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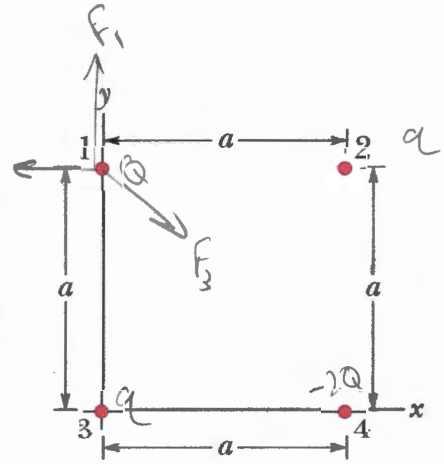
Lecture (1 or 2): 1   Section (A-F):

1. Four point charges form a square, as shown in the figure. The charges are $q_1 = Q$, $q_2 = q_3 = q$, and $q_4 = -2Q$. What is the ratio q/Q if the net electrostatic force on particle 1 is zero?



$$F_1 = \frac{kqQ}{a^2} \uparrow$$

$$F_2 = \frac{kqQ}{a^2}(-\hat{i})$$

$$F_3 = \frac{\sqrt{2}}{2} \cdot \frac{-2kQ^2}{2a^2}(-\hat{j}) + \frac{\sqrt{2}}{2} \cdot \frac{-2kQ^2}{2a^2}\hat{i}$$

$$\frac{kqQ}{a^2} = \frac{\sqrt{2}kQ^2}{2a^2}$$

$$q = \frac{\sqrt{2}Q}{2}$$

$$\frac{q}{Q} = \frac{\sqrt{2}}{2}$$

2. A nonconducting spherical shell, with an inner radius $R_1 = 4$ m and an outer radius $R_2 = 6$ m, has charge spread non-uniformly through its volume between its inner and outer surfaces. The volume charge density ρ is the charge per unit volume measured in coulombs per cubic meter. For this shell,

$$\rho(r) = \frac{b}{r^2},$$

where r is the distance to the center, and $b = 8.9 \times 10^{-12}$ C/m.

a) What is the electric field at distance 3 m from the center?

b) What is the electric field at distance 5 m from the center?

c) What is the electric field at distance 10 m from the center?

Gauss's Law

$\Phi_E = \int E\,d\vec{A} = 4\pi k Q_{enclosed}$

a) zero ✓

b) $EA = 4\pi k Q_{enclosed}$ ⟶ $E \cdot 4\pi r^2 = 4\pi k \int_4^5 \rho(r) \cdot 4\pi r^2 dr$

~~$E \cdot 4\pi r^2 = 4\pi k \cdot \int_4^5 \frac{b}{r^2} dr$~~

~~$E \cdot 25 = k \left[-\frac{b}{r} \right]_4^5$~~

~~$25E = -kb \left(\frac{1}{5} - \frac{1}{4} \right)$~~

~~$25E = -kb \left(-\frac{1}{20} \right)$~~

~~$E = \frac{kb}{25 \cdot 20} = 1.6 \cdot 10^{-4}$ N/C~~

$E \cdot r^2 = k \int_4^5 4\pi b\, dr$

$E \cdot 25 = 4\pi k b \cdot (5-4)$

$E = 0.0406$ N/C

c) $EA = 4\pi k Q_{enclosed}$ ⟶ $E \cdot 4\pi r^2 = 4\pi k \int_4^6 \rho(r) \cdot 4\pi r^2 dr$

~~$E \cdot 4\pi r^2 = 4\pi k \int_4^6 \frac{b}{r^2} dr$~~

~~$100E = -bk \left(\frac{1}{6} - \frac{1}{4} \right)$~~

~~$E = \frac{bk}{12 \cdot 100}$~~

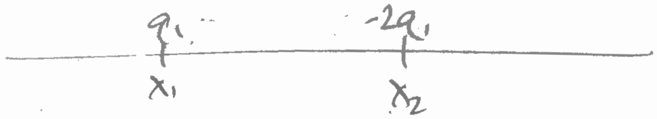
~~$E = 6.675 \cdot 10^{-5}$ N/C~~

$E \cdot r^2 = k \int_4^6 4\pi b\, dr$

$E \cdot 100 = 4\pi k b \cdot (6-4)$

$E = 0.0203$ N/C

3. Two particles are fixed to an x axis: particle 1 of charge $q_1 = 2.1 \times 10^{-8}$ C at $x = x_1$ and particle 2 of charge $q_2 = -2q_1$ at $x = x_2$. Assuming the potential is zero at infinity, at what coordinate x_0 on the axis is the net potential produced by the particles equal to zero?



potential $V = \frac{U}{q}$

$r_2 = x - x_2$

$r_1 = x - x_1$

particle 1

$V_1(r) = \frac{kq_1}{r_1}$

particle 2

$V_2(r) = -\frac{2kq_1}{r_2}$

$$\frac{kq_1}{r_1} = \frac{2kq_1}{r_2}$$

$$\frac{r_2}{r_1} = 2$$

$$\frac{|x - x_2|}{|x - x_1|} = -2$$

$$\frac{x - x_2}{x - x_1} = -2$$

$$x - x_2 = -2x + 2x_1$$

$$3x = 2x_1 + x_2$$

$$x = \frac{2x_1 + x_2}{3}$$

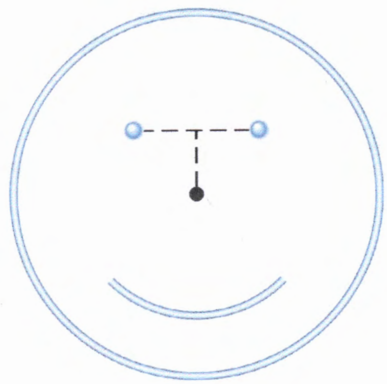
$$\frac{x - x_2}{x - x_1} = 2$$

$$x - x_2 = 2x - 2x_1$$

$$x = 2x_1 - x_2$$

4. The smiling face shown in the figure consists of three items:

- a thin ring of radius r_{ring} with charge Q_{ring}
- a thin rod of charge Q_{arc} that forms a circular arc of radius r_{arc} subtending an angle θ_{arc}.
- an electric dipole with a dipole moment p located so that each of the two charges is at distance r_{dipole} from the center.



What is the net electric potential at the center, assuming the potential is zero at infinity?

$V = V_{ring} + V_{arc} + V_{dipole}$

$V_{dipole} = 0$ because the two charges cancel each other out.

$$V_{ring} = \int \frac{K\,dQ}{r_{ring}} = \frac{KQ_{ring}}{r_{ring}}$$

$$V_{arc} = \int \frac{K\,dQ}{r_{arc}} = \frac{KQ_{arc}}{r_{arc}}$$

$$V = K\left(\frac{Q_{ring}}{r_{ring}} + \frac{Q_{arc}}{r_{arc}}\right)$$

5. An electron in vacuum passes through a ring of radius r, which has a negative charge Q. The speed of the electron at the center of the ring is v_0. The charge of the electron is q_e, and its mass is m_e. What is the speed of this electron at infinity (very far away from the ring)?

$KE_i + PE_i = KE_f + PE_f$

$\frac{1}{2}mv_0^2 + V = \frac{1}{2}mv_f^2$

$v_0^2 + \frac{2V}{m} = v_f^2$

$v_f = \sqrt{v_0^2 + \frac{2V}{m_e}}$

$V = \int_0^{2\pi r} \frac{k\,dq}{r}$ $\quad dq = \frac{Q}{2\pi r}dr$

$= \int_0^{2\pi r} \frac{kQ}{2\pi r^2}dr$

$= \left[\frac{-kQ}{2\pi r}\right]_0^{2\pi}$

$V = \int \frac{k\,dQ}{r} = \frac{kQ}{r}$

$v_f = \sqrt{v_0^2 + \frac{2kQ}{m_e r_{ring}}}$