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First Name: ____________________

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MIDTERM #4
Physics 1BH
Prof. David Saltzberg
March 10, 2016

Time: 50 minutes. Closed Notes. Closed Book. Allowed the standard "cheat sheet". Calculators are allowed. Show your work.

If a problem is confusing or ambiguous, notify the professor. Clarifications will be written on the blackboard. Check the board.

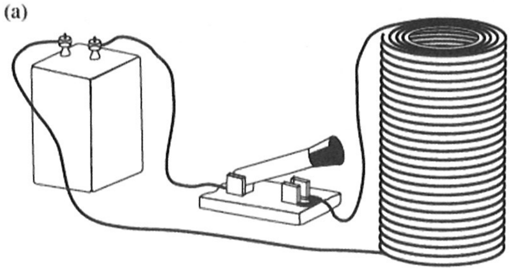
Extra workspace is given and extra paper is at the front of the room.

Problem	**Points**
1	25 /25
2	25 /25
3	25 /25
4	25 /25
TOTAL	100 /100

0.2 H

1) A coil with resistance of 0.02 Ω and self-inductance of 20 μH is connected across a 12 Volt battery. For simplicity, neglect the internal resistance of the battery.

(a)



a) How long after the switch is closed will the current reach 75% of its final value?

$I(t) = I_0 \left(1 - e^{-t/\tau}\right)$

$\tau = L/R = \frac{20\mu}{0.02} = 1 \text{ millisec}$

$0.75\, I_0 = I_0 \left(1 - e^{-t/\tau}\right)$

$e^{-t/\tau} = 0.25$

$t_1 = \tau \ln 4$

$t_1 = 10^{-3}(1.4)$

$t_1 = 1.4$ milliseconds

b) How much energy has been withdrawn from the battery during this time?

$\tau = 10^{-3}$ s

$$E = \int_0^{0.14\times10^{-3} \equiv t_1} \mathcal{E} I \, dt$$

$$= (12) I_0 \int_0^{t_1} \left(1 - e^{-t'/\tau}\right) dt'$$

$$I_0 = \frac{\mathcal{E}}{R} = \frac{12}{0.02}$$

$$= \frac{12^2}{0.02} \left[t + \tau e^{-t/\tau} \right]_0^{t_1}$$

$$= \frac{144}{0.02} \left[t_1 + \tau e^{-t_1/\tau} - \tau \right]$$

$$= 7200 \left[1.4\times10^{-3} + 10^{-3}\left(e^{-1.4}\right) - 1\times10^{-3} \right]$$

$$= 7200 \left[10^{-3}\right]\left[1.4 + 0.25 - 1\right]$$

$$= (7.2)(0.65)$$

$$= 4.7 \text{ J}$$

c) How much energy is stored in the magnetic field at this time ~~when the current reaches its steady state value?~~

$U = \frac{1}{2} L I^2$

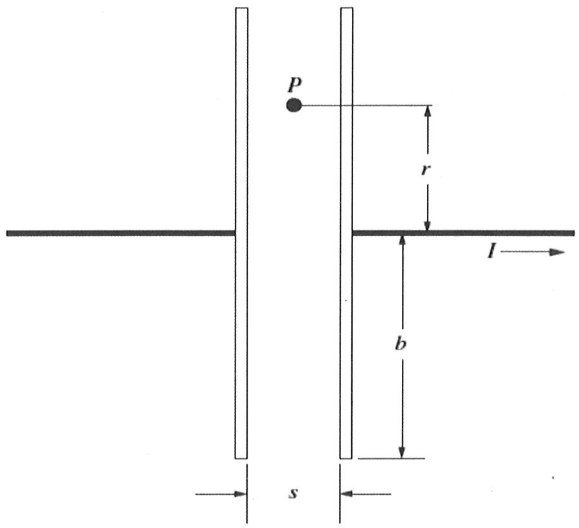
$U = \left(\frac{1}{2}\right)(20\times10^{-6})\left(0.75\ \frac{12}{0.02}\right)^2$

$U = 2.0\,J$

d)

Remander of energy heated up the wires

2) The capacitor shown below has circular plates with radius b. It is being discharged with a constant current I. The plates are separated by a much smaller distance, s which is much smaller than b. Point P is a distance r from the axis of the attached wires and $r < b$.



a) Prove that the total ``displacement current'' between the plates is equal to the conduction current in the wires, I.

$$\oint B \cdot dl = \mu_0 \Big(\underset{\substack{\uparrow \\ \text{conduction current}}}{I} + \underbrace{\epsilon_0 \frac{d\Phi_E}{dt}}_{\text{"displacement current" } I_D} \Big)$$

$$I_D = \epsilon_0 \frac{d\Phi_E}{dt} = \epsilon_0 \frac{d}{dt}(EA) = \epsilon_0 \frac{d}{dt}\left(\frac{\sigma}{\epsilon_0} A\right)$$

$$= \epsilon_0 \frac{d}{dt}\left(\frac{Q}{\epsilon_0}\right)$$

$$= \epsilon_0 \left(\frac{1}{\epsilon_0}\right)\frac{dQ}{dt}$$

$$= I \quad \checkmark$$

WWWWW

= "which was what we wanted"

b) What is the magnetic field at point P as a function of r?

Hint #1: Although this could be done with the Biot-Savart law that is extremely laborious, so do not use it.
Hint #2: There is a corresponding displacement current density J_D between the plates that makes magnetic field just like the usual J in a material does.

$$J_D = \frac{I_D}{\pi b^2} = \frac{I}{\pi b^2}$$ since E is uniform $\Rightarrow$ J_D is uniform

$$\int \vec{B} \cdot d\vec{l} = \mu_0 \int \vec{J}_D \cdot d\vec{A}$$

$$(B)(2\pi r) = \mu_0 \left(\frac{I}{\pi b^2}\right)(\pi r^2)$$

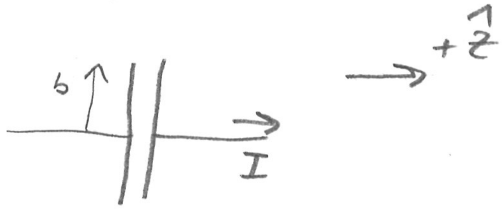
$$\boxed{B = \frac{\mu_0 I r}{2\pi b^2}}$$

(Note this gives expected result at $r=b$)

(J_D at greater radii than r all make $B=0$ by cylindrical symmetry)

Method II

Using Differential Maxwell's Equations!



$$\vec{E} = \frac{\sigma}{\epsilon_0}\hat{z} = \frac{Q}{\epsilon_0 A}\hat{z} = \frac{It\,\hat{z}}{\epsilon_0 A} = \frac{It\,\hat{z}}{\epsilon_0 \pi b^2}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Since $\vec{E}$ is along $\hat{z}$ we only need that component of the curl of $\vec{B}$ in cylindrical coordinates

$$\frac{1}{r}\left(\frac{\partial(rB_\theta)}{\partial r} - \frac{\partial B_r}{\partial \theta}\right) = \frac{\mu_0 \cancel{\epsilon_0} I}{\cancel{\epsilon_0} \pi b^2}$$

By cylindrical symmetry $\frac{\partial B_r}{\partial \theta} = 0$

$$\frac{1}{r}\frac{\partial(rB_\theta)}{\partial r} = \frac{\mu_0 I}{\pi b^2}$$

$$rB_\theta = \frac{1}{2}\frac{\mu_0 I r^2}{\pi b^2}$$

$$B_\theta = \frac{\mu_0 I r}{2\pi b^2}$$

$$\boxed{\vec{B} = \frac{\mu_0 I r}{2\pi b^2}\hat{\theta}}$$

3) An electromagnetic plane wave in free space has ~~intensity 100 W/m²~~ magnitude = 3 V/m and is traveling along the positive $\hat{x}$ direction. It is infinite in extent along the $\hat{y}$ and $\hat{z}$ directions. This is a radio wave with frequency f=30 MHz. The electric field points along the $\hat{y}$ direction. The magnetic field has its most negative values at x=~~10 m~~ 2.5 at t=0. Write down the equations describing the electric and magnetic fields as a function of time and space coordinates in a form proportional to $\sin(kx \pm \omega t + \phi_0)$, where you specify the values of wavenumber, k, angular frequency, ω, and phase constant, ϕ_0.

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^7} = 10\,\text{m}$$

Reminders: Make sure you also indicate the direction of the magnetic field. For this problem, signs matter!

$$k = \frac{\text{radians}}{\text{per wavelength}} = \frac{2\pi}{\lambda} = \frac{2\pi}{10} = 0.628\,\text{m}^{-1}$$

$$\omega = \frac{\text{radians}}{\text{per cycle}} = \frac{2\pi}{T} = 2\pi f = 6.28 \times 30\times10^6 = 1.9\times10^8\,\text{s}^{-1}$$

$$\sin(kx - \omega t + \phi_0) = -1$$

$$\sin\left(\left(\frac{2\pi}{10}\right)(2.5) - 0 + \phi_0\right) = -1$$

$$\sin\left(\frac{\pi}{2} + \phi_0\right) = 0$$

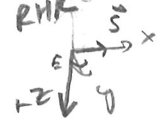
$$\Rightarrow \phi_0 = \frac{\pi}{2} \quad \text{since} \quad \sin\left(\frac{3\pi}{2}\right) = -1$$

$$\vec{E} = \hat{y}\,(3\,\text{V/m}) \sin\Big(\underset{\text{m}^{-1}}{0.628}\,x - \underset{\text{s}^{-1}}{1.9\times10^8}\,t + \pi/2\Big)$$

unitless

positive direction of propagation

$\vec{B}$ is in phase and points in $+\hat{z}$ direction so $\vec{S}$ points in $+\hat{x}$ direction by RHR



$$\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B}) \quad \text{R.H.R.}$$

$$\vec{B} = (+\hat{z})\left(\frac{3\,\text{V/m}}{3\times10^8\,\text{m/s}}\right) \sin(\text{same thing})$$

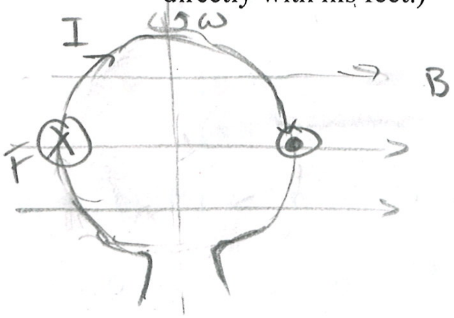
$$\boxed{\vec{B} = +\hat{z}\,(10^{-8}\,\text{T}) \sin(\text{same as above})}$$

Student Answer

4) Essay question with diagram:

In class we performed a demo where a student pedaled a bicycle at a constant angular speed which turned a coil (armature) inside a region of constant magnetic field to illuminate some light bulbs. As bulbs were added in parallel, he had to push harder to keep the bulbs as bright, as required by the concentration of energy. Using the forces involved, why did he have to push harder? Your explanation should include the concepts of ``back-EMF'' or ``back-torque'' (sometimes called ``counter-EMF'', ``counter-torque'') where relevant.

(For simplicity ignore any possible bicycle chains and assume he was turning the coil directly with his feet.)



when spinning a coil of wire in the magnetic field, a current is induced in the wire to oppose the change in magnetic flux. (Lenz's Law)

Here the coil is spun with the left side coming out and the right side going into the page. This spinning is done by the student. Magnetic flux through the coil increases out of the loop, so current is induced in a clockwise direction (facing the coil). Force = $q\vec{v} \times \vec{B}$, with charges flowing up on the right side and down on the left. The force points into the page on the left and out on the right. This creates a counter-torque that opposes the direction the student tries to spin the coil.

equivalenty you could say
Force = $I\vec{L} \times \vec{B}$

Nice job

25/25