Physics 1B - Winter 2017

MIDTERM EXAM #2

READ THIS BEFORE YOU BEGIN

- You are allowed to use only yourself and a writing instrument on this exam.
- If you finish more than 5 minutes before the end of the exam period, please raise your hand and a proctor will collect your exam. Otherwise, please stay in your seat until the end of time is called.
- When the exam is finished, please remain in your seat, pass your exam to the aisle, and the proctor(s) will come around and collect your exam. Once your exam is collected, you may leave the room.
- Show all work. The purpose of this exam is primarily to test how you think; you will get more partial credit for a logical, well-thought-out response, and **you will get little or no credit for an answer without convincing reasoning.** Points will be given specifically for the quality of your reasoning which includes clarity and conciseness.
- Please **box all of your final answers** to computational problems.
- Use the space provided to give detailed, readable answers. **Don't try to cram your answers on the page containing the problem statement!**
- You may use the back of any exam paper as room for extra work.

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Discussion Section # Wednesday 5-6 in Geology

Problem 1. (22 points) +20

Consider a solid, spherically-symmetric conductor of radius r_b with a spherical cavity of radius r_a cut out from its center. The result is a spherical conducting shell of thickness $r_b - r_a$. An electron having charge $q = -e$ is fixed at a position precisely in the center of the cavity. The conductor is neutral – it has no excess charge on it.

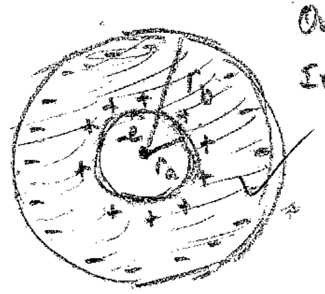
a. **(4 points)** Describe and draw the charge density everywhere inside and on the surfaces of the conductor, and justify your description using Gauss's Law.

b. **(6 points)** Let r denote the distance of a given point in space from the center of the cavity. Determine an expression for the electric field in each of the three regions $r < r_a$, $r_a < r < r_b$, and $r > r_b$.

c. **(9 points)** Sketch the electric potential $V(r)$ as a function of distance from the center of the cavity for all values of $r > 0$, and determine an explicit expression for this potential everywhere in space.

d. **(3 points)** Qualitatively describe what, if anything, would change if the charge in the cavity were moved to a different point inside the cavity that is off-center. What, if anything, would stay the same? No math is required here, but diagram(s) are recommended.

a)

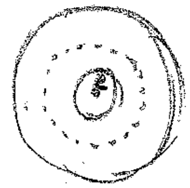


Outer surface $q = -e$

Inner surface $q = +e$

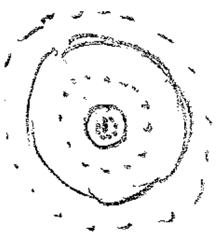
+4

From Gauss law, we know that along a spherical surface between r_a & r_b, the net electric flux is 0, we also know that E inside a conductor should be 0. Drawing it gives



If flux = 0, Q_{inside} must also equal 0, for this to happen, charge on inner surface must be +e. The conductor is neutral so the outer shell must be −e.

b) Applying Gauss Law to the three regions



When $r < r_a$

$$\int E\,dA = \frac{Q_{in}}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{-e}{\varepsilon_0}$$

$$\boxed{E = \frac{-e}{4\pi r^2 \varepsilon_0}}$$

When $r_a < r < r_b$

$$\boxed{E = 0}$$

When $r > r_b$

$\frac{1}{2}QV^2$

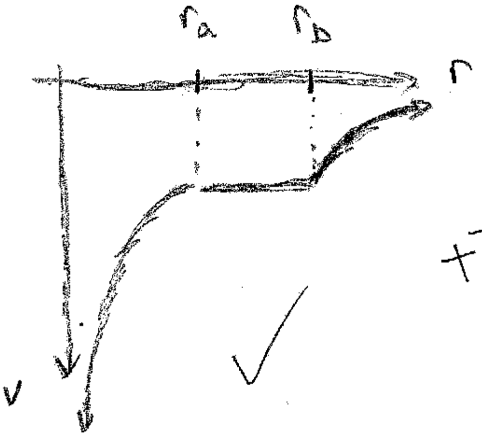
$$\int E\,dA = \frac{Q_{in}}{\varepsilon_0}$$

+6

$$E(4\pi r^2) = \frac{-e}{\varepsilon_0}$$

$$\boxed{E = \frac{-e}{4\pi r^2 \varepsilon_0}}$$

c)



+7

$$V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{l}$$

Let $V_a = V$ at $r = \infty$

$$V_b = V_\infty - \int_\infty^b \vec{E}\cdot d\vec{l}$$

Change variables to be more consistent

$$V_r = V_\infty - \int_\infty^r \vec{E}\cdot d\vec{l}$$

When $r < r_a$

$$V_r = 0 - \int_\infty^r \frac{-e}{4\pi r^2 \varepsilon_0}\,dr$$

$$= -\frac{e}{4\pi\varepsilon_0}\left(\frac{1}{r}\right)\Big|_\infty^r$$

$$= \boxed{\frac{-e}{4\pi\varepsilon_0 r}}$$

When $r_a < r < r_b$

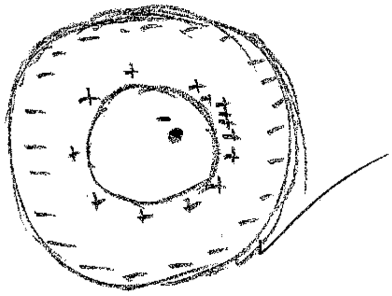
$$\boxed{\frac{-e}{4\pi\varepsilon_0 r_a}}$$

When $r > r_b$

$$V_r = 0 - \int_\infty^r \frac{-e}{4\pi r^2 \varepsilon_0}\,dr$$

$$= \boxed{\frac{-e}{4\pi\varepsilon_0 r}}$$

d) Charge density on the inner surface would no longer be uniform. Charge density on the outer surface would remain the same. E between r_a & r_b and E outside r_b would be the same. E inside r_a would still be the same, but Gauss surface is not a concentric sphere.



+3

Problem 2. **(27 points)**

A thin, uniformly-charged ring of radius R and charge per unit length $\lambda > 0$ is fixed in the x-y plane with its center at the origin.

a. **(4 points)** What is the electric potential V at a given point $(0, 0, z)$?

b. **(6 points)** Using this electric potential, compute all three components E_x, E_y, E_z of the electric field at a given point $(0, 0, z)$.

c. **(3 points)** On physical grounds, what would you expect the electric potential to look like when z is much larger than R?

d. **(4 points)** Use a Taylor expansion to determine an approximate expression for the z-dependence of the electric potential when z is much larger than R. Does this expression agree with your expectations on physical grounds? condition to expand about

e. **(4 points)** Use a Taylor expansion to determine an approximate expression for the z-dependence of the electric potential when z is much smaller than R.

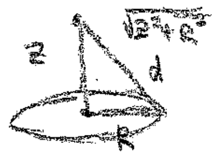
f. **(6 points)** Suppose that an electron is released from rest at $t = 0$ from a point with $z > 0$ along the z-axis such that z is much smaller than R. Determine an expression for the position as a function of time $z(t)$ of the electron for all $t > 0$.

a)

$$V = \int \frac{k\,dq}{d} \qquad \lambda\, ds = dq$$

$$= k\int \frac{\lambda\, ds}{d}$$

Using property $r = 2\pi\theta$, $dr = d\theta\,(2\pi)$



$$= \frac{k\lambda}{\sqrt{z^2+R^2}} \int_0^{2\pi R} ds$$

$$= \frac{k\lambda}{\sqrt{z^2+R^2}} \int_0^{2\pi} 2\pi\, d\theta = \frac{k\lambda}{\sqrt{z^2+R^2}}\, 4\pi^2 = \boxed{\frac{\lambda\pi}{\varepsilon_0\sqrt{z^2+R^2}}}$$

-1

Using

$$-\frac{dV}{dr} = E$$

b) $E_x = 0$

$E_y = 0$

$$E_z = \boxed{\frac{\lambda\pi}{\varepsilon_0 (z^2+R^2)}\,\hat{z}}$$

-1

Assuming a) was correct

c) Electric potential would approach zero as z got larger and farther away from the ring. -1

d) Taylor Expansion V about $z=\infty$

$$V = \frac{\lambda\pi}{\varepsilon_0\sqrt{z^2+R^2}}$$

Mercy

$$= f + \frac{f'\,x}{1!} + \frac{f''\,x^2}{2!} + \ldots$$

$$= 0 + 0 + 0$$ -2

$= \boxed{0}$ Yes this agrees that V should approach 0 as $z \to \infty$

e) Taylor Expansion of V about $z=0$

$$V = \frac{\lambda\pi}{\varepsilon_0\sqrt{z^2+R^2}}$$

$$= f + \frac{f'\,x}{1!} + \frac{f''\,x^2}{2!} + \ldots$$

Approaches a constant

$$\boxed{\frac{\lambda\pi}{\varepsilon_0 R}}$$

Have mercy

f) The motion would be the same as SHM.

$$y(t) = A\cos(\omega t + \Phi)$$
$$= z\cos(\omega t)$$

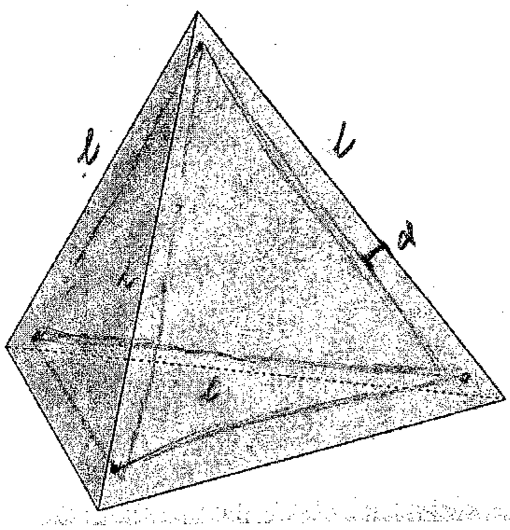
where ω is found through Taylor expansion from d or e

-3

Problem 3. (13 points)

A tetrahedron is a four-sided pyramid-like surface constructed by gluing together four equilateral triangles edge-to-edge.

+12



Suppose you fabricate two such tetrahedra from some conducting material, one of which is *only slightly smaller* than the other and you nest the smaller one inside the larger one so that there is a separation distance d between all four of their faces. Because they are nearly the same size, the distance d between the almost-touching faces of the inner and outer tetrahedra is much smaller than the edge length ℓ of each face. Suppose that the inner conductor has excess charge $Q > 0$ on it, and the outer conductor has excess charge $-Q$.

a. **(4 points)** The electric field in the region between the conductors will have a nearly constant value throughout. Why? What is the value of this field? Justify.

b. **(3 points)** Taking the value of the field to be exactly the constant value determined in part a., what is the potential difference between the inner and outer tetrahedra?

c. **(3 points)** What is the capacitance of this system?

d. **(3 points)** Suppose that we hook up an infinite number of these tetrahedral capacitors labeled $1, 2, 3, \ldots$ in ~~series~~ // such that the n^{th} capacitor is a perfect 1/2 scale copy (1/2 the size in all physical dimensions) of the $(n-1)^{\text{th}}$ capacitor. What is the capacitance of this infinite ~~series~~ // of progressively smaller tetrahedral capacitors?

a) The conductors will form a capacitor.

The E field is constant throughout because the surfaces all have uniform charge distribution (they are conductors). Since the seperation throughout is uniform as well, and d is much smaller than ℓ, through Gauss law we can show how E is nearly uniform when d is much smaller than ℓ.

$$\int E\,dA = \frac{Q_{in}}{\varepsilon_0}$$

$\sqrt{l'^2 - \frac{l'^2}{4}}$

$\frac{l'}{2}\sqrt{l'^2 - \frac{l'^2}{4}}$ $\frac{3}{4}$

$\frac{l'^2}{4}\sqrt{3}$

$$E\left(4\frac{l'^2}{4}\sqrt{3}\right) = \frac{Q_{in}}{\varepsilon_0}$$

$$E = \frac{Q}{\varepsilon_0 l'^2\sqrt{3}}$$

Gauss surface is a tetrahedron between the outer & inner

As $l' \to l$ since d is very small and approaching 0

$$E = \boxed{\frac{Q}{\varepsilon_0 l^2\sqrt{3}}}$$

+3

b) $V_0 - V_d = -\int_d^0 \frac{Q}{l^2\sqrt{3}}\,dr$

Using

$V_b - V_a = \int \vec{E}\cdot d\vec{l}$

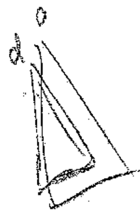
or

$-\frac{dV}{dr} = E$

$$= -\left.\frac{Qr}{l^2\sqrt{3}}\right|_d^0$$

$$= \boxed{\frac{Qd}{l^2\sqrt{3}}}$$

+3 Assuming a) was right



c) Using $C\Delta V = Q$

$$C = \frac{Q}{\Delta V}$$

+3

$$\boxed{C = \frac{l^2\sqrt{3}}{d}}$$

Assuming part b was right

d) $C = \varepsilon\frac{A}{D}$ If A is $\frac{1}{4}$th'd & If D is $\frac{1}{2}$ th'd

then C is $\frac{1}{2}$ th'd

$C_{//} = C_1 + C_2 + C_3$ +1

$= C + \frac{1}{2}C + \left(\frac{1}{2}\right)^2 C + \ldots$

Infinite geometric series

On back

$S = a_1\left(\frac{1}{1-r}\right)$ Sum of infinite geometric series

$= 2(a_1)$

$= \boxed{2c}$ +1

$= \boxed{\dfrac{2L^2\sqrt{3}}{d}}$ Assuming b & c were correct

+1