PHYSICS 1B Winter 2015 February 24, 2015 Shoko Sakai

YOUR NAME: SOLUTIONS DISCUSSION SECTION: ___________

YOUR ID: ______________________________

Please write your answer ***clearly***.

Partial credits are given, so show as much work as possible. You may use diagrams where appropriate.

Make sure you have all 8 pages.

A calculator and one 3”x5” index card with notes is allowed.

Problem	
#1 (8 pts)	
#2 (13 pts)	
#3 (6 pts)	
#4 (10 pts)	
#5 (5 pts)	
#6 (13 pts)	
Total: 55 pts	

1. Two small, identical conducting spheres A and B are a distance R apart; each carries the same charge Q. [8 pts]

(a) What is the force sphere B exerts on sphere A? [2 pts]

$$F = \frac{1}{4\pi\epsilon_0}\frac{Q^2}{R^2}$$

(b) An identical sphere with zero charge, sphere C, makes contact with sphere B and is then moved very far away. What is the force sphere B now exerts on sphere A? [3 pts]

$$Q_B = \frac{1}{2}Q$$

$$F = \frac{1}{4\pi\epsilon_0}\frac{Q^2}{2R^2} = \frac{1}{8\pi\epsilon_0}\frac{Q^2}{R^2}$$

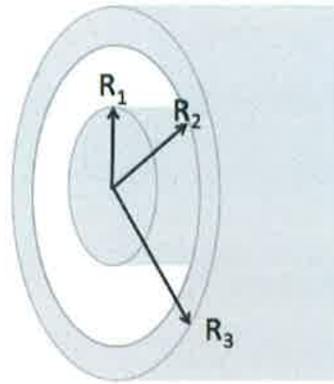
(c) Sphere C is brought back and now makes contact with sphere A and is then moved far away. What is the force sphere B exerts on sphere A in this third case? [3 pts]

$$Q_A = (Q + \tfrac{1}{2}Q)/2 = \tfrac{3}{4}Q$$

$$F = \frac{1}{4\pi\epsilon_0}\frac{Q^2}{R^2}\underbrace{\left(\frac{3}{4}\right)}_{\leftarrow A}\underbrace{\left(\frac{1}{2}\right)}_{\leftarrow B}$$

$$F = \frac{1}{32\pi\epsilon_0}\frac{Q^2}{R^2}$$

2. A very long solid nonconducting cylinder of radius R_1 is uniformly charged with a charge density r_E. It is surrounded by a concentric cylindrical tube of inner radius R_2 and outer radius R_3 as shown in the figure, and it too carries a uniform charge density r_E. [13 pts]



(a) What is the electric field for $R_1 < r < R_2$ as a function of r? [4 pts]

$$E(2\pi r l) = \frac{Q_{encl}}{\epsilon_0}.$$

$$E = \frac{Q_{encl}}{2\pi \epsilon_0 r l}$$

$$E = \frac{\rho_E \pi R^2 l}{2\pi \epsilon_0 r l} = \boxed{\frac{\rho_E}{2\epsilon_0} \frac{R_1^2}{r}}$$

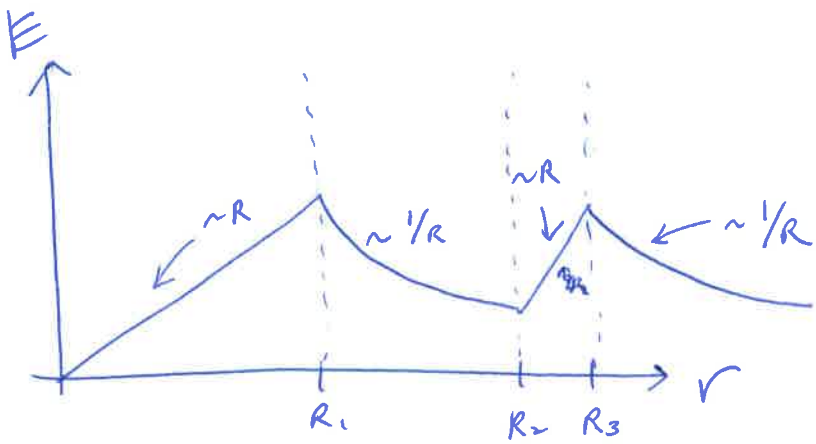
(b) What is the electric field for $r < R_1$? [3 pts]

$r < R_1$

$$E = \frac{\rho_E \pi r^2 l}{2\pi \epsilon_0 r l}$$

$$\boxed{E = \frac{\rho_E}{2\epsilon_0} R}$$

(c) Sketch the electric field as a function of the distance r from the center of the cylinders. Indicate clearly where $r=R_1$, R_3 and R_3. Also indicate what function of r the electric field is (for e.g., $E \sim 1/r$, etc). [6 pts]



3. A conducting spherical shell has inner radius = 10.0cm, outer radius = 15.0cm, and has a +3.0μC point charge at the center. A charge of -3.0μC is put on the conductor. [6 pts]

(a) Where on the conductor does the -3.0 μC end up? Explain. [2 pts]

Since $E=0$ inside the conductor,

$Q_{encl} = 0$ for $10cm < r < 15cm$

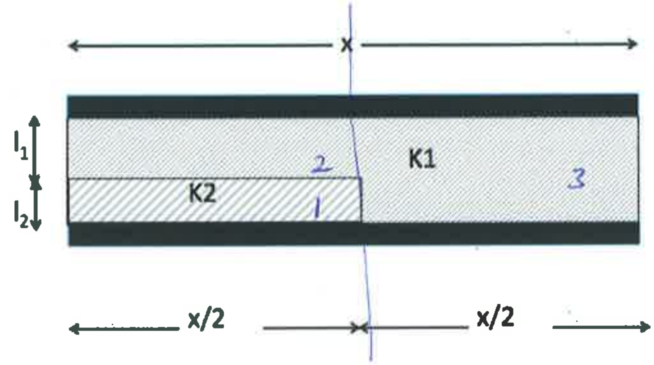
$\Rightarrow$ $-3\mu C$ needs to be on the inner surface of the shell.

(b) What is the electric field both inside and outside the shell? [4 pts]

inside: $E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$

outside: $E = 0$

4. A parallel-plate capacitor of area A has a dielectric inside, with dielectric constant K1, except for one corner of it which has a different dielectric of dielectric constant K2 as shown below. Determine the capacitance as a function of K1, K2 and thicknesses l_1 and l_2. [10 pts]



Left half:

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2}.$$

$$= \frac{1}{\left(\frac{K_2 \epsilon_0 A/2}{l_2}\right)} + \frac{1}{\left(\frac{K_1 \epsilon_0 A/2}{l_1}\right)}$$

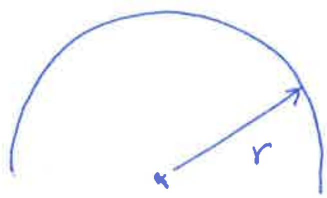
$$= \frac{2 l_2}{K_2 \epsilon_0 A} + \frac{2 l_1}{K_1 \epsilon_0 A}$$

$$\boxed{C' = \frac{\epsilon_0 A}{2} \cdot \frac{K_1 K_2}{l_1 K_2 + l_2 K_1}}$$

$$C = C' + C_3$$

$$\boxed{C = \frac{\epsilon_0 A}{2} \frac{K_1 K_2}{l_1 K_2 + l_2 K_1} + \frac{K_1 \epsilon_0 A/2}{(l_1 + l_2)}}$$

5. A total charge Q is uniformly distributed on a thread of length l. The thread forms a semicircle. What is the potential at the center as a function of Q and l? Assume V = 0 at large distances. [5 pts]



$\pi r = l$

$r = l/\pi$

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r} = \frac{1}{4\pi\epsilon_0 r}\int dq$$

$$= \frac{Q}{4\pi\epsilon_0 r} = \boxed{\frac{Q}{4\epsilon_0 l} = V}$$

6. Two parallel-plate air-gap capacitors, each having a capacitance of 2.00μF, are connected in parallel across a 12.0-V battery. The battery is disconnected from the parallel combination, and then a slab that has a dielectric constant K=2.50 is inserted between the plates of one of the capacitors, completely filling the gap. [13 pts]

(a) Before the dieletric slab is inserted, find the charge on each capacitor. [4 pts]

$Q_1 = CV$

$= (2\mu F)(12V)$

$Q_1 = 24\mu F$

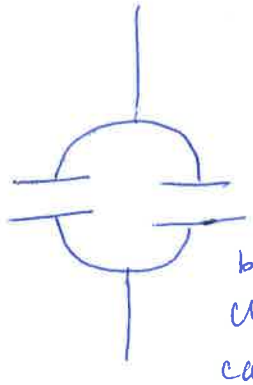
(b) After the dielectric is inserted, find the potential across each capacitor and the total energy stored in the capacitors. [9 pts]

$C_{eq} = C_1 + KC_2 = (2\mu F) + (2.5)(2\mu F) = 7\mu F.$

$V = \frac{Q_{total}}{C_{eq}} = \frac{48}{7} = 6.86V.$

$Q_1 = (2\mu F)(6.86) = 13.7\mu C.$

$Q_2 = (5\mu F)(6.86) = 34.3\mu C$



$V_1 = V_2$

but charge can move around.

$U = U_1 + U_2 = \frac{1}{2}Q_1V + \frac{1}{2}Q_2V$

$= \frac{1}{2}(Q_1 + Q_2)V$

$= \frac{1}{2}(13.7 + 34.3)(6.86)$

$U = 165\mu J$