Midterm exam 1

Physics 1B, Spring 2016

Name:
UCLA ID number:
Lecture 5, Section (number, meeting time, or TA name): :

Please write solutions with some minimal derivation in the space provided below each problem; it is not sufficient to give just the final answer. The level of detail should be such that a grader, or your fellow classmate would understand how you solved the problem.

Problem 1.

A rectangular flat-bottom barge with a bottom area $A = 100\,\mathrm{m}^2$ is loaded so that the bottom is at $H = 1$m below the surface. The density of water is $\rho = 10^3\mathrm{kg/m}^3$, and the water surface is perfectly still.

(a) Calculate the mass of the barge.

Solution. The force of buoyancy is equal and opposite the force of gravity:

$$Mg = V\rho g = AH\rho g.$$

Therefore, the mass of the barge M is given by

$$M = AH\rho = 100\,\mathrm{m}^2 \times 1\,\mathrm{m} \times 10^3\ \mathrm{kg/m}^3 = 1.0 \times 10^5\ \mathrm{kg}$$

(b) A round hole with radius $r = 2$ cm is made in the bottom of the barge, and the water starts leaking in. When the water level reaches $h = 5$ cm, a bilge alarm will alert the barge operator. How long will it take for the water to reach the level 5 cm? (Assume that the Bernoulli's equation is applicable.)

Solution. The Bernoulli's equation is

$$p + \rho g h + \frac{1}{2}\rho v^2 = \mathrm{const.}$$

Let us consider two points: the point at the water level outside the barge, and the point at the hole in the barge. We have

$p_{\mathrm{atm}} + \rho g H = p_{\mathrm{atm}} + \frac{1}{2}\rho v^2$, so that $v = \sqrt{2gH}$.

The volume that flows through the hole in time t is given by

$$V = \pi r^2 v t$$

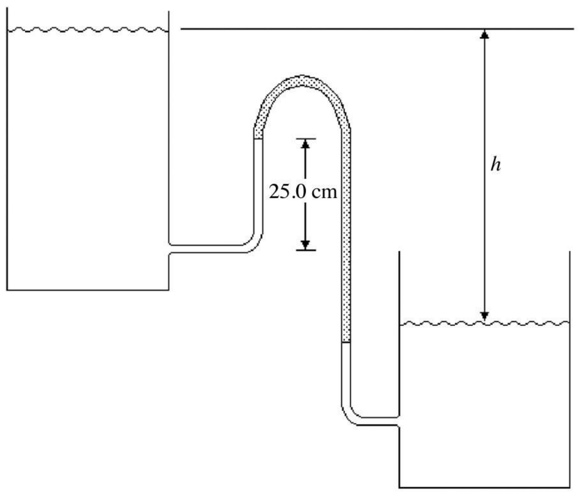
This must be equal to Ah if the water is to fill the barge up to h, and so the time needed for that to happen is given by

$$t = \frac{Ah}{\pi r^2 v} = \frac{Ah}{\pi r^2\sqrt{2gH}} = \frac{100\ \mathrm{m}^2 \times 0.05\ \mathrm{m}}{\pi \times (0.02)\ \mathrm{m}^2 \times \sqrt{2 \times 9.8\ \frac{\mathrm{m}}{\mathrm{s}^2} \times 1\ \mathrm{m}}}$$

$$= 898\ \mathrm{s} \approx 15\ \mathrm{min}.$$

Problem 2

The two water reservoirs shown in the figure are open to the atmosphere, and the water has density 1000 kg/m^3. The manometer contains incompressible oil with a density of 820 kg/m^3. What is the difference in elevation *h* if the manometer reading *m* is 25.0 cm?

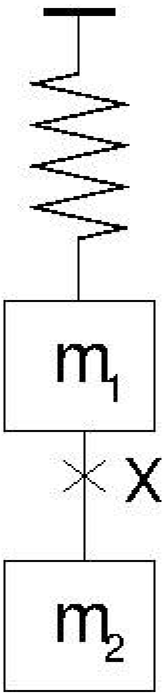


Solution. The pressure difference $\rho_{\text{water}} g h$ between the two reservoirs is balanced by the difference between the column pressures of oil and water in the pipe. This can be written as

$\rho_w g h = \rho_w \, g \, m - \rho_{oil} \, g \, m$. This yields

$$h = \frac{\rho_w - \rho_{oil}}{\rho_w} m = 4.5\,\text{cm} = 0.045\,\text{m}$$

Problem 3



The two weights, $m_1 = 1 \text{ kg}$ and $m_2 = 2 \text{ kg}$ hang on the spring as shown in the figure. Initially the system is in equilibrium, and the weights are at rest. When the string connecting the two weights is cut at point X, the lower one falls, and the upper one begins to oscillate with an amplitude $A = 0.2$ m.

What is the period of these oscillations?

Solution. The upper weight starts the oscillation at rest, which means the amplitude A is equal the initial displacement. This initial displacement is caused by the force $(m_2 g)$, and, therefore,

$$kA = m_2 g. \Rightarrow k = \frac{m_2 g}{A}$$

The period of oscillations is

$$T = 2\pi\sqrt{\frac{m_1}{k}} = 2\pi\sqrt{\frac{m_1 A}{m_2 g}} = 0.6s$$

Problem 4

A simple pendulum has a length of 120 cm.

(a) What is its period of oscillations?

Solution.

$$T = 2\pi\sqrt{\frac{L}{g}} = 2.2\,\mathrm{cm}$$

(b) What is the period of oscillations inside an elevator moving up with an acceleration 1.2 m/s^2

Solution.

The force of tension is equal to $F = mg + ma = m(g + a)$, where $a = 1.2\mathrm{m/s}^2$. Therefore,

$$T = 2\pi\sqrt{\frac{L}{g + a}} = 2.1\,\mathrm{s}$$

(c) What is the period of the same pendulum on Mars, where the acceleration of gravity is about 0.37 that on Earth?

Solution.

$$T = 2\pi\sqrt{\frac{L}{0.37g}} = 3.6\,\mathrm{s}$$

Problem 5
Two violinists are trying to tune their instruments in an orchestra. One is producing the desired frequency of 440.0 Hz. The other is producing a frequency of 448.4 Hz. By what percentage should the out-of-tune musician change the tension in his string to bring his instrument into tune at 440.0 Hz?

Solution.

Frequency

$$f = \frac{n}{2L}v = \frac{n}{2L}\sqrt{\frac{F}{\mu}}$$

This implies that F is proportional to f^2:

$$F = \text{const} \times f^2$$

The fractional change in the force is

$$\frac{F_{\text{untuned}} - F_{\text{tuned}}}{F_{\text{untuned}}} = 1 - \frac{f^2_{\text{tuned}}}{f^2_{\text{untuned}}} = 0.037 = 3.7\%$$

Problem 6

Standing waves of frequency 50 Hz are produced on a string that has mass per unit length 0.025 kg/m. With what tension must the string be stretched between two supports if adjacent nodes in the standing wave are to be 0.8 m apart?

The adjacent nodes are separated by distance l which is a half of a wavelength. Then $\lambda = 2l$.

Speed of the wave
$v = f\lambda = 2fl$

On the other hand,

$$v = \sqrt{\frac{F}{\mu}}.$$

Therefore,

$$2fl = \sqrt{\frac{F}{\mu}}.$$

Then

$$F = 4\mu f^2 l^2 = 160\,N$$