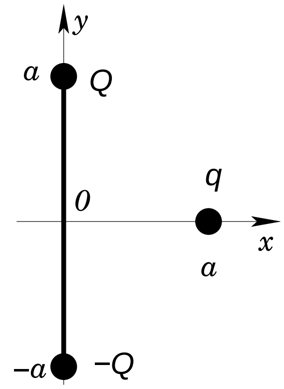
Midterm 2
Physics 1B

Name:

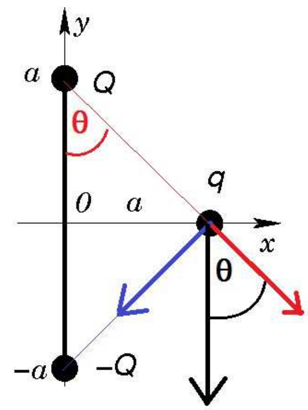
ID number:

Lecture 4

1. A dipole consists of a positive charge Q and negative charge $-Q$ positioned at $y = a$ and $y = -a$ and connected by a rigid rod, as shown. A positive charge q is located on the x-axis at $x = a$.



(a) What is the direction of the force acting on the charge q? (Draw a vector showing the direction.)



(b) What is the magnitude of the force acting on charge q?

The x-components of the forces coming from $+Q$ and $-Q$ cancel out, and the y-components add up (and are of equal magnitudes). The distance between Q and q is $\sqrt{2}a$. Hence,

$$F_{\text{tot}} = 2F_y = 2 \times \frac{1}{4\pi\epsilon_0} \frac{qQ}{\left(\sqrt{2}a\right)^2} \cos\theta = 2 \times \frac{1}{4\pi\epsilon_0} \frac{qQ}{2a^2} \frac{\sqrt{2}}{2} = \frac{1}{4\pi\epsilon_0} \frac{qQ}{a^2} \frac{\sqrt{2}}{2} \ . \quad (1)$$

The same solution can be obtained without $\cos\theta$, using Pythagoras' theorem, because the two components of the force are orthogonal to each other.

2. A very long, solid, non-conducting cylinder has radius R. The volume charge density depends on the distance r from the central axis as

$$\rho(r) = \rho_0 \left(\frac{R}{r}\right),\ r \le R;$$

$$\rho(r) = 0,\ r > R.$$

a) What is the electric field at distance $r = R/2$ from the axis?

Let us use the Gaussian surface that is a cylinder of length L and radius $R/2$. The Gauss's law states that

$$\Phi_E = \oint \boldsymbol{E} \cdot d\boldsymbol{A} = \frac{Q_{\text{encl}}}{\epsilon_0}\ . \tag{2}$$

Because of the symmetry, the electric field is perpendicular to the side surface of the cylinder and it is constant on that surface. Hence,

$$\Phi_E = \oint \boldsymbol{E} \cdot d\boldsymbol{A} = E \oint d\boldsymbol{A} = E\, 2\pi \left(\frac{R}{2}\right) L = E\, \pi R L\ . \tag{3}$$

On the other hand, the charge within this radius is given by

$$\begin{aligned} Q\left(\frac{R}{2}\right) &= \int_0^{R/2} dV\, \rho(r) = \int_0^{R/2} (2\pi r L\, dr)\, \rho(r) = 2\pi L \rho_0 R \int_0^{R/2} dr = \\ &= \pi L \rho_0 R^2\ . \end{aligned} \tag{4}$$

We can use this in equations (2) and (3) to obtain

$$E\, \pi R L = \frac{1}{\epsilon_0} \pi L \rho_0 R^2, \tag{5}$$

which yields

$$E = \frac{\rho_0 R}{\epsilon_0}. \tag{6}$$

b) What is the electric field at distance $r = 3R$ from the axis?

The flus through the Gaussian surface at distance $(3R)$ from the axis is

$$\Phi_E = E \oint d\boldsymbol{A} = E\, 2\pi (3R) L. \tag{7}$$

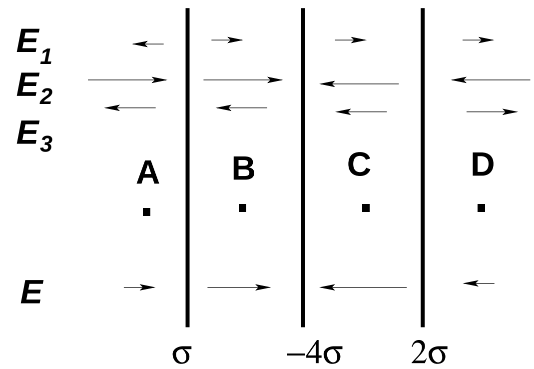
The enclosed charge is

$$\begin{aligned} Q &= \int_0^{R} dV\, \rho(r) = \int_0^{R} (2\pi r L\, dr)\, \rho(r) = 2\pi L \rho_0 R \int_0^{R} dr = \\ &= 2\pi L \rho_0 R^2\ . \end{aligned} \tag{8}$$

Then

$$E\, 6\pi R L = \frac{2}{\epsilon_0} \pi L \rho_0 R^2 \quad \Rightarrow \quad E = \frac{1}{3\epsilon_0} \rho_0 R\ . \tag{9}$$

3. Three very large square planes are arranges parallel to each other, as shown (view from edge). The first plane is charged positively and has charge per unit area $\sigma_1 = \sigma$. The other two planes have charge densities $\sigma_2 = -4\sigma$ and $\sigma_3 = 2\sigma$. Express all results below in terms of σ. Show direction of the field by an arrow in the diagram.



(a) What is the direction and the magnitude of the electric field at point A?

$$|\vec{E}_1| = \frac{\sigma}{2\epsilon_0}, \ |\vec{E}_2| = \frac{4\sigma}{2\epsilon_0}, \ |\vec{E}_3| = \frac{2\sigma}{2\epsilon_0},$$

$$|\vec{E}_A| = |-1+4-2| \ \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{2\epsilon_0},$$

pointing to the **right**.

(b) What is the direction and the magnitude of the electric field at point B?

$$|\vec{E}_B| = |1+4-2| \ \frac{\sigma}{2\epsilon_0} = \frac{3\sigma}{2\epsilon_0},$$

pointing to the **right**.

(c) What is the direction and the magnitude of the electric field at point C?

$$|\vec{E}_C| = |1-4-2| \ \frac{\sigma}{2\epsilon_0} = \frac{5\sigma}{2\epsilon_0},$$

pointing to the **left**.

(d) What is the direction and the magnitude of the electric field at point D?

$$|\vec{E}_D| = |1-4+2| \ \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{2\epsilon_0},$$

pointing to the **left**.

4. Two particles are fixed to an x axis: particle 1 of charge $(4q)$ at $x = 0$ and particle 2 of charge $(-q)$ at $x = a$.

(a) Assuming the potential vanishes at infinity, at what finite coordinate on the axis is the net potential produced by the particles equal to zero? (Find all such points if there are more than one.)

The electric potential is the sum of two contributions from the two charges:

$$V = \frac{1}{4\pi\epsilon_0} q \left(\frac{4}{|x|} - \frac{1}{|x-a|} \right) = 0 \quad \Rightarrow \quad \frac{|x|}{|x-a|} = 4 \tag{10}$$

For $x > a$, $|x| = x$ and $|x - a| = x - a$. Then $x = 4(x - a)$, or

$$x = 4a/3.$$

For $0 < x < a$, $|x| = x$ and $|x - a| = a - x$. Then $x = 4(a - x)$, or

$$x = 4a/5.$$

It is easy to see that there is no solution for $x < 0$. Therefore, the only points where the potential is zero are $x_1 = 4a/5$ and $x_2 = 4a/3$.

(b) At what finite coordinate on the axis is the electric field equal to zero?

Unlike the electric potential, the electric field is a *vector*, which points away from a positive charge and toward a negative charge. The contributions of the two charges add as two vectors. At any point between the two charges, the two vectors point in the same direction and, therefore, they cannot cancel each other. For any point with $x < 0$, the stronger charge $(4q)$ is also the closer charge, so it produces a bigger field, which cannot be canceled by the field of the smaller charge $(-q)$, which is also more remote. Therefore, the two fields can cancel each other only for $x > a$, where

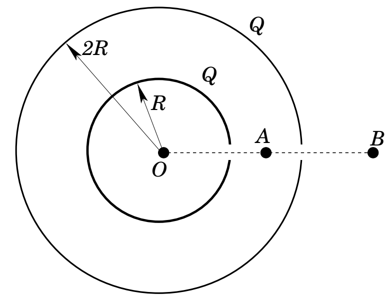
$$\frac{1}{4\pi\epsilon_0} q \left(\frac{4}{x^2} - \frac{1}{(x-a)^2} \right) = 0 \quad \Rightarrow \quad \frac{x^2}{(x-a)^2} = 4 \quad \Rightarrow \quad \frac{x}{x-a} = 2 \tag{11}$$

This implies $x = 2(x - a)$, or

$$x = 2a,$$

which satisfies the condition $x > a$ and is the only possible solution.

5. Two concentric thin non-conducting spherical shells have radii R and $2R$, and each has a small hole along the line OB, as shown. Each shell carries a positive charge Q, uniformly distributed with a constant charge density.



(a) What is the electric potential at point A, at distance $(3R/2)$ from the center, assuming the potential is zero at infinity? (Any effect of the holes should be neglected.)

The potential created by a sphere with charge Q and radius R for $r > R$ is $V_1(r) = kQ/r$. Inside the sphere, the potential is constant (because the electric field is zero) and equal $V_1(R) = kQ/R$ for all $r \le R$. The potential due to the second sphere is $V_2(r) = kQ/r$ for $r > 2R$, and a constant equal $kQ/(2R)$ for all $r \le (2R)$. The total potential is a superposition of V_1 and V_2:

$$V_A = V_1(3R/2) + V_2(3R/2) = k\frac{Q}{(3R/2)} + k\frac{Q}{2R} = \frac{7kQ}{6R}. \tag{12}$$

(b) A proton with charge q_p and mass m_p is released from point O in the direction of point B with an initial speed v_0. What is the speed of the proton at point B at distance $(3R)$ from the center? (Assume the motion is non-relativistic, and there are no forces other than the electrostatic forces.)

The change in the proton's potential energy is $(q_pV_O - q_pV_B)$. The point O is inside both spheres, where the potential is

$$V_O = V_1(0) + V_2(0) = kQ\left(\frac{1}{R} + \frac{1}{2R}\right) = \frac{3kQ}{2R}. \tag{13}$$

The potential at point B is

$$V_B = V_1(3R) + V_2(3R) \quad = \quad \frac{kQ}{3R} + \frac{kQ}{3R} = \frac{2kQ}{3R}. \tag{14}$$

The conservation of energy requires that the sum of the proton's kinetic energy, $mv^2/2$, and its potential energy, q_pV, remains constant:

$$\frac{1}{2}mv_0^2 + \frac{3kq_pQ}{2R} = \frac{1}{2}mv^2 + \frac{2kq_pQ}{3R}\ , \tag{15}$$

which yields

$$v^2 = \sqrt{v_0^2 + \frac{5}{3m}\frac{kq_pQ}{R}}\ . \tag{16}$$