Name:

Student I

Discussic

Physics 1C, Spring 2019, Lecture 2

Midterm 1

Time allotted: 50 minutes

No calculators or notes allowed.

No phones out during the exam.

All work must be your own.

Partial credit will be awarded for correct work.

Problem 1	20 /20
Problem 2	19 /20
Problem 3	17 /20
Total	56 /60

Inspirational quote:

But still, try, for who knows what is possible? —Michael Faraday (1870)

Potentially useful formulas:

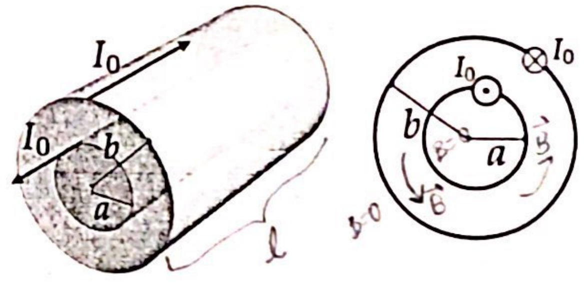
Vacuum magnetic permeability

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}} \tag{1}$$

Field due to a ring (radius a) of current I on axis

$$\mathbf{B}(z) = \frac{\mu_0 I}{2} \frac{a^2}{(z^2 + a^2)^{3/2}}\,\hat{\mathbf{z}} \tag{2}$$

Problem 1. A coaxial cable consists of a long, cylindrical, conducting shell of radius a surrounded by another, coaxial shell of radius $b > a$. The two shells both carry current I_0 distributed evenly over their surface area, but in opposite directions.



⊙ is positive

(a) Use Ampere's law to find the magnetic field everywhere as a function of radius.

(b) Calculate the self-inductance L of a length ℓ of such a cable.

(c) A length of this coaxial cable is connected as an inductor to a fixed capacitance C, forming a series LC circuit. Explain how you might vary the geometry of the cable to make the LC oscillation frequency ω_0 large.

a) $r < a$

$\oint B \cdot dl = \mu_0 I_{enc}$ $\quad I_{enc} = 0$

$B = 0$

$r > b$

$\oint B \cdot dl = \mu_0 I_{enc}$

$I_{enc} = I_0 - I_0$

$B = 0$

$a < r < b$

$\oint B \cdot dl = \mu_0 I_{enc}$

$\oint B \cdot dl = \mu_0 I_0$

$B \cdot 2\pi r = \mu_0 I_0$

$B = \dfrac{\mu_0 I_0}{2\pi r}$

$$B(r) \begin{cases} 0 & r < a \\ \dfrac{\mu_0 I_0}{2\pi r} & a < r < b \ * B \text{ is CCW} \\ 0 & r > b \end{cases}$$

$$\boxed{L = \frac{\mu_0 \ell}{2\pi} \ln\left(\frac{b}{a}\right)}$$

b) $L = \dfrac{\Phi_B}{I}$

$\Phi_B = \int \vec{B} \cdot dA \qquad A = \ell \cdot r \quad dA = \ell\, dr \qquad r = a \text{ to } r = b$

$= \displaystyle\int_a^b \frac{\mu_0 I_0}{2\pi r} \cdot \ell\, dr$

$= \dfrac{\mu_0 I_0 \ell}{2\pi} \displaystyle\int_a^b \frac{1}{r}\, dr = \dfrac{\mu_0 I_0 \ell}{2\pi}\left[\ln(b) - \ln(a)\right]$

$= \dfrac{\mu_0 I_0 \ell}{2\pi} \ln\left(\dfrac{b}{a}\right)$

c) $\omega = \dfrac{1}{\sqrt{LC}}$

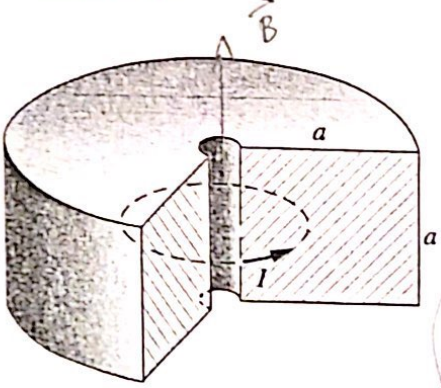
To make the ω_0 large, make L smaller.

To make L smaller make the length ℓ of the coaxial cable shorter. Also, you could try to increase b and decrease a
outer radius
inner radius

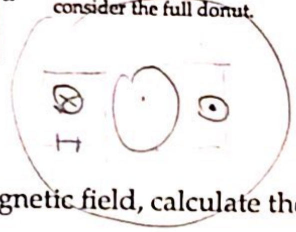
20/20

Problem 2. Magnetic fields inside good conductors cannot change quickly. We found that current in a simple inductive circuit decays exponentially with characteristic time L/R. In a large conducting body such as the core of the earth, a circuit is not easy to identify. Nevertheless we can find an order-of-magnitude estimate for the decay time τ with some simplifying approximations.

We will model the core of the earth as a conducting "donut" of square cross section (side length a) and total resistance R.



This image shows a portion of the donut cut out to display the cross-section. In your answer, consider the full donut.



(a) As an estimate of the magnitude B of the magnetic field, calculate the field at the center of a conducting ring of radius $a/2$ carrying current I.

(b) Treat the magnetic field in the donut as uniform, with the magnitude you found in (a). Estimate the magnetic field energy U stored in the donut. Treat the donut as a cylinder for the purpose of finding its volume.

(c) Suppose the magnetic energy stored in the donut depends on time roughly as $U \approx U_0 e^{-t/\tau}$, with U_0 constant. Use the fact that the power is

$$\frac{dU}{dt} = -I^2R \tag{3}$$

and your answer to (b) to obtain an expression for τ in terms of a, R, and fundamental constants. Verify that your expression is dimensionally correct.

a) $r = a/2 \quad I = I \quad B(z) = \frac{\mu_0 I}{2} \frac{a^2}{(z^2+a^2)^{3/2}} \hat{z}$

field due to ring of current

$= \frac{\mu_0 I}{2} \frac{\frac{a^2}{4}}{(z^2+\frac{a^2}{4})^{3/2}} \hat{z}$ ↙ up

$z=0$, so

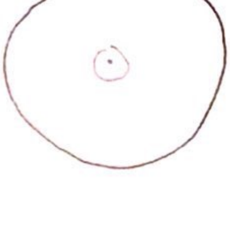
$B = \frac{\mu_0 I}{2} \frac{\frac{a^2}{4}}{(\frac{a^2}{4})^{3/2}}$

$B = \frac{\mu_0 I}{2} \frac{\frac{a^2}{2^2}}{\frac{a^3}{2^3}}$

$B = \frac{\mu_0 I}{2} \cdot \frac{a^2}{2^2} \cdot \frac{2^3}{a^3}$

$B = \frac{\mu_0 I}{2} \cdot \frac{2}{a} = \frac{\mu_0 I}{a}$

$\boxed{B = \frac{\mu_0 I}{a}}$ ✓ 6



b) $u = \frac{B^2}{2\mu_0} \quad U = \frac{B^2}{2\mu_0} \cdot V$

$V = \pi r^2 h$

$r = a$
$h = a$

$U = \frac{\left(\frac{\mu_0 I}{a}\right)^2}{2\mu_0} \cdot \pi a^2 \cdot a$

$U = \frac{\frac{\mu_0^2 I^2}{a^2}}{2\mu_0} \cdot \pi \cancel{a^2}\, a$

$\boxed{U = \frac{\mu_0 I^2 a \pi}{2}}$ ✓ 8

$I^2 = \frac{2U}{\mu_0 a \pi}$

c) $\frac{dU}{dt} = U_0 e^{-t/\tau} \cdot -\frac{1}{\tau}$

$U_0 e^{-t/\tau} \cdot -\frac{1}{\tau} = -I^2 R$

$U_0 e^{-t/\tau}\left(-\frac{1}{\tau}\right) = -\frac{2U}{\mu_0 a \pi} R$

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c) (continued)

$$\cancel{U_0 e^{-t/\tau}}\left(-\frac{1}{\tau}\right) = \frac{-2(\cancel{U_0 e^{-t/\tau}})}{\mu_0 a \pi} R$$

$$-\frac{1}{\tau} = \frac{-2R}{\mu_0 a \pi}$$

$$1 = \frac{2R\tau}{\mu_0 a \pi}$$

$$\mu_0 a \pi = 2R\tau$$

$$\boxed{\tau = \frac{\mu_0 a \pi}{2R}}$$

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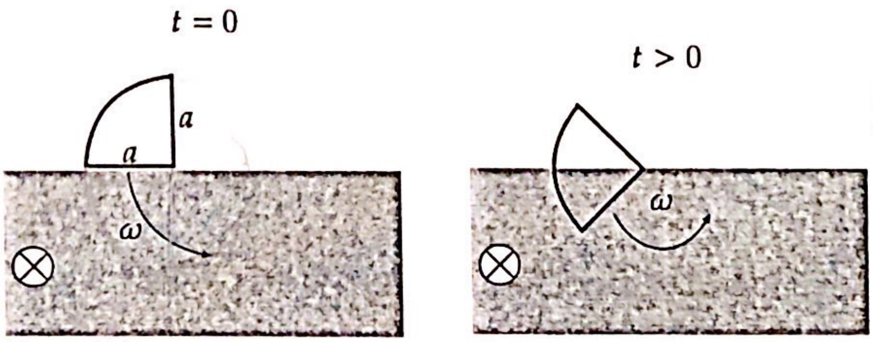
Problem 3. A conducting wire loop in the shape of a quadrant of a circle with radius a rotates at constant angular velocity $d\theta/dt = \omega$ as shown in the figure below. The shaded area indicates a constant, uniform magnetic field of magnitude B directed into the page.



$\mathbf{B} = -B\,\hat{\mathbf{z}}$

$\frac{d\theta}{dt} = \omega$

(a) Calculate the emf $\mathcal{E}(t)$ around the wire as a function of time for one full period, $0 \le t < 2\pi/\omega$. Your answer will be a piecewise function with four time segments. The area of a circular sector of radius r subtending angle θ is Area $= r^2\theta/2$.

(b) For some time segments of its motion, the loop will experience a magnetic force. Without detailed calculations, give the direction of this force, if any, during each segment of time.

a) $\frac{3\pi}{2\omega} < t < \frac{2\pi}{\omega}$ $\mathcal{E}(t)$, out of field → change in flux is zero

$\mathcal{E}(t) = \frac{-d\Phi_B}{dt}$

$0 < t < \frac{\pi}{2\omega}$, $\mathcal{E}(t)$, going into field

$\frac{\pi}{2\omega} < t < \frac{\pi}{\omega}$ $\mathcal{E}(t)$, in the field → change in flux is zero

$\frac{\pi}{\omega} < t < \frac{3\pi}{\omega}$, $\mathcal{E}(t)$, going out of field

$$-\frac{d\Phi_B}{dt} = -\frac{\Delta\Phi_B}{\Delta t} = \frac{-(-B)\cdot\Delta A}{\Delta t} = \frac{B\cdot r^2}{2}\cdot\frac{\Delta\theta}{\Delta t} = \frac{Br^2\omega}{2}$$

$$\Phi_B = \int \vec{B}\cdot d\vec{A} = -B\int dA \qquad \frac{d\theta}{dt} = \omega$$

$$= -B\int \frac{r^2\theta}{2}\cdot\frac{d\theta}{dt} \qquad \frac{d\theta}{\omega}\,dt$$

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$$\mathcal{E}(t) = \begin{cases} \frac{Ba^2\omega}{2} & 0 < t < \frac{\pi}{2\omega} \\ 0 & \frac{\pi}{2\omega} < t < \frac{\pi}{\omega} \\ -\frac{Ba^2\omega}{2} & \frac{\pi}{\omega} < t < \frac{3\pi}{2\omega} \\ 0 & \frac{3\pi}{2\omega} < t < \frac{2\pi}{\omega} \end{cases}$$

as a function of time???

b) $V = 2\pi r\omega$

Force is directed towards the center of the circle quadrant by right hand rule.

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