Midterm 2

Physics 1B (Lec 4)

Name: [redacted]

ID number:

Discussion section: 4

Time to complete the exam: 90 min

Each problem is worth 20 points. If a problem has parts (a) and (b), they are 10 points each. It is not sufficient to present the final answer. You need to show the solution and justify your steps at the level of detail that would be sufficient for your fellow classmate (or grader) to understand how you arrived at the final answer. Please write your solutions in the spaces below each question. You can use the back sides of the pages as scrap paper. Numerical answers need not have more significant figures than the numbers provided in the problem.

1	2	3	4	total
20	20	20	20	80

$\mu = \left(2 \frac{g}{cm}\right)\left(\frac{100 cm}{1 m}\right)\left(\frac{1 kg}{1000 g}\right) = .2 \frac{kg}{m}$

Problem 1

A standing wave is generated on a string with linear mass density $\mu = 2$ g/cm and the tension force $F = 100$ N. The distance between the first and the fourth nodes is 0.3 m.

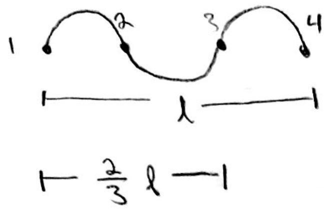
7 (a) Find the wavelength of the standing wave

Given

$\mu = .2 \frac{kg}{m}$

$F_T = 100 N$

$l_{1-4} = 0.3 m$



$\lambda = \frac{2}{3} l = \frac{2}{3}(0.3 m) = \boxed{0.2 m}$

7 (b) Find the frequency of the wave on the string

$v = \sqrt{\frac{F_T}{\mu}} = \lambda f$

$f = \frac{1}{\lambda}\sqrt{\frac{F_T}{\mu}} = \frac{1}{0.2 m}\sqrt{\frac{100 N}{.2 \frac{kg}{m}}} = 111.803 \text{ Hz}$

$= \boxed{100 \text{ Hz}}$

6 (c) Find the wavelength of the sound wave generated by the string if the speed of sound in air is 340 m/s. *(Hint: the sound wave must have the same frequency as the string, but not necessarily the same wavelength.)*

Given

$f = 111.803 Hz$

$= 100 Hz$

$V_s = 340 \frac{m}{s}$

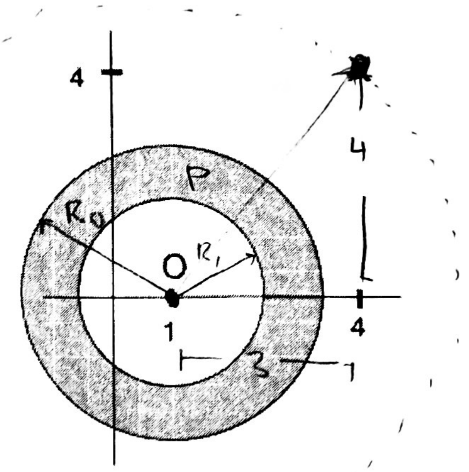
$V_s = f \lambda_s$

$\lambda_s = \frac{V_s}{f} = \frac{340 \frac{m}{s}}{111.803 Hz} = 3.04106 m$

$= \boxed{3 m}$

Problem 2

A spherical shell with inner and outer radii $R_i = 1.5\ m$ and $R_o = 2.5\ m$, respectively, is centered at a point with coordinates (1,0), as shown. The shell carries a constant charge density $\rho = 2 \times 10^{-9}$ C/m^3. All coordinates are in meters.



(a) Calculate the electric field at the center of the sphere O(1,0) (Justify your answer.)

$\boxed{\vec{E} = 0\ \frac{N}{C}}$ because if you take a sphere of radius $r < R_i$, the charge enclosed by that sphere is zero. Since $\Phi = \oint \vec{E} \cdot d\vec{A} = E \cdot A = \frac{Q_{encl}}{\varepsilon_0}$ and thus $E = \frac{Q_{ENCL}}{A\,\varepsilon_0}$, $\vec{E} = 0\ \frac{N}{C}$ since the enclosed charge is zero.

+5

(b) Calculate the electric field at point X with coordinates (4,4)

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{ENCL}}{\varepsilon_0}$$

$$EA = \frac{Q_{ENCL}}{\varepsilon_0} \Rightarrow E = \frac{Q_{ENCL}}{A\,\varepsilon_0}$$

+15

$$Q_{ENCL} = \int \rho(r)\,dV = \int_{R_i}^{R_o} 4\pi r^2 \rho\,dr = \frac{4}{3}\pi r^3 \rho \Big|_{R_i}^{R_o}$$

$$= \frac{4}{3}\pi\rho\left[R_o^3 - R_i^3\right] = \frac{4}{3}\pi\rho\left[2.5m^3 - 1.5m^3\right]$$

$$= 1.02625 \times 10^{-7}\ C$$

$$A = 4\pi r^2 \Rightarrow r = \sqrt{(3m)^2 + (4m)^2} = \sqrt{25m} = 5m$$

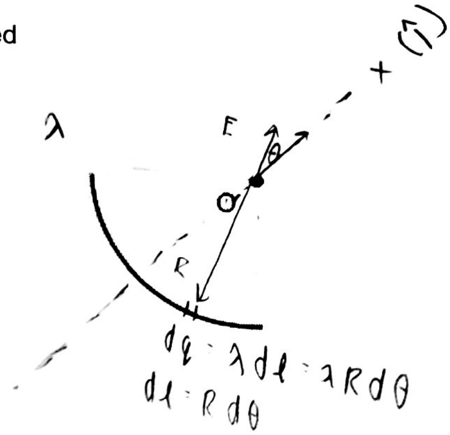
$$= 4\pi(5m)^2 = 100\pi\ m^2$$

$$E = \frac{Q_{ENCL}}{A\,\varepsilon_0} = \frac{1.02625 \times 10^{-7}\ C}{(100\pi\ m^2)(8.85 \times 10^{-12}\ C^2/N \cdot m^2)}$$

$$= 36.9115\ \frac{N}{C} = \boxed{37\ \frac{N}{C}}$$

Problem 3 R = 1m

A thin thread carrying a constant charge density $\lambda = 4 \times 10^{-9}$ C/m is shaped as 1/4 of a circle. Calculate the electric field at the center of the circle O.



$$dE_x = k\frac{dq}{R^2}\cos\theta = \frac{k(\lambda\, dl)}{R^2}\cos\theta$$

$$= \frac{k(\lambda R\, d\theta)}{R^2}\cos\theta$$

$$= \frac{k\lambda\, d\theta}{R}\cos\theta$$

$$E_x = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{k\lambda}{R}\cos\theta\, d\theta$$

$$= \frac{k\lambda}{R}\sin\theta \Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$$

$$= \frac{k\lambda}{R}\left(\sin\frac{\pi}{4} - \sin\left(-\frac{\pi}{4}\right)\right)$$

$$= \frac{k\lambda}{R}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right)$$

$$= \frac{k\lambda}{R}\left(\sqrt{2}\right)$$

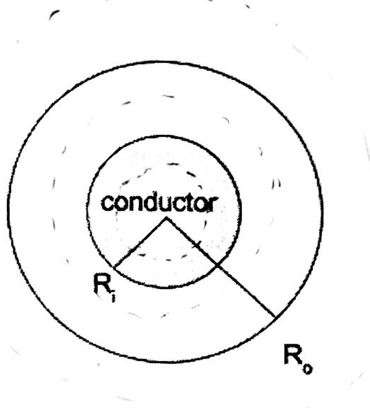
$$\vec{E} = \sqrt{2}\,\frac{k\lambda}{R}\,\hat{i} = \sqrt{2}\left(\frac{(8.99\times10^{9})(4\times10^{-9}\frac{C}{m})}{(1m)}\right)\hat{i}$$

$$= 50.8894\,\frac{N}{C}\,\hat{i}$$

$= \boxed{50\,\frac{N}{C}\,\hat{i}}$, where $\hat{i}$ is the direction indicated on the illustration above

Problem 4

A filled conducting sphere with zero charge and radius R_i is surrounded by a spherical shell with the inner and the outer radii R_i and R_o, respectively. The shell carries the charge density $\rho(r) = \rho_0(R_i/r)$, $R_i < r < R_o$. There is no charge outside R_o.



(a) Calculate the electric field $E(r)$ for $r < R_i$

$$\Phi = \oint \vec{E}\cdot d\vec{A} = \frac{Q_{ENCL}}{\varepsilon_0} \qquad Q_{ENCL} = 0\ C$$

$$EA = \frac{Q_{ENCL}}{\varepsilon_0} \Rightarrow E = \frac{Q_{ENCL}}{\varepsilon_0 A} = \boxed{0\ \frac{N}{C}}$$

(b) Calculate the electric field $E(r)$ for $R_i < r < R_o$

$$\Phi = \oint \vec{E}\cdot d\vec{A} = \frac{Q_{ENCL}}{\varepsilon_0} = E\cdot A$$

$$Q_{ENCL} = \cancel{\int_0^{R_i} \rho(r)\,dV}^{\,0} + \int_{R_i}^{r} \rho(r)\,dV = \int_{R_i}^{r} \left[\rho_0\left(\frac{R_i}{\cancel{r}}\right)\right]\cdot\left[4\pi r^{\cancel{2}}\,dr\right]$$

$$= \int_{R_i}^{r} (4\pi\rho_0 R_i)\,r\,dr = \overset{2}{\cancel{4}}\pi\rho_0 R_i \frac{r^2}{\cancel{2}}\Big|_{R_i}^{r} = 2\pi\rho_0 R_i\left[r^2 - R_i^2\right]$$

$$A = 4\pi r^2$$

$$E = \frac{Q_{ENCL}}{A\cdot\varepsilon_0} = \frac{\cancel{2\pi}\rho_0 R_i[r^2 - R_i^2]}{2\,\cancel{4\pi} r^2\cdot\varepsilon_0} = \boxed{\frac{\rho_0 R_i[r^2 - R_i^2]}{2\varepsilon_0 r^2}\ \frac{N}{C}}$$

(c) Calculate the electric field $E(r)$ for $r > R_o$

$$\Phi = \oint \vec{E}\cdot d\vec{A} = \frac{Q_{ENCL}}{\varepsilon_0} = E\cdot A$$

$$Q_{ENCL} = \cancel{\int_0^{R_i} \rho(r)\,dV}^{\,0} + \int_{R_i}^{R_o} \rho(r)\,dV = \int_{R_i}^{R_o} \left[\rho_0\left(\frac{R_i}{\cancel{r}}\right)\right]\cdot\left[4\pi r^{\cancel{2}}\,dr\right]$$

$$= \int_{R_i}^{R_o} (4\pi\rho_0 R_i)\,r\,dr = 2\pi\rho_0 R_i r^2\Big|_{R_i}^{R_o} = 2\pi\rho_0 R_i\left[R_o^2 - R_i^2\right]$$

$$A = 4\pi r^2$$

$$E = \frac{Q_{ENCL}}{A\,\varepsilon_0} = \frac{\cancel{2\pi}\rho_0 R_i[R_o^2 - R_i^2]}{2\,\cancel{4\pi} r^2\,\varepsilon_0} = \boxed{\frac{\rho_0 R_i[R_o^2 - R_i^2]}{2\varepsilon_0 r^2}\ \frac{N}{C}}$$