Last Name: Brun
First Name: Jake
University ID:

Midterm #1, Version C
Physics 1B
Prof. David Saltzberg
April 29, 2014

Time: 50 minutes

Closed Notes. Closed Book. Allowed one 3"x5" index card.
Calculators are allowed. Show your work.

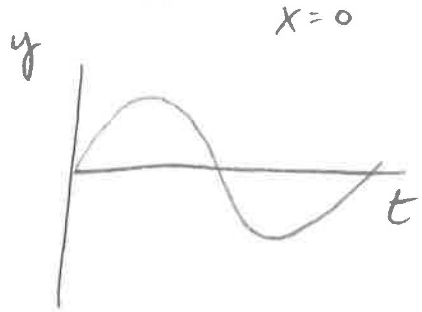
If a problem is confusing or ambiguous, notify the professor

Clarifications will be written on the blackboard. Check the board.

There are 1[illegible] pages including this cover sheet. Make sure you have them all.
Extra workspace is given and extra paper is at the front of the room.

Problem	**Points**	**Problem**	**Points**
1	/15	6	/25
2	/15	EC	/10
3	/15		
4	/15	--------	------------------------
5	/15	**TOTAL**	/100

1. ([illegible] pts.) Write an equation describing a transverse sinusoidal wave on a string that has a wave speed (phase velocity) of 314 m/s, a frequency of 100 Hz, an amplitude of 10 meters and is traveling in the negative *x* direction, where at time *t=0* the wave has a displacement of zero meters and is becoming positive.



$$\lambda = \frac{314\,m/s}{100\,Hz} = 3.14\,m$$

$$k = \frac{2\pi}{\lambda} = 2\,m$$

$$\omega = 2\pi f = 2\pi(100) = 628\,rad/s$$

$$y(x,t) = 10 \sin(kx + \omega t)$$

$$\boxed{y(x,t) = 10 \sin(2x + 628t)}$$

or equivalently

$$= 10 \cos\left(2x + 628t - \tfrac{\pi}{2}\right)$$

or $+\frac{3\pi}{2}$

3. (15 [redacted] pts.) Suppose the velocity, v, of an under-damped harmonic oscillator is given as a function of time, t, by:

$$\mathrm{v}(t) = 7e^{-0.1t}\cos(4t + \frac{\pi}{2})$$

with all numbers in SI units. How long does it take for the total energy stored in the oscillator to drop to 1% of what its value was at 5 seconds?

Only need to look at the envelope $E \propto v^2$

$$\frac{1}{100}\left(\cancel{7}e^{-0.1(5)}\right)^2 = \left(\cancel{7}e^{-0.1(5+t)}\right)^2$$

$$\frac{1}{100}\left(\cancel{e^{-0.5}}\right)^2 = \left(\cancel{e^{-0.5}}\right)^2\left(e^{-0.1t}\right)^2$$

Fun fact: Answer does not depend on where you start! Only true for an exponential

$$\frac{1}{100} = e^{-0.2t}$$

$$100 = e^{0.2t}$$

$$0.2t = \ln(100)$$

$$t = 5\ln(100)$$

$$t = 23 \text{ seconds}$$

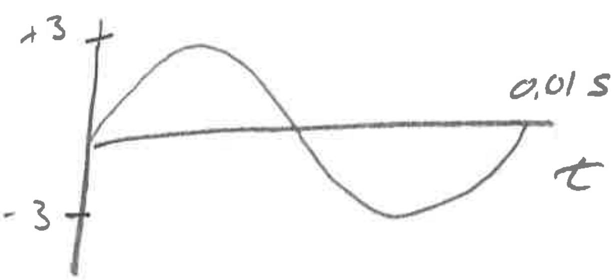
(Note: If you added 5 seconds to count from $t=0$, we gave you full credit)

5. (15 pts.) Using the convention in our class, an oscillating mass on a spring is described by the complex number $z=-3i$, where $i=\sqrt{-1}$. The oscillator is known to undergo a full cycle in 10 milliseconds as simple harmonic motion. Sketch the oscillation (displacement vs. time) from 0 to 10 milliseconds and label the amplitude.

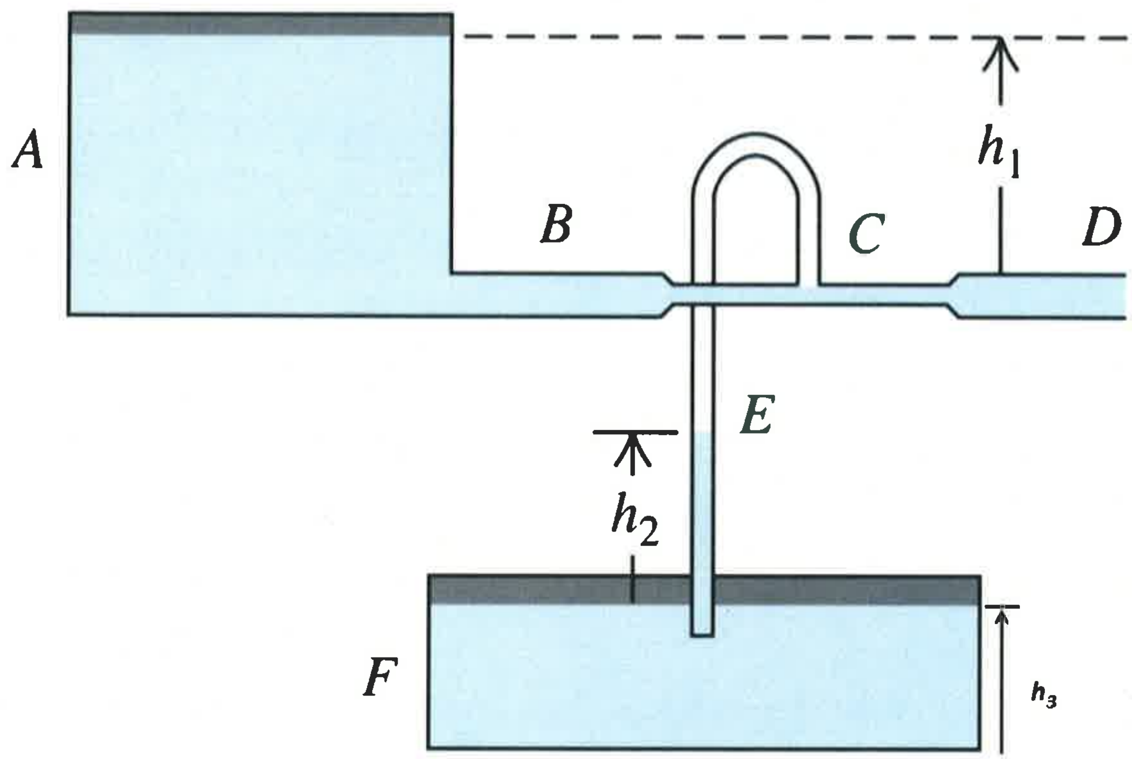
$$x(t) = \mathrm{Re}\left[-3i\,e^{i\omega t}\right]$$

$$= \mathrm{Re}\left[-3i\left(\cos\omega t + i\sin\omega t\right)\right]$$

$$= 3\sin(\omega t)$$



6. Two large tanks are open to atmospheric pressure, p_{atm}, and contain water as shown. A pipe passes from point B, past point C, and flows out into the air at point D. The pipe becomes narrower at point C where the cross-section (area) drops by a factor of 2. A tube opens into pipe at point C and dips into the liquid in the lower tank as shown. The depth of the water in the first tank is h_1. Its depth in the second tank is h_3. (For simplicity, the radius of the pipe is small compared to h_1, h_2 and h_3.)



The questions are on the next pages.

I am setting $y=0$ at A

a) (10 pts) What is the speed of the water flow when it exits the tube at point D?

A P_{atm}

B $P_{atm} + \rho g(-h_1) + \frac{1}{2}\rho v_D^2$

$$\cancel{P_{atm}} = \cancel{P_{atm}} - \rho g h_1 + \frac{1}{2}\rho v_D^2$$

$$\cancel{\rho} g h_1 = \frac{1}{2}\cancel{\rho} v_D^2$$

$$\boxed{v_D = \sqrt{2gh_1}}$$

b) (15 pts) To what height h_2 does the water rise in the tube above the surface of the second tank? [Hints: 1) The upper and lower bodies of water are separate. 2) The pressure of the air of the tube connecting C to E is uniform.]

By continuity $V_c = 2V_D = 2\sqrt{2gh_1}$

A = C

$$P_{atm} = P_c + \rho g(-h_1) + \tfrac{1}{2}\rho\left[2\sqrt{2gh_1}\right]^2$$

$$P_{atm} = P_c - \rho g h_1 + 4\rho g h_1$$

$$P_{atm} = P_c + 3\rho g h_1$$

$$P_c = P_{atm} - 3\rho g h_1$$

F = E (take $y=0$ at F)

F P_{atm}

E $P_c + \rho g h_2$

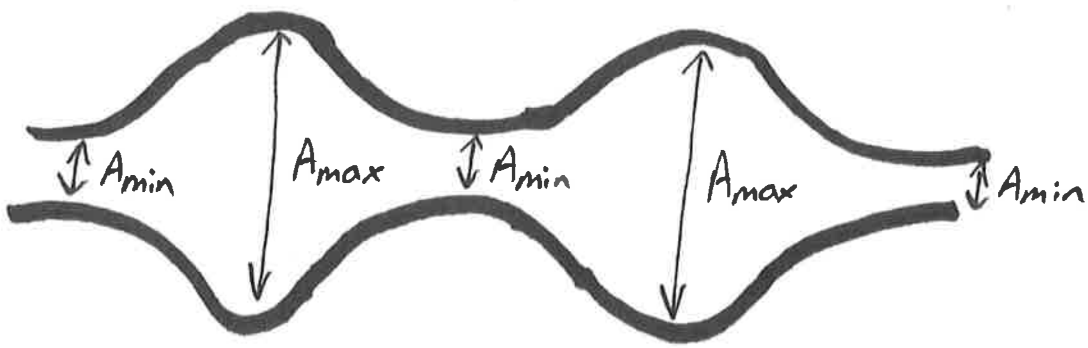
$$\cancel{P_{atm}} = \cancel{P_{atm}} - 3\rho g h_1 + \rho g h_2$$

$$3h_1 = h_2$$

$$\boxed{h_2 = 3h_1}$$

$h_2 = 1.5m$

<u>Extra Credit</u> (10 pts.) If an incident wave is only partially reflected from a boundary (such as sound from a soft wall) the resulting superposition of the two waves has an envelope that does not go completely to zero, as shown:



$$SWR \equiv A_{max}/A_{min}$$

Therefore a very important quantity in engineering and physics is the "<u>standing wave ratio</u>" or SWR which is defined as SWR=A_{max}/A_{min}. Suppose the reflected wave is in phase and has 50% of the incident amplitude. Find the ratio of the envelopes between the places where a node and an antinode would have been if the reflection had been 100% and in phase.

$$y(x,t) = A\cos(kx-\omega t) + \tfrac{1}{2}A\cos(kx+\omega t)$$

$$= A\Big[\cos(kx)\cos(\omega t) + \sin(kx)\sin(\omega t) + \tfrac{1}{2}\cos(kx)\cos(\omega t) - \tfrac{1}{2}\sin(kx)\sin(\omega t)\Big]$$

$$= A\Big[\tfrac{3}{2}\underbrace{\cos(kx)}_{\text{1 at former nodes}}\cos(\omega t) + \tfrac{1}{2}\underbrace{\sin(kx)}_{\text{1 at former antinodes}}\sin(\omega t)\Big]$$

$$\text{ratio} = \frac{3/2}{1/2} = \boxed{3}$$