized sunglasses block (absorb) light waves that are polarized horizontally (with electric fields that point horizontally). Light that is reflected at a shallow angle is polarized by reflection: for angle of incidence equal to Brewster's angle, only light with horizontal polarization is reflected.

(c) (9 pts) An object is placed near a pair of converging lens as shown below (placed vertically to give you plenty of space); the focal length and twice the focal length of each lens is marked on the diagram. Construct the image produced by the lenses graphically.



(d) (9 pts) The electric field of an electromagnetic wave is given by:

$$\vec{E} = E_o \hat{y} \cos\left(\pi/2 - kx - \omega t\right)$$

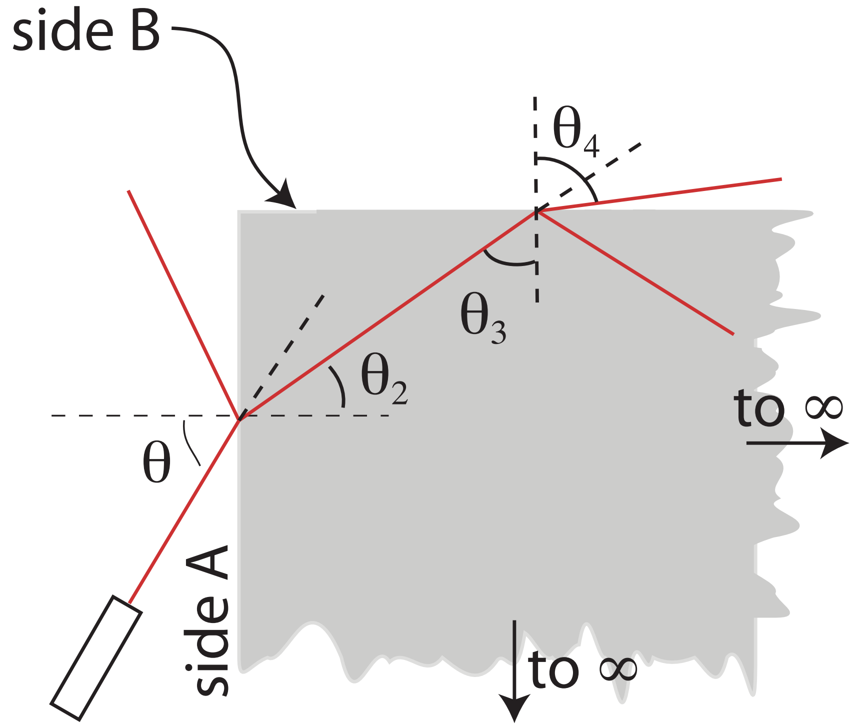
i. In which direction is this wave propagating?

$-\hat{x}$

ii. Give an expression for the magnetic field associated with this wave.

$-\frac{E_o}{c} \hat{z} \cos\left(\pi/2 - kx - \omega t\right)$

[2.] (32 pts) A laser beam is shone onto a block of ice ($n = 1.31$) as shown in the figure below (the block is surrounded by air, and you can assume that it is semi-infinite, extending to infinity in the directions indicated).



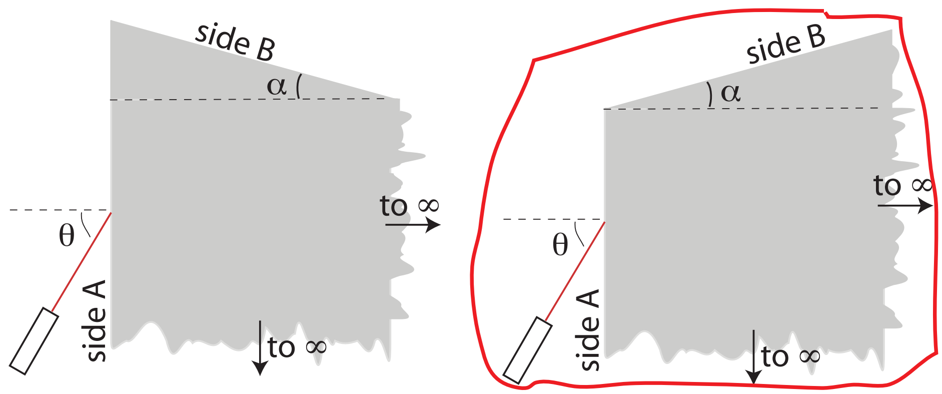
(a) Draw the path of the ray through the block, **including both refracted and reflected rays at each interface**. Be careful to indicate the change in direction of the refracted rays as they pass through the interfaces at side A and side B. (use the image above for your drawing) see above

(b) Calculate the angles of refraction at the two interfaces (at side A and side B), given that the angle of incidence on side A is $\theta = 70°$.

At the first interface, we use Snell's law to get the refracted angle: $\sin(70) = 1.31 \sin(\theta_2)$, which yields $\theta_2 = 45.83°$. This angle of refraction into the block leads to an incident angle at top interface of $\theta_3 = 90 - \theta_2 = 44.17$. Applying Snell's law again, we have $1.31 \sin(44.17) = \sin(\theta_4)$, which gives $\theta_4 = 65.9°$.
Note that the calculated angles are consistent with intuition: that at the first interface, the ray refracts toward the normal (because the ice has bigger index of refraction than the air), leading to a refracted angle which is smaller than the incident angle. Also at the second interface the opposite occurs, because the ray is now starting in the high index material and going into a low index material.

(c) You want to modify the block of ice in order to prevent a refracted ray from emerging from the top of the block (side B). Which of the following two shapes should you choose for the top surface in order to acheive this goal?

> You pick the surface circled below, which increases the angle of incidence of the second ray (at the top interface). By increasing the angle of incidence the ray can exceed the critical angle.



(d) What is the minimum angle of the slanted surface (α in the figure) in order to prevent the ray from emerging from the surface at side B?

> By slanting the surface at angle α as shown, you add the angle α to the angle of incidence at the top interface. In the flat-top case, $\theta_3 = 90 - \theta_2$ is the incident angle at the top interface. Here, the angle of incidence will be $90 - \theta_2 + \alpha$. We want this new angle of incidence to be equal to the critical angle. First we calculate the critical angle: $\theta_c = \sin^{-1}(1./1.31) = 49.8°$. In the case of a flat top of the block, we found $\theta_3 = 90 - \theta_2 = 44.16°$. So, we only need to tilt the block by $\alpha = \theta_c - 90 + \theta_2 = 5.63°$ to make the ray experience TIR at the top interface.

[3.] (32 pts)

(a) A solenoid is created by winding copper wire on the outer surface of a cylinder of length l and radius a. The copper wire is wound with a total number of turns N. What is the self-inductance of this solenoid (you may assume that the solenoid is very long, $l \gg a$).

The self-inductance of a multi-turn coil can be defined as:

$$N\Phi = LI$$

Where N is the number of turns, Φ is the magnetic flux (due to the field generated by the coil itself) through ONE turn of the coil, and I is the current flowing in the coil. We'll assume we can approximate the field to be the same as produced by an infinitely-long solenoid:

$$B = \mu_o n I = \mu_o \frac{NI}{l}$$

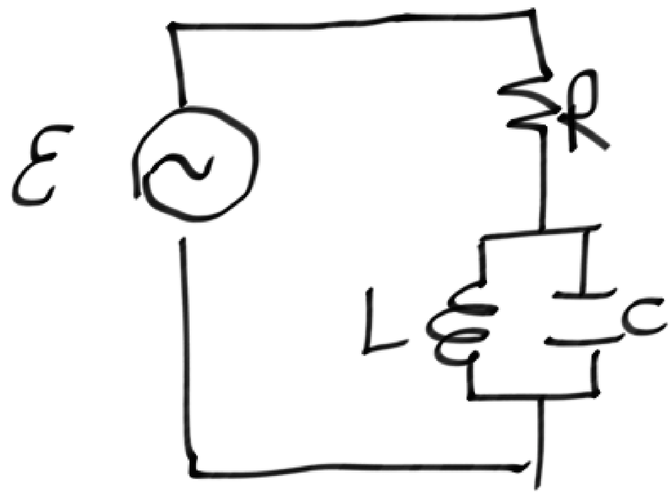
This field exists inside the solenoid and is uniform (and points along the axis of the solenoid). The flux through one turn of the coil is the flux through a circle f radius a:

$$\Phi = \mu_o \pi a^2 \frac{NI}{l}$$

The inductance is then:

$$L = \frac{N\Phi}{I} = \mu_o \frac{N^2 \pi a^2}{l}$$

(b) This solenoid is connected to a circuit as shown below (from this point on, you can denote the inductance of the solenoid as L). The generator provides EMF $\mathcal{E} = \mathcal{E}_o \cos(\omega t)$.



Write down the total impedance (complex) seen by the generator.

You have a resistor, with impedance R, in series with a parallel combination of our inductor, with impedance $i\omega L$, and a capacitor, with inductance, $1/(i\omega C)$. The impedance of the parallel combination is:

$$\frac{1}{Z_{LC}} = \frac{1}{i\omega L} + i\omega C$$

or

$$Z_{LC} = \frac{i\omega L}{1 - \omega^2 LC}$$

This adds in series with the resistor to give a total impedance:

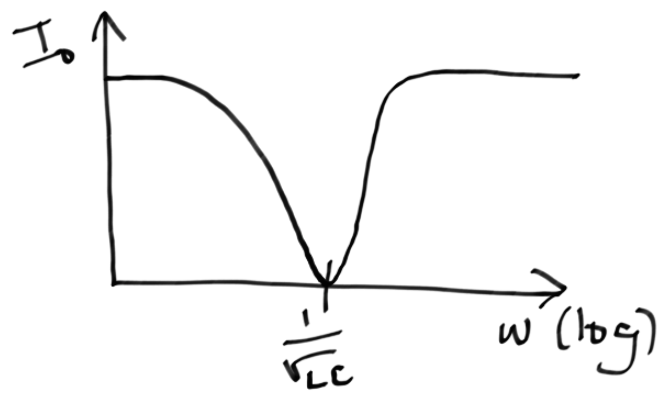
$$Z_{\text{tot}} = R + \frac{i\omega L}{1 - \omega^2 LC}$$

(c) Give an expression for the peak value of the current flowing from the generator and sketch a plot of the current as a function of frequency.

We can get the peak value of the current by just dividing the peak EMF by the magnitude of the impedance:

$$I_o = \frac{\mathcal{E}_o}{|Z_{\text{tot}}|} = \frac{\mathcal{E}_o}{R^2 + \dfrac{\omega^2 L^2}{1 - \omega^2 LC}}$$

We know that the parallel L-C subcircuit has zero impedance at very low frequency (inductor is a short) and at very high frequency (capacitor is a short). So at both low and high frequency, the current is $\mathcal{E}_o/R$. In between, the current will be less (L-C subcircuit adds finite impedance) and in fact will be zero at $\omega = 1/\sqrt{LC}$ (impedance of L-C subcircuit is infinite).



(d) At what frequency is the the current flowing from the generator the smallest? Why?

The current is smallest at the resonant frequency, $\omega = 1/\sqrt{LC}$. At that frequency, the impedance of the parallel inductor capacitor combination is infinite! At this frequency, there is a resonant oscillation between the capacitor and the inductor; the current through the inductor is equal and opposite to that flowing through the capacitor. So no net current flows from and to the generator, but there can be a large current "trapped" in an oscillation between the capacitor and inductor in this circuit.