Final exam solutions

Physics 1B, Spring 2016

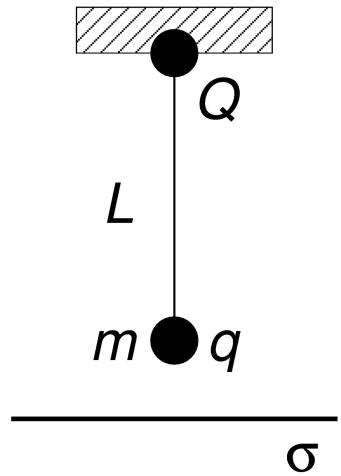
Name:
UCLA ID number:
Lecture: ☐ 4 ☐ 5
Section (number, meeting time, or TA name):

Please write solutions with some minimal derivation in the space provided below each problem; it is not sufficient to give just the final answer. The level of detail should be such that a grader, or your fellow classmate would understand how you solved the problem. You may use the back sides of each page as scrap paper.

1	**2**	**3**	**4**	**5**	**6**	**total**

Problem 1.

A small ball of mass m is attached to a massless string of length L and charged with a positive charge q. A positive point charge Q is placed at the pivot of the pendulum. An infinite horizontal plane with a uniform positive charge density σ is placed below the pendulum. The charges are such that, when the pendulum is at rest, the string is under tension.



(a) Calculate the force of tension in the string.

The force of tension is equal

$$F = mg + \frac{1}{4\pi\epsilon_0}\frac{Qq}{L^2} - \frac{\sigma q}{2\epsilon_0}$$

(b) Calculate the period of small oscillations of this pendulum, taking into account both the force of gravity and the electrostatic forces.

The restoring force is a combination of the force of gravity and the force of repulsion from the plane. The force due to the point charge Q plays no role because it is always orthogonal to the displacement, and it does not contribute to the restoring force. For a small displacement $\theta \approx x/L$,

the restoring force is

$$F = -(mg - \frac{\sigma q}{2\epsilon_0})\sin\theta \approx -(mg - \frac{\sigma q}{2\epsilon_0})\theta \approx -(mg - \frac{\sigma q}{2\epsilon_0})\frac{x}{L}$$

The equation of motion is

$$m\frac{d^2}{dt^2}x = -k_{\text{eff}}x$$

where

$$k_{\text{eff}} = (mg - \frac{\sigma q}{2\epsilon_0})\frac{1}{L}$$

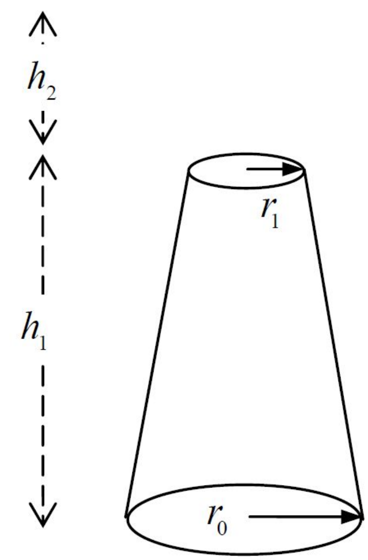
This is identical to oscillations of a mass attached to a spring.

The period of oscillations is

$$T = 2\pi\sqrt{\frac{m}{k_{\text{eff}}}} = 2\pi\sqrt{\frac{mL}{mg - \sigma q/(2\epsilon_0)}} = 2\pi\sqrt{\frac{L}{g - \sigma q/(2\epsilon_0 m)}}$$

Problem 2.

A drinking fountain shoots water to the height h_2 up in the air from a nozzle of radius r_1. The pump at the base of the unit, at height h_1 below the nozzle, pushes water into a pipe of radius r_0.



(a) What is the speed of the water coming out of the nozzle?

Using Bernoulli's equation for the water going from height h_1 to height $h_1 + h_2$, one can write:

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g(h_1 + h_2) + \frac{1}{2}\rho v_2^2,$$

where $v_2 = 0$ and $P_1 = P_2 = P_{\text{atm}}$. Therefore,

$$v_1 = \sqrt{2gh_2}$$

(b) What gauge pressure does the pump have to provide? (Ignore viscosity.)

Using continuity equation, $\pi r_1^2 v_1 = \pi r_0^2 v_0$, we obtain $v_0 = (r_1^2/r_0^2)v_1$. Now let us use Bernoulli's equation

$$P_0 + \rho g h_0 + \frac{1}{2}\rho v_0^2 = P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2,$$

where $P_1 = P_{\text{atm}}$ and $h_0 = 0$, and $v_1 = \sqrt{2gh_2}$. This gives

$$P_0 - P_{\text{atm}} = \rho g h_1 + \frac{1}{2}\rho(v_1^2 - v_0^2) = \rho g h_1 + \frac{1}{2}\rho\left(1 - \frac{r_1^4}{r_0^4}\right)v_1^2 = \rho g\left\{h_1 + h_2\left(1 - \frac{r_1^4}{r_0^4}\right)\right\}$$

Problem 3.

On a remote planet, the atmosphere is different from Earth, but it allows life to exist. The speed of sound on this planet is $v_s = 100$ m/s.

(a) A spacecraft is flying horizontally at a height of 1 km with a speed of 200 m/s. How far will the spacecraft be from an animal on the surface, when the animal hears the sonic boom? (Calculate the actual distance, not the projection on the ground.)

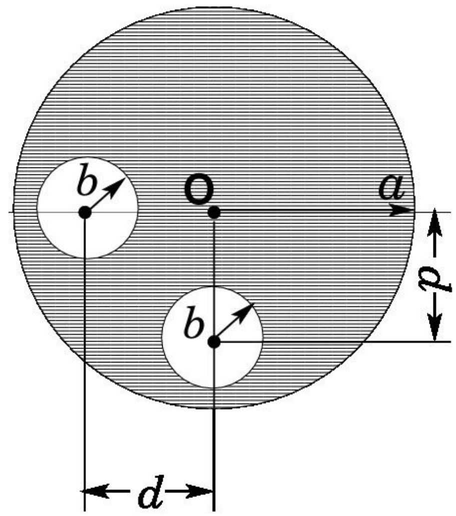
The angle of the shock wave is given by $\sin\theta = v_s/v_{\text{spacecraft}} = 1/2$. The distance to the spacecraft is
$L = h/\sin\theta = 2\,\text{km}$

(b) When the spacecraft preparing for landing slows down to a speed of 10 m/s heading directly toward an animal, it emits a sound of frequency 1.0 kHz to warn and to scare the animal away. What frequency will the animal hear?

The frequency is Doppler shifted to 1.1 kHz

Problem 4.

A non-conducting sphere of radius a with a constant positive charge density ρ has two identical hollow spherical cavities (with zero charge density), as shown in the figure.



(a) What is the magnitude of the electric field at point O in the center of the large sphere?

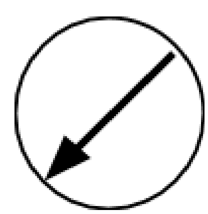
Each cutout is equivalent to a negative charge $4\pi b^3\rho/3$. The sphere without the cutouts contributes zero to the electric field in the center. Each cutout contributes

$$|\vec{E}_1| = \frac{1}{4\pi\epsilon_0}\frac{4\pi\rho b^3/3}{d^2} = \frac{1}{3\epsilon_0}\frac{\rho b^3}{d^2}$$

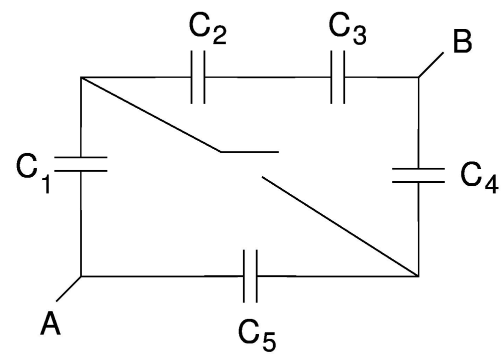
The total electric field is a vector sum of two identical vectors at 45 degrees to each other:

$$E = \frac{\sqrt{2}}{3\epsilon_0}\frac{\rho b^3}{d^2}$$

(b) Show the direction of the electric field by drawing an arrow in this circle:



Problem 5. Every capacitor in the circuit has the same capacitance C : $C_1 = C_2 = C_3 = C_4 = C_5 = C$



(a) Calculate the equivalent capacitance between points A and B when the switch is open.

$C_{eq} = C/3 + C/2 = (5/6)\ C$

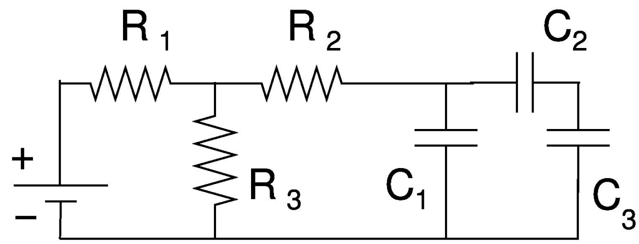
(b) Calculate the equivalent capacitance between A and B when the switch is closed.

The circuit is equivalent to $C_1 + C_5 = 2C$ and $(1/2)C + C = (3/2)\ C$ connected in a series. Hence, $C_{eq} = (6/7)\ C$.

(c) If the potential difference $V_{AB} = V$ is applied between points A and B when the switch is open, what is the charge Q_1 on capacitor C_1?

The three equal capacitors are connected in a series and share voltage V. This means the voltage on C_1 is equal $V/3$. Therefore, $Q_1 = CV/3$.

Problem 6.



Three identical resistors $R_1 = R_2 = R_3 = R$ and three identical capacitors $C_1 = C_2 = C_3 = C$ are connected to an ideal battery with emf $\mathcal{E}$ as shown in the figure. Express all answers in terms of $R, C, \mathcal{E}$.

(a) Assuming that enough time has elapsed for the currents to be constant, calculate the currents I_1, I_2, I_3 in the resistors R_1, R_2, R_3, respectively.

$$I_1 = I_3 = \frac{\mathcal{E}}{2R}$$

I_2 =0

(b) Calculate the total charge stored in the three capacitors combined ($Q = Q_1 + Q_2 + Q_3 = ?$).

The voltages on the capacitors are $\mathcal{E}/2$, $\mathcal{E}/4$, and $\mathcal{E}/4$, respectively, and so the sum of the charges is

$Q = Q_1 + Q_1 + Q_3 = C(\mathcal{E}/2 + \mathcal{E}/4 + \mathcal{E}/4) = C\mathcal{E}$. This is the correct answer.

(There is no ambiguity in the problem; it explicitly asks for $Q = Q_1 + Q_1 + Q_3$. However, some students could potentially get confused and calculate the charge on the "equivalent" capacitor $C_{\text{eq}} = C + C/2 = \frac{3C}{2}$. The voltage is equal the potential drop on R_3 (there is no current in R_2): $V = R_2 I_2 = \mathcal{E}/2$. So, $Q_{\text{eq}} = C_{\text{eq}} \times V = \frac{3C\mathcal{E}}{4}$.
To be fair to all, **we will accept both solutions as correct solutions**.)

(c) A slab of dielectric with the dielectric constant K is inserted into the parallel-plate capacitor C_3, so that it fills the entire space between conducting plates. Calculate the electrostatic potential energy stored in capacitor C_3 with the dielectric inside.

The new capacitance is $C_3' = KC$. The charges on C_2 and C_3 are equal to each other and equal q, but the voltages are not equal, and

$V = \mathcal{E}/2 = V_2 + V_3 = \frac{q}{C} + \frac{q}{KC} = \frac{K+1}{K}\frac{q}{C}$. From this equation, one can find

$q = \frac{KCV}{1+K} = \frac{KC\mathcal{E}}{2(K+1)}$. Therefore, $PE = \frac{q^2}{2KC} = \frac{KC\mathcal{E}^2}{8(K+1)^2}$.