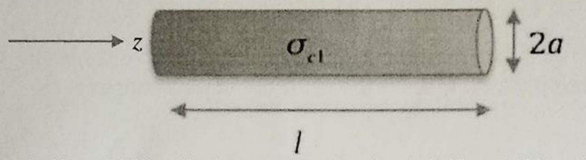
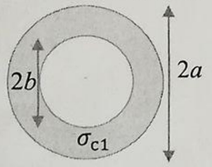
Physics 1BH – Prof. J. Rosenzweig – Winter 2018
Midterm 2
March 6, 2018

Use only the paper provided for you. Show all of your work for full credit. Write your name on each sheet of paper in your answers, then staple all together in order. You have 1 hour and 50 minutes to complete this exam.
You are permitted one sheet of paper as notes, with writing on both sides.

1. Consider a long ($l >> a$) resistor having conductivity σ_{c1} with a voltage V applied across it, as shown below, where you may in the usual manner approximate the current density along the resistor J_z as uniform and Ohm's law applies $J_z = \sigma_{c1} E_z$ and σ_{c1} is the conductivity

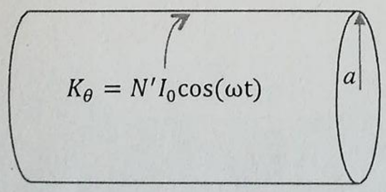


   (a) (5 pts) What is the current density J_z? (5 pts) What is the resistance R?
   (b) (10 pts) Approximating the current flow as infinite in length, what is the magnetic field inside of the cylinder, $r<a$?
   (c) Now consider a cylindrical resistor of the same length, but with a hole cut in it, up to the radius $r=b$ and resistive material from $b<r<a$ ($\sigma_c = \sigma_{c1}$), as shown.



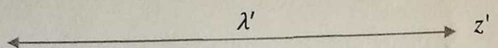
   (10 pts) What is the resistance of this new geometry? (10 pts) Again approximating the current density as infinite in length, what is the magnetic field for all $r<a$?

2. An infinitely long cylindrical solenoid of charge of constant density has a time varying azimuthal current $I(t) = I_0 \cos(\omega t)$.



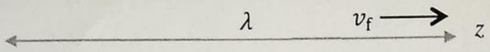
   (a) (10 pts) If we simply consider that is the magnetic field inside of the solenoid is the same as a static current, but with sinusoidal variation $\cos(\omega t)$ what is the magnetic field $\vec{B}$ inside of the solenoid?
   (b) (10 pts) Find the magnetic vector potential $\vec{A}$ inside the solenoid.
   (c) (10 pts) What is the induced electric field inside of the solenoid? You may for extra credit (10 pts) do this two ways – using Faraday's law of induction, and/or directly differentiating $\vec{A}$.

3. An infinite uniform line charge lying on the z'-axis is observed to be stationary in its rest frame, and has linear charge density λ' .



(a) (10 pts) What is the electrostatic potential associated with this static linear charge density?

(b) Now consider the charge to be in motion in the positive z direction with speed $v_f = 0.8c$. (10 pts) What are the observed line charge density and current in this "lab" frame, considering the Lorentz transformation-induced length contraction? (10 pts) What is the longitudinal component of the vector potential A_z?



(c) (15 pts) A particle of charge q is travelling parallel to this line charge, in the same direction and with the same speed v_f. What is the net force on this charge?

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a) $R = \frac{\rho L}{A} = \frac{L}{\sigma A} = \frac{L}{\sigma \pi a^2}$ ✓

$I = \frac{V}{R} = \frac{V\sigma A}{L}$

$J_z = \frac{I}{A} = \frac{V\sigma}{L}$ ✓ $E = \frac{V}{L}$ $E\sigma = \frac{V\sigma}{L}$ 10/10

b) $B = \frac{\mu_0 I}{2\pi r}$ $I = \pi r^2 J$

$B = \frac{\mu_0 r J}{2} = \frac{\mu_0 r V \sigma}{2L} \hat{\theta}$ 9/10

c) $R = \frac{L}{\sigma \pi (a^2 - b^2)}$ ✓ 10/10

$\int B \cdot da = \mu_0 J_{enc}$

$r < b$ $J_{enc} = 0$

$\vec{B} = 0$ for $r < b$ ✓

$b < r < a$ $B = \frac{\mu_0 I}{2\pi r}$ $I = (\pi r^2 - \pi b^2) J$

$B = \frac{\mu_0 (r^2 - b^2) J}{2r} = \frac{\mu_0 V \sigma (r^2 - b^2)}{2 r L} \hat{\theta}$ 9/10

2. a) B of solenoid $B = \mu_0 n I$

$B = \mu_0 N' I_0 \cos(wt)\,\hat{z}$ ✓ 8/10

b) $\int B \cdot da = \int_c A \cdot dl$

$\pi r^2 B = 2\pi r A$

$A = \frac{rB}{2}$

$A = \frac{\mu_0 N' I_0 \cos(wt)\, r}{2}\,\hat{\theta}$ 9/10

c) $\int E ds = -\frac{d}{dt} \int B \cdot da$

$\int E ds = -\frac{d}{dt} \pi r^2 \mu_0 N' I_0 \cos(wt)$

$= \pi r^2 \mu_0 N' I_0 \sin(wt) w$

$2\pi r E = \pi r^2 \mu_0 N' I_0 \sin(wt) w$

$E = \frac{r \mu_0 N' I_0 w \sin(wt)}{2}\,\hat{\theta}$ 9/10

$\int B \cdot da = \int_c A \cdot dl$

$\int E ds = -\frac{d}{dt} \int_c A \cdot dl$

$\Rightarrow E = -\frac{d}{dt} A = -\frac{d}{dt}\left(\frac{\mu_0 N' I_0 \cos(wt) r}{2}\right)$

$= \frac{\mu_0 N' I_0 \sin(wt) w r}{2}\,\hat{\theta}$ 9/10

a) $E = \frac{\quad}{2\pi\epsilon_0 r}$

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$\phi = -\int_{r}^{r_0} \frac{\lambda}{2\pi\epsilon_0 r} = \frac{\lambda}{2\pi\epsilon_0}\ln r\Big|_{r_0}^{r}$?

$\frac{\lambda}{2\pi\epsilon_0}\ln r - \phi(r_0)$ set $\phi(r_0) = 0$

$\phi(r) = \frac{-\lambda'}{2\pi\epsilon_0}\ln r$ 9/10

b) $\lambda = \gamma\lambda'$ $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-(0.8)^2}} = \frac{1}{0.6} = \frac{5}{3}$

$\lambda = \frac{5}{3}\lambda'$ ✓

$I = \lambda' v_f = \frac{5}{3}\lambda'\frac{4}{5}c = \frac{4}{3}\lambda' c$ ✓ 10/10

$B = \text{curl}\, A$

$\frac{\mu_0 I}{2\pi r} = \left(\overset{0}{\frac{\partial A_r}{\partial z}} - \frac{\partial A_z}{\partial r}\right)\hat{\theta}$

$\int \frac{\mu_0 I}{2\pi r} = \int -\frac{\partial A_z}{\partial r}$

$-\frac{\mu_0 I}{2\pi}\ln r = A_z$ ✓ 10/10

c) $E_\perp = \gamma E_\perp'$ $E' = \frac{\lambda}{2\pi\epsilon_0 r}$

$E_\perp = \frac{\gamma\lambda}{2\pi\epsilon_0 r}$

$\vec{F} = \frac{\gamma\lambda q}{2\pi\epsilon_0 r}\hat{r}$

$cB_\perp = \gamma(c\overset{0}{B_\perp'} - \beta \times E_\perp')$

$B_\perp = -\frac{\gamma\beta}{c}E_\perp$

$= \frac{-\gamma\beta\lambda}{c2\pi\epsilon_0 r}$

$\vec{F} = qv\times B = -\frac{\gamma\beta^2\lambda q}{2\pi\epsilon_0 r}\cancel{\hat{\theta}}\ \hat{r}$

$\vec{F} = \frac{\gamma\lambda q}{2\pi\epsilon_0 r}\hat{r} - \frac{\gamma\beta^2\lambda\cancel{q}}{2\pi\epsilon_0 r}\hat{\theta}$

10/15